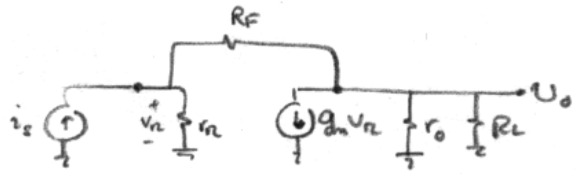
۱.

الف)



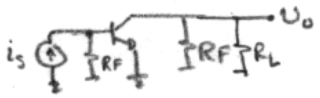
$$\begin{cases} i_s = \frac{v_\pi}{r_\pi} + \frac{v_\pi - v_o}{R_F} \xrightarrow{①} i_s = v_o \left(\left(\frac{1}{r_\pi \| R_F}\right)\left(\frac{1}{r_o \| R_L \| R_F}\right) \frac{1}{g_m + \frac{1}{R_F}} - \frac{1}{R_F} \right) \rightarrow \\ g_m v_\pi = \frac{v_o}{r_o \| R_L} + \frac{v_o - v_\pi}{R_F} \rightsquigarrow \frac{v_o}{r_o \| R_L \| R_F} = v_\pi \left(g_m + \frac{1}{R_F}\right) \quad ① \end{cases}$$

$$g' = \left(g_m (r_\pi \| R_F)(r_o \| R_L \| R_F)\right)^{-1}$$

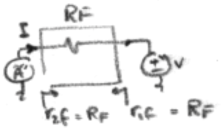
$$i_s = \frac{-v_o}{R_F}\left(1 - \frac{R_F}{g_m + \frac{1}{R_F}} g_m g'\right)$$

$$= \frac{-v_o}{R_F}\left(1 - \frac{R_F^2 g_m}{1 + g_m R_F} g'\right) \longrightarrow \frac{v_o}{i_s} = -R_F\left(1 - \frac{g_m R_F^2}{1 + g_m R_F} g'\right)^{-1}$$

ب)



$f = \frac{-1}{R_F}$



$$a = \frac{v_o}{i_s} = -g_m (R_F \| r_\pi)(R_L \| R_F \| r_o) = -\frac{1}{g'}$$

FB نابدار $\quad A = \frac{v_o}{i_s} = \frac{a}{1+af} = \frac{-1/g'}{1 + \frac{1}{g' R_F}} = \frac{-R_F}{1 + g' R_F} = -R_F(1 - R_F g')$

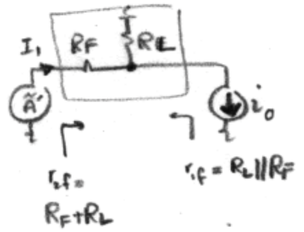
با $R_F \gg r_\pi, R_L$

$$g' \sim \frac{1}{g_m r_\pi R_L} \sim \frac{1}{\beta R_L}$$

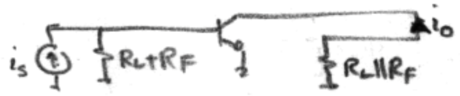
الف و ب) نزدیک بهم هستند اگر $g' R_F \ll 1 \quad , \quad g_m R_F \gg 1$

معادل این دو شرط باید $\beta R_L \gg R_F \gg r_m$ باشد

ج)



$f = \frac{I_1}{i_o} = \frac{R_L}{R_L + R_F}$



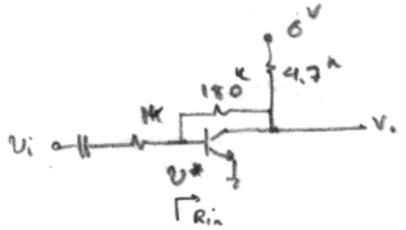
$$a = \frac{i_o}{i_s} = g_m \times \left((R_L + R_F) \| r_\pi\right) \times \frac{(r_o)}{R_L \| R_F + r_o} \frac{(R_L \| R_F)}{(R_L \| R_F)} = \frac{1}{g''(R_L \| R_F)}$$

$$\frac{i_o}{i_s} = \frac{a}{1+af} = \frac{1}{\frac{1}{a} + f} = \frac{1}{\frac{R_L \times R_F}{(R_L + R_F) R_F} + g''(R_L \| R_F)} = \frac{1}{R_L \| R_F}\left(\frac{R_F}{1 + g'' R_F}\right)$$

$\left(R_F g'' \ll 1 \text{ وقتی}\right) \rightarrow \frac{i_o}{i_s} \simeq \frac{R_F}{R_L \| R_F}\left(1 - R_F g''\right) \qquad v_o = -i_o (R_L \| R_F) \rightarrow \frac{v_o}{i_s} = -R_F\left(1 - R_F g''\right)$

باز هم با همان فرض گفته شده $R_F g' \ll 1$ و اگر $\beta R_L \gg R_F \gg (r_m ; R_L)$ باشد الف و ب و ج) معادل هم می‌شوند. در واقع اگر $\beta \gg 1$ باشد (بهره جریان در حالت ج) می‌توان از بهرهٔ مستقیم جریان عبوری از دو مقاومت R_F و R_L در مقایسه با β صرف نظر کرد و آنچه از جنبه‌های فیدبک خوانده‌ایم را اعمال کنیم. والا نمی‌شود.

$\begin{cases} V_A = 100^V \\ \beta = 300 \end{cases}$ در صورت سؤال فراموش شده. خازن کوپلاژ ورودی هم داریم!



$$I_c = \frac{6-0.7}{4.7^k + \frac{180^k}{300}} = 1^{mA} \qquad r_m = 25^{\Omega} \qquad r_\pi = 7.5^k$$

شرط $R_F \gg r_m$ برقرار است و $R_F \ll \beta R_L$ هم برقرار است

$= 1.4^M$

$R_F \ll \beta R_L$, $R_F \gg R_L$, $R_F \gg r_m$ می‌توان به جای الف) از ب) استفاده کرد ولی شرط ج) هم تقریباً برقرار است

لذا برای سادگی از ب) استفاده می‌کنیم

$$\begin{cases} a = \frac{-1}{g'} \\ \quad = \frac{-1}{25^{\Omega}} 7.2^k \times 4.38^k \\ \quad = -1.26^{M\Omega} \end{cases} \qquad \left| f = \frac{-1}{R_F} = -5.5^{\mu S} \right. \qquad \left| 1 + af = 7.93 \right.$$

$$\Big\{ R_{in} = \frac{r_i \xrightarrow{} 180^k \| r_\pi}{1+af} = 0.9^k$$

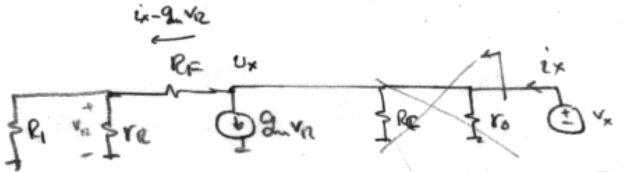
لذا $\dfrac{v_o}{v_i} = \overbrace{\dfrac{a}{1+af}}^{v_o/i_s} / (R_{in} + 1^k) = -83.62$

$$\text{SPICE} \begin{cases} a_v = -80 \\ R_{in} = 0.9^k \\ R_{out} = 2.4^{k\Omega}! \end{cases}$$

$$R_{out} = \frac{r_o \xrightarrow{} R_F \| R_L \| r_o}{1+af} = \quad = 577^{\Omega}$$

! R_{out} را با SPICE [illegible] تفاوت دارد

R_{out} را با KVL ، KCL حل



$$R_{out} = r_o \| R_C \| (..R')$$

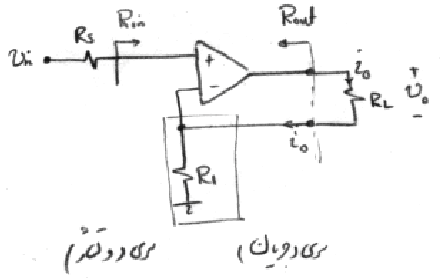
$$R' = \frac{v_x}{i_x}$$

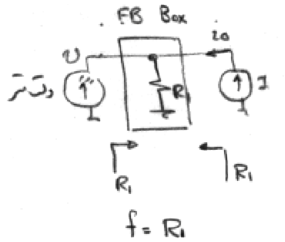
$$\begin{cases} (i_x - g_m v_\pi)(r_\pi \| R_1) = v_\pi \\ v_x = v_\pi + R_F (i_x - g_m v_\pi) \end{cases} \longrightarrow R' = \frac{(r_\pi \| R_1) + R_F}{1 + g_m (r_\pi \| R_1)} = 5^k \Bigg\} \; R_{out} = 100^k \| 4.7^k \| 5^k \cong 2.4^{k\Omega}$$

برای مطالعه بیشتر محاسبه مقاومت خروجی را می‌توانید با روش return ratio مقایسه کنید (مثلاً بخوانید ص ۵۹۹ گری - مایر فصل ۸-۸)

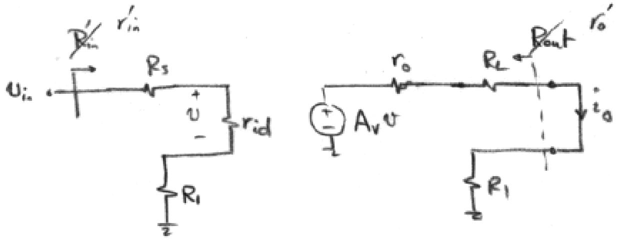
برای دیدن همین مسأله را بدون تئوری فیدبک با استفاده از قضیه میلر، رجوع شود به جزوه دکتر علوی (وبسایت) صفحهٔ ۳۵، مثال ۴-۱

۴. برای حل دو قسمت (*) دارید به جزوه کلاس مراجعه کنید





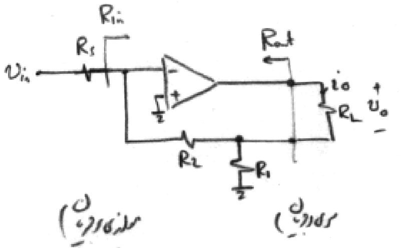
$f = R_1$

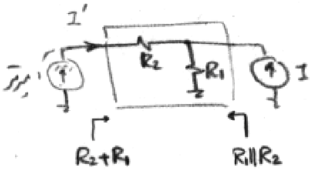


$$r_i' = r_{id} + R_S + R_1$$
$$r_o' = r_o + R_L + R_1$$
$$a' = \frac{r_{id}}{r_i'} \times A_v \times \frac{1}{r_o'}$$
$$A = \frac{i_o}{v_{in}} = \frac{a'}{1+a'f} = \left(R_1 + \frac{r_i' r_o'}{A_v r_{id}}\right)^{-1} \simeq \frac{1}{R_1}\left(1 - \frac{r_i' r_o'}{A_v r_{id} R_1}\right)$$
$$\downarrow$$
$$\frac{v_o}{v_{in}} = R_L \times \frac{i_o}{v_{in}} = \frac{R_L}{R_1}\left(1 - \frac{r_i' r_o'}{A_v r_{id} R_1}\right)$$

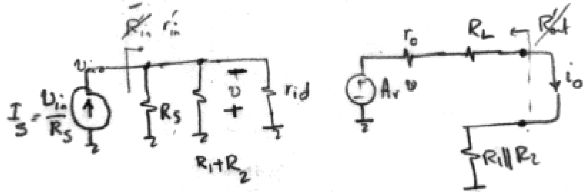
$$R_{in}' = r_i'(1+a'f) \longrightarrow R_{in} = R_{in}' - R_S = r_{id} + R_1 + A_v \frac{r_{id}}{r_o'} R_1 \simeq A_v \frac{r_{id}}{r_o'} R_1$$

$$R_{out}' = r_o'(1+a'f) \longrightarrow R_{out} = R_{out}' - R_L = r_o + R_1 + A_v \frac{r_{id}}{r_i'} R_1 \simeq A_v \frac{r_{id}}{r_i'} R_1$$





$$f = \frac{-R_1}{R_1+R_2}$$



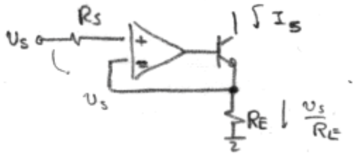
$$r_i' = R_S \| (R_1+R_2) \| r_{id}$$
$$r_o' = r_o + R_L + R_1 \| R_2$$
$$a' = \frac{i_o}{I_s} = -r_i' \times A_v \times \frac{1}{r_o'}$$

$$A = \frac{i_o}{I_s} = \frac{a'}{1+a'f} = -\left(\frac{R_1}{R_1+R_2} + \frac{r_o'}{A_v r_i'}\right)^{-1} \simeq -\left(1+\frac{R_2}{R_1}\right)\left(1 - \left(1+\frac{R_2}{R_1}\right)\frac{1}{A_v}\frac{r_o'}{r_i'}\right) = \frac{v_o/R_L}{v_{in}/R_S}$$
$$\longrightarrow A \cong \frac{-R_L}{R_S}\left(1+\frac{R_2}{R_1}\right)\left[1 - \left(1+\frac{R_2}{R_1}\right)\frac{r_o'}{A_v r_i'}\right]$$

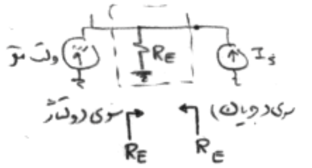
$$R_{in}' = \frac{r_i'}{1+a'f} = \frac{r_i'}{1 + \frac{R_1}{R_1+R_2} \times A_v \frac{r_i'}{r_o'}} \longrightarrow R_{in} = \frac{1}{\frac{1}{R_{in}'} - \frac{1}{R_S}} \cong \left(1+\frac{R_2}{R_1}\right)\frac{r_o'}{r_i' A_v} \quad (*)$$

$$R_{out}' = r_o'(1+a'f) = r_o'\left(1 + \frac{R_1}{R_1+R_2} A_v \frac{r_i'}{r_o'}\right) \longrightarrow R_{out} = R_{out}' - R_L \cong A_v r_i' \frac{R_1}{R_1+R_2}$$

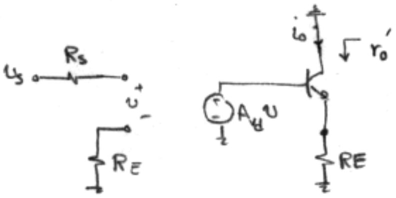
(*) اینجا چرا R_S در مقاومت ورودی ماند؟!



$$I_s = \alpha \frac{v_s}{R_E}$$



$f = R_E$



$$r_o' = r_o\left(1+\frac{\beta R_E}{r_\pi + R_E}\right)$$

$$\simeq r_o\left(1+g_m R_E\right)$$

$$R_o = r_o'(1+a'f) \qquad a' = \frac{i_o}{v_s} = A_{vd} \times \frac{g_m}{1+g_m R_E}$$

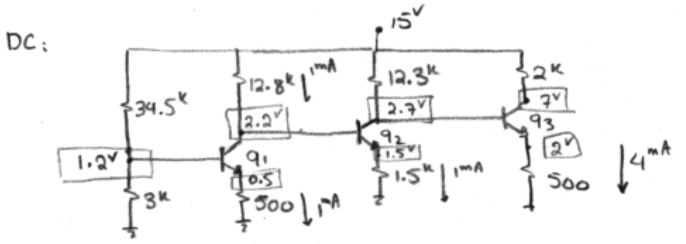
$$R_o = r_o(1+g_m R_E)\left(1+A_{vd}\frac{g_m R_E}{1+g_m R_E}\right) \approx r_o\left(1+g_m R_E + A_{vd}\, g_m R_E\right) \approx r_o\, g_m R_E\, A_{vd}$$

اگر با KVL - KCL نوشته می‌شد که

چرا تفاوت دارد؟ $R_{out} \simeq \beta r_o$ $\qquad$ $(A_v \gg 1)$ می‌شود R_{out}

۴. با صرف نظر از جریان‌های بیس

DC:

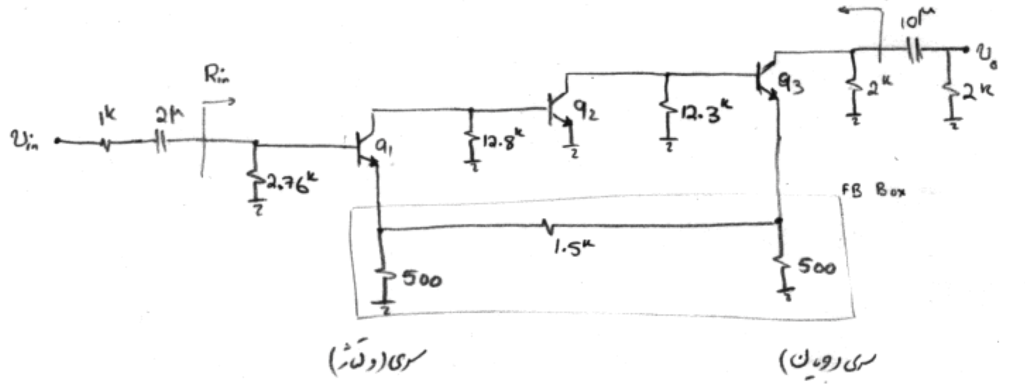


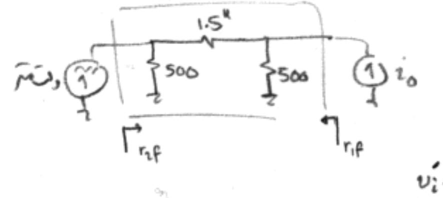
$I_{c_1} = I_{c_2} = 1^{mA} \qquad I_{c_3} = 4^{mA}$

$V_{CE_1} = 1.7^{V} \qquad V_{CE_2} = 1.2^{V} \qquad V_{CE_3} = 5^{V}$

$r_{\pi_1} = r_{\pi_2} = 2.5^{k},\ r_{\pi_3} = 0.625^{k}$

مدار معادل ac

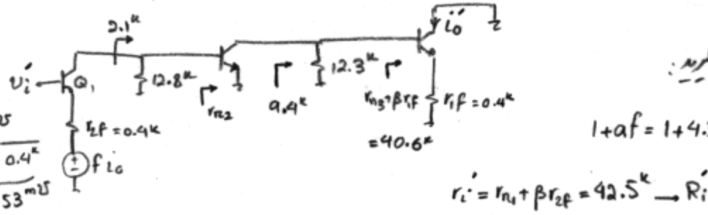




$r_{1f} = 500 \parallel 2^{k} = 400$

$r_{2f} = 500 \parallel 2^{k} = 400$

$$f = \frac{5\cancel{00}}{25\cancel{00}} \times 500^{\Omega} = 100^{\Omega}$$



برای سادگی مدار را این طور در نظر می‌گیریم:

$$a = \frac{i_o}{v_i} = \underbrace{\frac{-2.1^{k}}{0.4^{k}+25}}_{4.94} \times \underbrace{\frac{-9.4^{k}}{25}}_{376} \times \underbrace{\frac{40^{m\mho}}{1+40^{m}\times 0.4^{k}}}_{2.353^{m\mho}}$$

$= 4.37\ \mho$

$$1+af = 1 + 4.37^{\mho} \times 100^{\Omega} = 438$$

$$\frac{i_o}{v_i} = \frac{a}{1+af} = \frac{4.37^{\mho}}{438} = 0.01^{\mho}$$

$r_i' = r_{\pi_1} + \beta r_{2f} = 42.5^{k} \longrightarrow R_i' = 18.6^{M}$

$r_o = \infty$

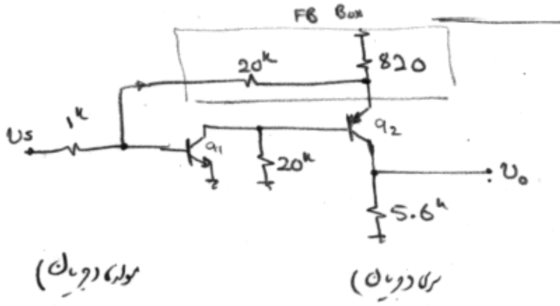
حال مدار را کامل می کنیم

$R_{in} = 2.76^k \parallel R'_i = 2.76^k \parallel 18.6^M \simeq \boxed{2.76^k}$

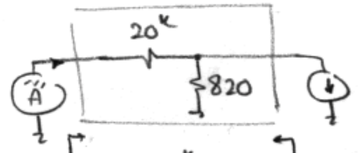
$R_{out} = \infty \parallel 2^k = \boxed{2^k}$

$\frac{i_o}{v_{in}} = \frac{i_o}{v'_i} \times \frac{v'_i}{v_{in}} = 0.01^{℧} \times \frac{R_{in}}{R_{in}+1^k} = \frac{-v_o/1^k}{v_{in}} \longrightarrow \frac{v_o}{v_{in}} = -10 \times \frac{2.76}{3.76} = \boxed{-7.34 = A_v}$

$$f_{-3dB}: \left\{ \begin{array}{l} C_1 = 2^{\mu F} \quad \blacktriangle \quad R = 1^k + R_{in} = 3.76^k \;;\; \tau_1 = RC = 7.5^{ms} \to f_1 = 21.1^{Hz} \\ C_2 = 10^{\mu F} \quad \blacktriangle \quad R = 2^k + 2^k = 4^k \;;\; \tau_2 = 40^{ms} \to f_2 = 4^{Hz} \end{array} \right\} \to f_{-3dB} = \sqrt{f_1^2 + f_2^2} = 21.5^{Hz}$$

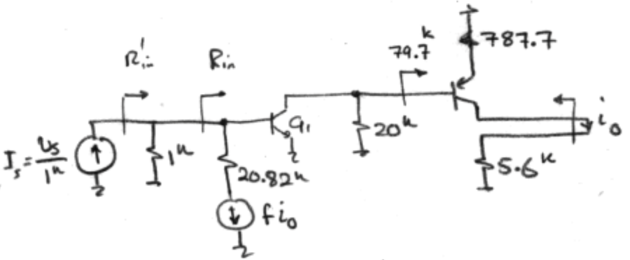


۵ با جعبه فیدبک سروکار لذا



$r_{2f} = 20.82^k$ $\quad r_{1f} = 20^k \parallel 820$

$f = \frac{820}{20820} = 0.0394$



$r'_i = r_{\pi_1} \parallel 20.82^k \parallel 1^k = 488.3^{\Omega}$

$a' = \frac{i_o}{i_s} = \frac{1^k}{1^k + 1^k} \times \beta \times \frac{20}{20+79.7} \times \beta = 1003$

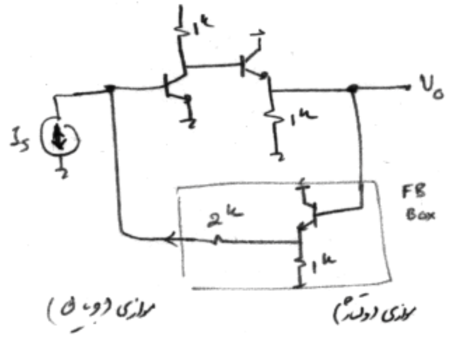
$\longrightarrow 1+T = 49.5$

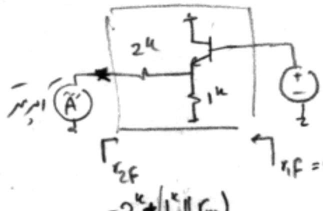
$A = \frac{i_o}{I_s} = \frac{a'}{1+a'f} = 20.26$

$20.26 = \frac{v_o/5.6^k}{v_s/1^k} \longrightarrow \frac{v_o}{v_s} = 5.6 \times 20.26 = 113.4$

$R'_{in} = r'_i \div (1+T) = 9.8^{\Omega} \longrightarrow R_{in} \cong 9.8^{\Omega}$ $\quad R_{out} = 5.6^k \parallel \infty = 5.6^k$

در این مدار ابتدا باید مدار که فیدبک دارد را معادل ۲-قطبی می کنیم



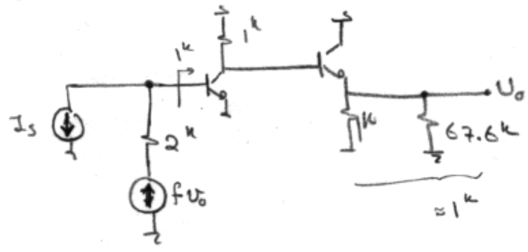


$f = \frac{i}{V} = \frac{0.66}{0.66+0.01} \times \frac{1}{2^{k\Omega}} = 0.49^{m℧}$

$r_{2f} = 2^k + (1^k \parallel r_m) \cong 2^k$

$r_{1f} = r_{\pi_1} + \beta(1^k \parallel 2^k) = 67.6^k$

$r_\pi < \frac{1}{r_m}$



$$a' = \frac{v_o}{I_s} = \underbrace{(2^k \| 1^k)}^{0.66^k} \underbrace{(g_m \times 1^k)}^{100} \times \overbrace{\frac{1^k}{1^k + r_m}}^{\approx 1}$$

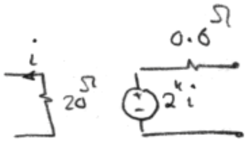
$$= 66^{k\Omega}$$

لذا $1 + T' = 1 + a'f = 33.4$

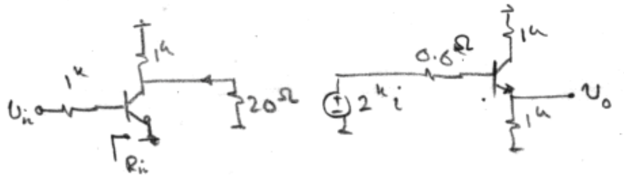
$r_i' = 0.66^k \longrightarrow R_{in} = 20^{\Omega}$

$r_o' = 1^k \| \underbrace{\frac{1^k + 1^k}{\beta}}_{20^{\Omega}} \approx 20^{\Omega} \longrightarrow R_{out} = \frac{20}{1+T'} = 0.6^{\Omega}$

مدار معادل ≡

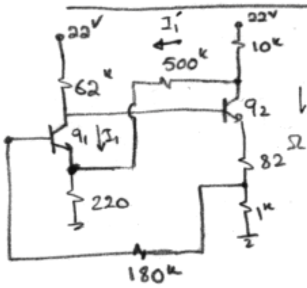


$A = \frac{a'}{1+a'} = \frac{v_o}{I_s} = \frac{66^{k\Omega}}{33.4} \approx 2^{k\Omega}$



$\frac{v_o}{v_{in}} = + \frac{1}{\underbrace{r_m}_{10} + \underbrace{\frac{1^k}{\beta}}_{10}} \times 2^k \times \underbrace{\frac{1^k}{1^k + r_m}}_{\approx 1} = +100$, $R_{in} = r_\pi = 1^k$, $R_o = 1^k \| r_m = 10^{\Omega}$

DC



٦. 226^{Ω} باید باشه $220^{k\Omega}$

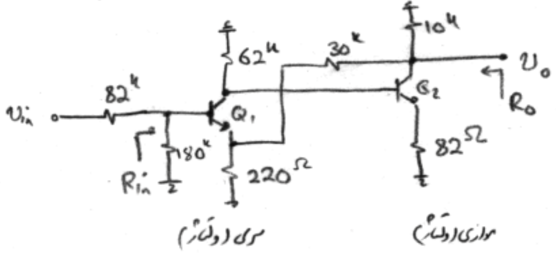
ابتدا صرف نظر از جریان بیس ها و I_1' داریم

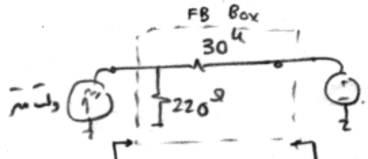
$(22^V - 62^k I_1 - 0.7 - 0.7) = 0.22^k I_1 \longrightarrow I_1 = 0.33^{mA}$

$I_2 = \frac{0.22^k \times I_1 + 0.7}{1^k} = 0.773^{mA}$

$V_{C_2} = 22 - 10 \times I_2 = 14.27^V \longrightarrow I_1' = 0.03^{mA}$ ✓

سعی کردم با در نظر گرفتن جریان بیس ها و I_1' [illegible] ها جواب بده

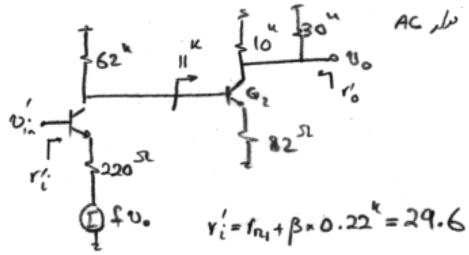




$r_{2f} = 220 \| 30^k \approx 220^{\Omega}$ $\quad r_{1f} = 220 + 30^k \cong 30^k$

$f = \frac{220}{220 + 30^k} = 0.0073$

مدار AC



$r_i' = r_{\pi_1} + \beta \times 0.22^k = 29.6^k$

$r_o' = 7.5^k$

$a' = \frac{62^k \| 11^k}{220^{\Omega} + 76^{\Omega}} \times \frac{7.5^k}{82^{\Omega} + 31^{\Omega}} = 2085$

$a'f = 15.2$

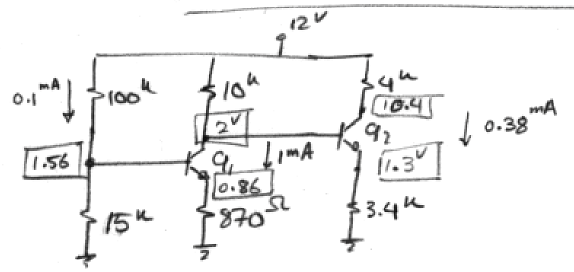
1.ب

$R'_{in} = r'_i(1+a'f) = 479.5^M$ $\qquad R'_{out} = \frac{r'_o}{1+a'f} = 463^{\Omega}$

$A' = \frac{v_o}{v'_{in}} = \frac{a'}{1+a'f} = 128.7$

$R_{in} = 180^k \| R'_{in} = \boxed{131^{k\Omega}}$ $\qquad R_{out} = R'_{out} = \boxed{463^{\Omega}}$ $\qquad A_v = \frac{v_o}{v_{in}} = A' \times \frac{R_{in}}{R_{in} + 82^k} = \boxed{79}$

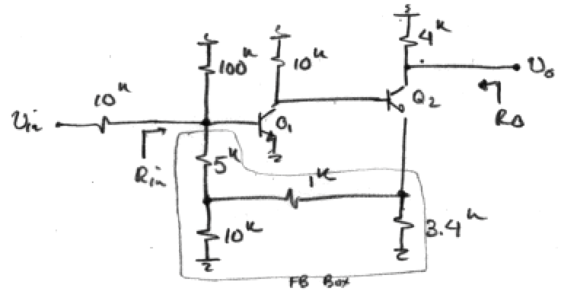
DC:

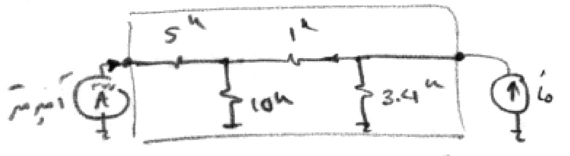


$I_{C_1} = 1^{mA}$ $\qquad V_{CE_1} = 1.1^V$

$I_{C_2} = 0.38^{mA}$ $\qquad V_{CE_2} = 9.1^V$

AC:



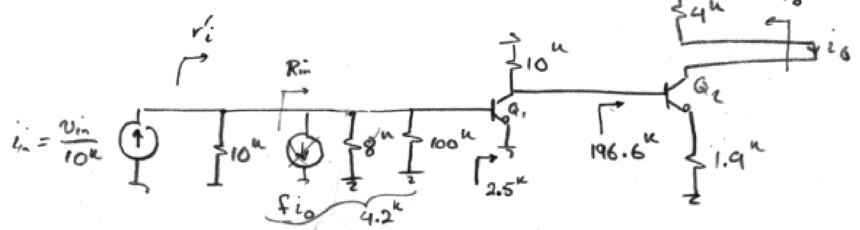


$r_{2f} = 5^k + (10^k \| 4.4^k) = 8^k$

$r_{1f} = 3.4^k \| (\underbrace{1^k + 5^k \| 10^k}_{4.3^k}) = 1.9^k$

$f = \frac{-3.4}{3.4+4.3} \times \frac{10}{15} = 0.3$

موازی (جریان) سری (جریان)



$(f=0)\quad a' = \frac{i_o}{i_{in}} = \frac{4.2^k}{4.2^k + 2.5^k} \times 100 \times \frac{-10}{10+196.6} \times 100 = -303.4$ $\qquad a'f = 91$

$r'_i = 1.57^k$ $\qquad r'_o = 4^k + \infty = \infty$

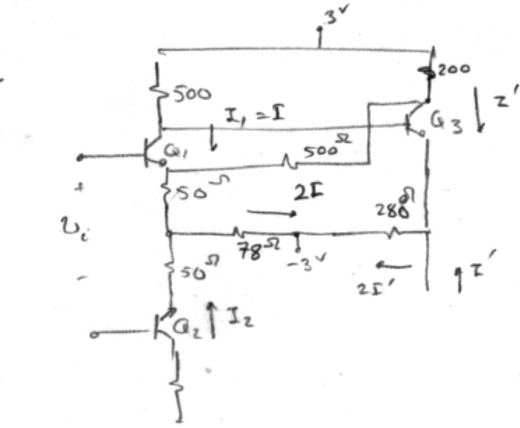
$A = \frac{a'}{1+a'f} = -3.3 = \frac{-\left(\frac{v_o}{4^k}\right)}{\left(\frac{v_{in}}{10^k}\right)} \xrightarrow{\frac{10}{4}} \frac{v_o}{v_{in}} = 1.32^V$

$R'_{in} = \frac{r'_i}{1+T} = 17^{\Omega} = R_{in} \| 10^k$

$\rightarrow R_{in} \cong 17^{\Omega}$

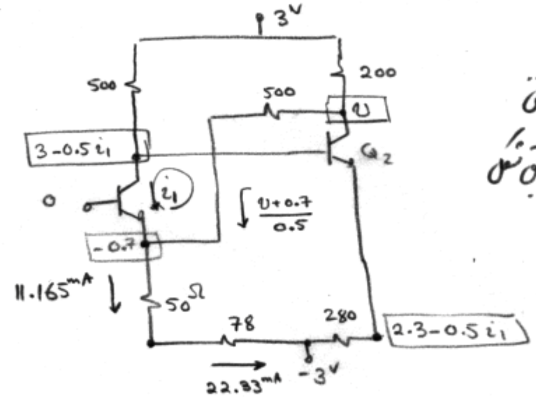
$R_{out} = 4^k \| \infty = 4^k$

DC



بنا به تقارن در طبقه دیفرانسیل ورودی $I_1 = I_2 = I$ ،

با فرض $V_{DC_{in}} = 0$ ولتاژ بیس $V_{B_1} = V_{B_2} = 0$ لذا



لذا با محاسبه دقیق i_1 و v مطابق شکل داریم

$$\frac{1}{2}\,\frac{2.3-0.5i_1+3}{0.28} = i_2 = 9.46-0.89i_1$$

$$\left(\begin{array}{c}\text{لذا kcl در}\\ \text{کلکتور } Q_2\end{array}\right) \rightarrow 9.46-0.89i_1+\frac{v+0.7}{0.5} = \frac{3-v}{0.2}$$

$$(\text{kcl در بیس } Q_1) \rightarrow 11.165-\frac{v+0.7}{0.5} = i_1$$

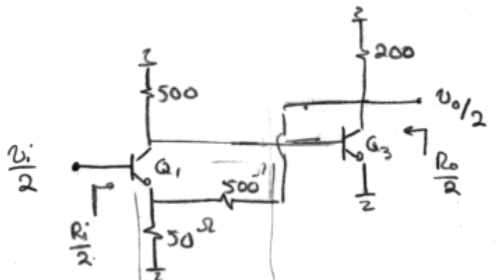
$\xrightarrow{\text{حل دو معادله}}$

$v = 1.464^V$

$i_1 = 6.85^{mA}$ ، $v_{CE_1} = 0.28^V$

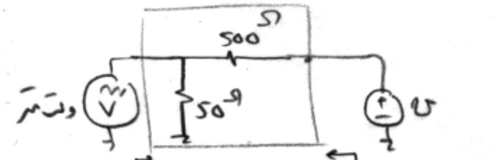
$i_2 = 3.35^{mA}$ ، $v_{CE_2} = 2.6^V$

مدار معادل نصف مدار AC



سری (ولتاژ)

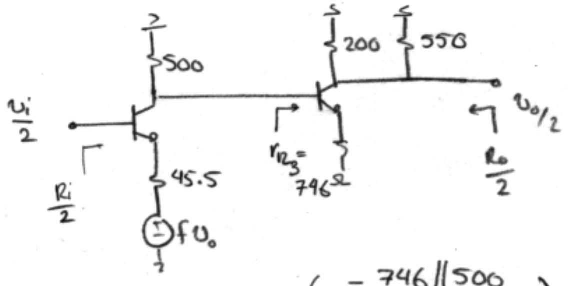
موازی (ولتاژ)



$r_{2f} = 50 \| 500 = 45.5^{\Omega}$

$r_{1f} = 550^{\Omega}$

$$f = \frac{50}{50+500} = 0.0909$$



$r_o' = 200 \| 500 = 143^{\Omega}$

$r_i' = r_{\pi_1} + \beta \times 45.5 = 4.9^k$

$$a' = \left(\frac{-746\|500}{\underbrace{45.5+3.65}_{6.1}}\right) \times \left(\underbrace{-\frac{143}{7.46}}_{19.17}\right) = 117 \quad \rightarrow a'f = T = 10.6$$

$R_i = 2 \times r_i'(1+T) = 113.7^k$

$R_o = 2 \times \frac{r_o'}{1+T} = 24.6^{\Omega}$

$$A = \frac{v_o}{v_i} = \frac{v_o/2}{v_i/2} = \frac{a'}{1+T} = 10$$

۷. تحلیل DC شاخه مرجع جریان ها Q_8 :

$$I_8 = \frac{12-0.7}{11.4} = 1^{mA}$$

لذا $I_7 = \frac{1.4 \times 1^m}{0.3} = 4.7^{mA} = I_9 \longrightarrow I_1 = I_2 = I_3 = I_4 = 2.35^{mA}$

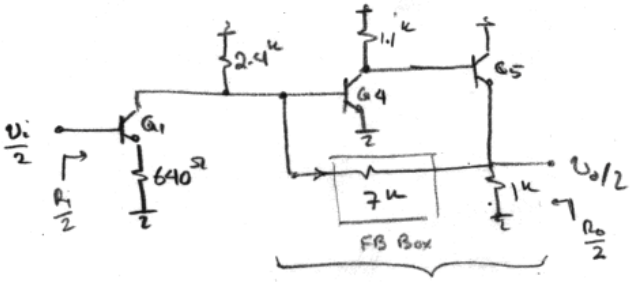
$$I_{10} = I_{11} = \frac{1.4 \times 1^m}{0.4} = 3.5^{mA}$$

$$V_{C3} = V_{C4} = 6 - 1.1 \times 2.35^{mA} = 3.5^V \longrightarrow V_{B5} = V_{B6} = V_{C10} = V_{C11} = 3.5 - 0.7 = 2.8^V$$

KCL at Q_1 کلکتور : $I_1 = \frac{6 - V_{C1}}{2.4^k} + \frac{2.8 - V_{C1}}{7^k} = 2.35^{mA} \longrightarrow V_{C1} = 1^V$

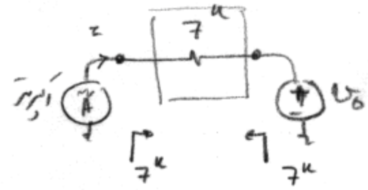
$$I_{7^k} = 0.27^{mA} \longrightarrow I_5 = I_6 = I_{10} + 0.27^{mA} = 3.8^{mA}$$

همه جریان‌ها تعیین شد. همه در ناحیه فعال!



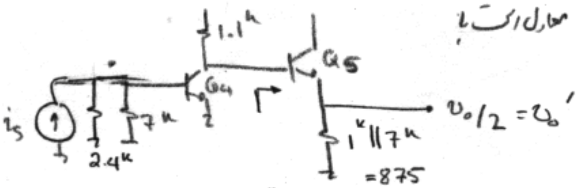
مدار معادل ac نصف مدار

ابتدا قسمت شامل Q_4, Q_5 را با FB مدل می‌کنیم



$$f = -\frac{1}{7 k\Omega}$$

معادل است با



$$r'_i = 2.4 \parallel \overset{1^k}{r_{\pi 4}} \parallel 7^k = 641^{\Omega} \qquad r'_o = 875^{\Omega} \parallel \underbrace{\frac{1.1^k + r_{\pi 5}}{\beta}}_{17.6} = 17.2^{\Omega}$$

$$a' = \frac{v'_o}{i_s} = 875^{\Omega} \times \left(-g_{m4} \underbrace{\left(1.1^k \parallel (87.5^k + r_{\pi 5})\right)}_{1^k}\right) \times \left(\frac{875}{875 + 6.6}\right)$$

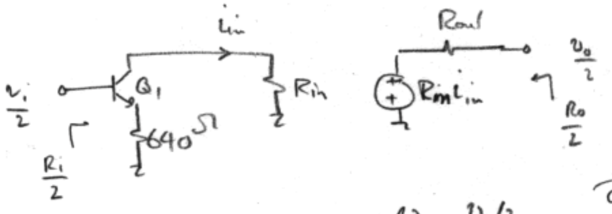
$$= -60^{k\Omega}$$

$$\rightarrow a'f = 8.6$$

لذا $\frac{v'_o}{i_s} = R_m = \frac{-60}{9.6} = -6.25$

$$R_{in} = r'_i (1+T) = 6.1^{k\Omega}$$

$$R_{out} = r'_o \div (1+T) = 1.8^{\Omega}$$



لذا $\frac{v_o}{v_i} = \frac{v_o/2}{v_i/2} = \frac{\overset{1.5^m}{g_{m1}}}{1 + g_{m1} R_E} \times \overset{6.1^k}{R_m} \longrightarrow \boxed{\frac{v_o}{v_i} = 9.4}$

$$R_{in} = 2 \times (r_{\pi 1} + \beta \times 640^{\Omega}) = 130^{k\Omega}$$

$$R_{out} = 2 \times 1.8^{\Omega} = 3.6^{\Omega}$$