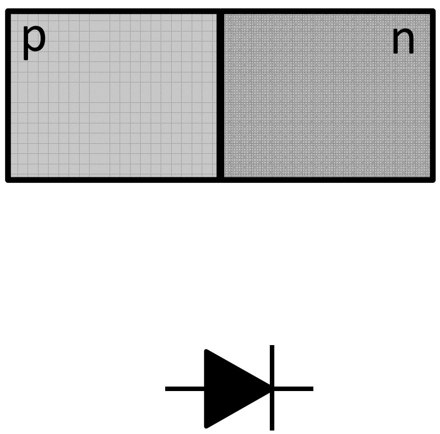
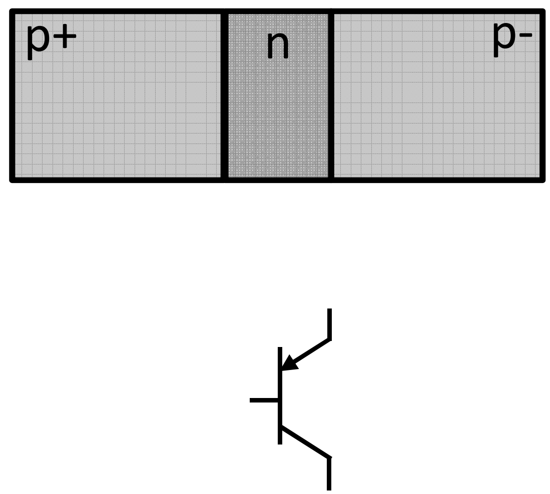
Session 0:
Principles of Electronics
Review of Solid State Devices

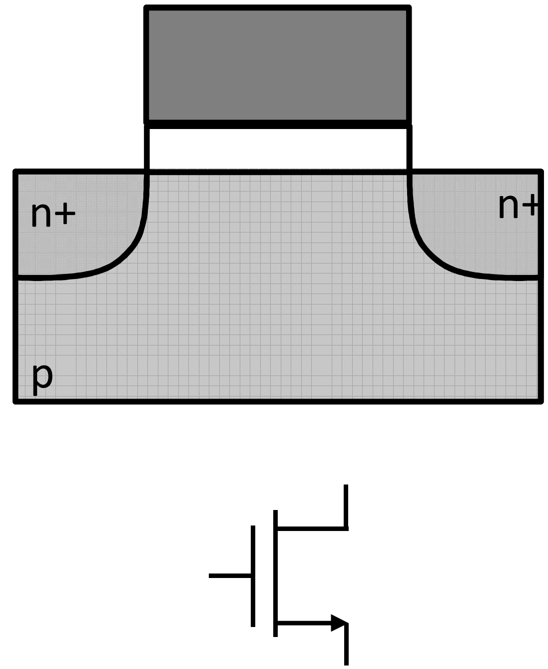
From Atom to Transistor

Objective

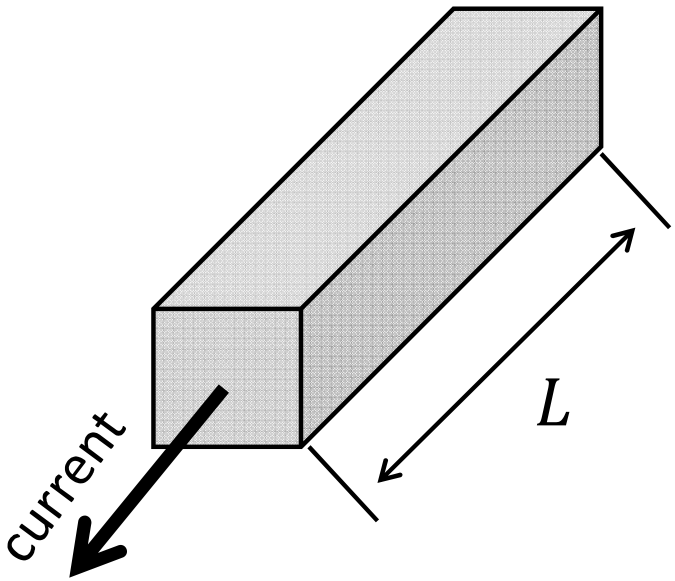
To Understand: how “Diodes,” and “Transistors” operate!







21 Century Alchemy!



Ohm's law

$$R = \frac{V}{I} \;\rightarrow\; \rho = R\frac{A}{L}$$ resistivity

Resistivity is characteristic of the material

Art of VLSI design is:
to put together materials with different resistivity's next to each other to perform a certain task.

Conductor

Al , Cu ⟷ SiO_2

$\rho_{Al} \approx 10^{-6}\ [\Omega cm]$ $\rho_{SiO_2} \approx 10^{16}\ [\Omega cm]$

Insulator

Periodic Table of Elements

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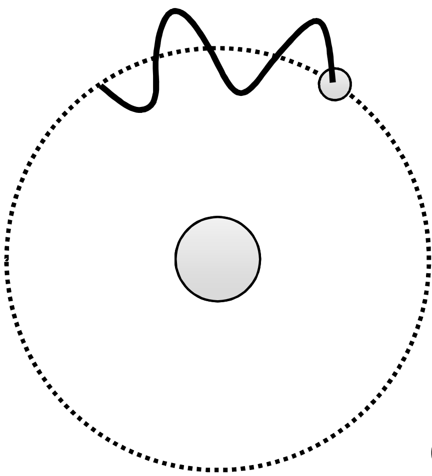


Bohr Atomic Model

wave-particle duality

$$\lambda = h/p$$

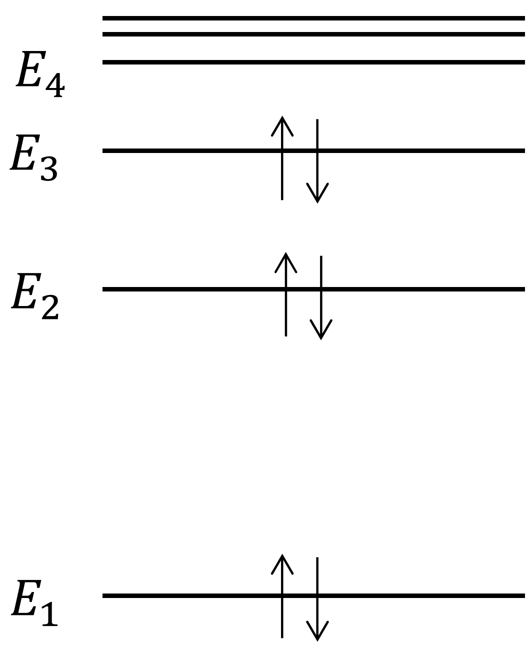
$$mvr = n\hbar$$

de Broglie standing wave

Abbreviated Periodic Table

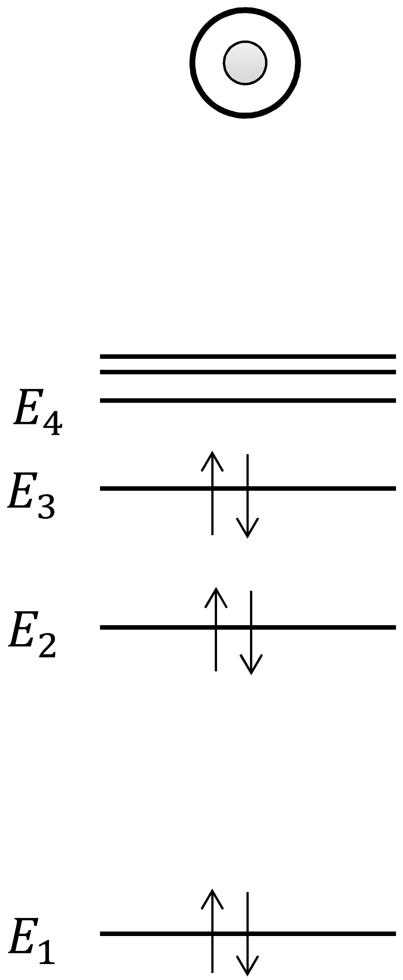
II	III	IV	V	VI
4 Be	5 B	6 C	7 N	8 O
12 Mg	13 Al	14 Si	15 P	16 S
30 Zn	31 Ga	32 Ge	33 As	34 Se
48 Cd	49 In	50 Sn	51 Sb	52 Te
80 Hg	81 Tl	82 Pb	83 Bi	84 Po

Energy Bands:

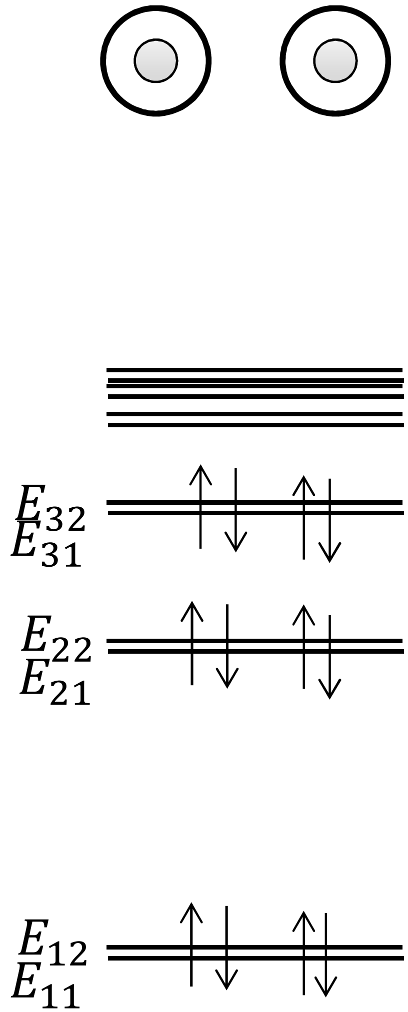


Bohr Atomic Model

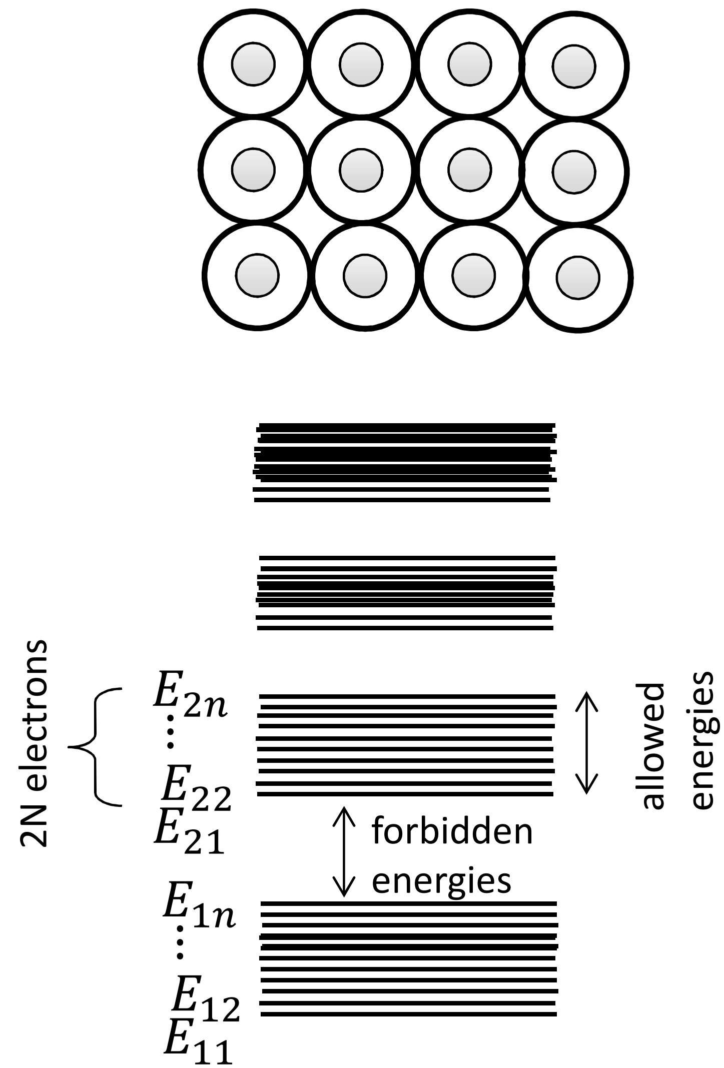
Single atom:



2 atoms:

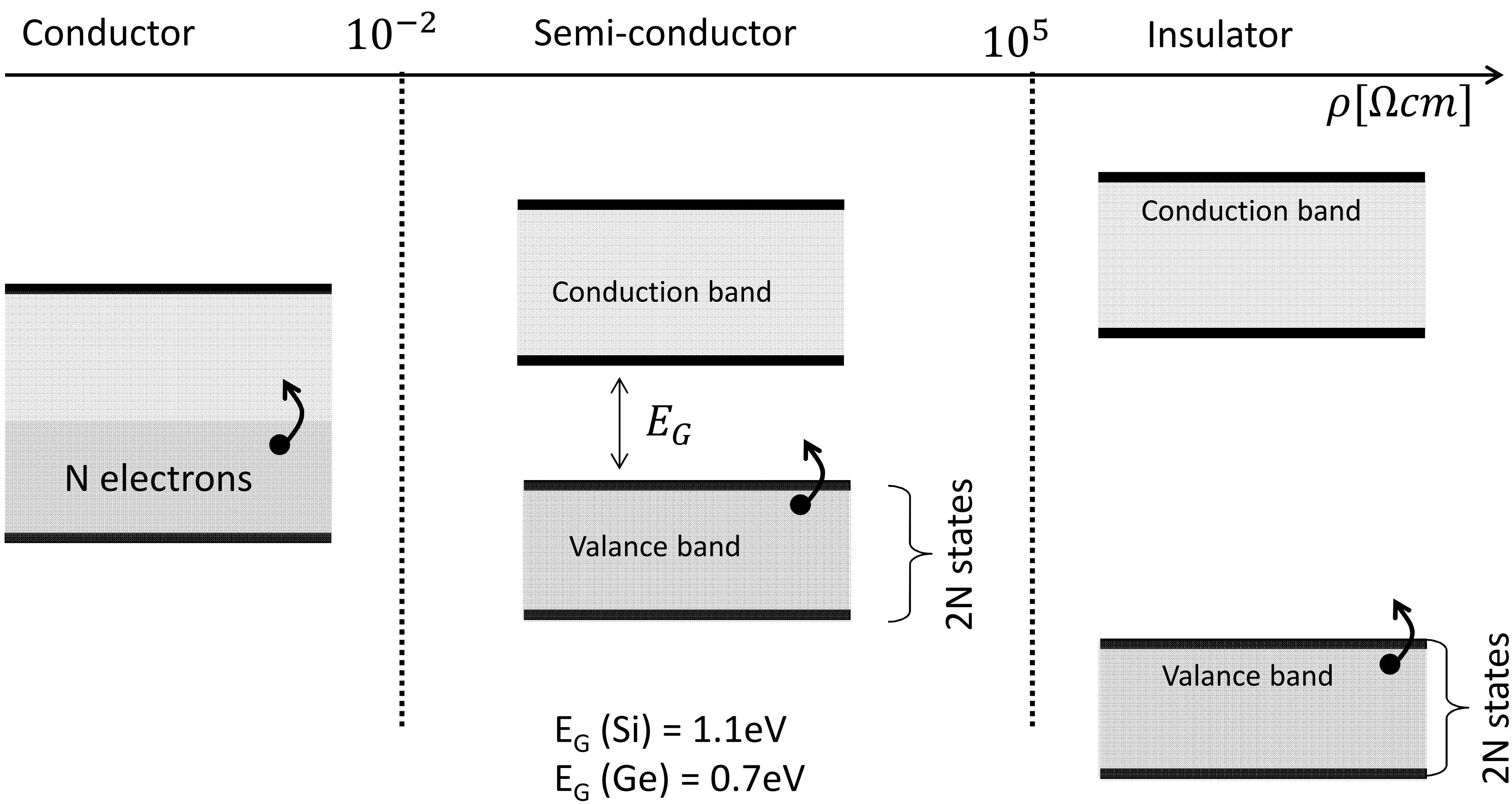


N atoms:



Pauli exclusion principle

Materials

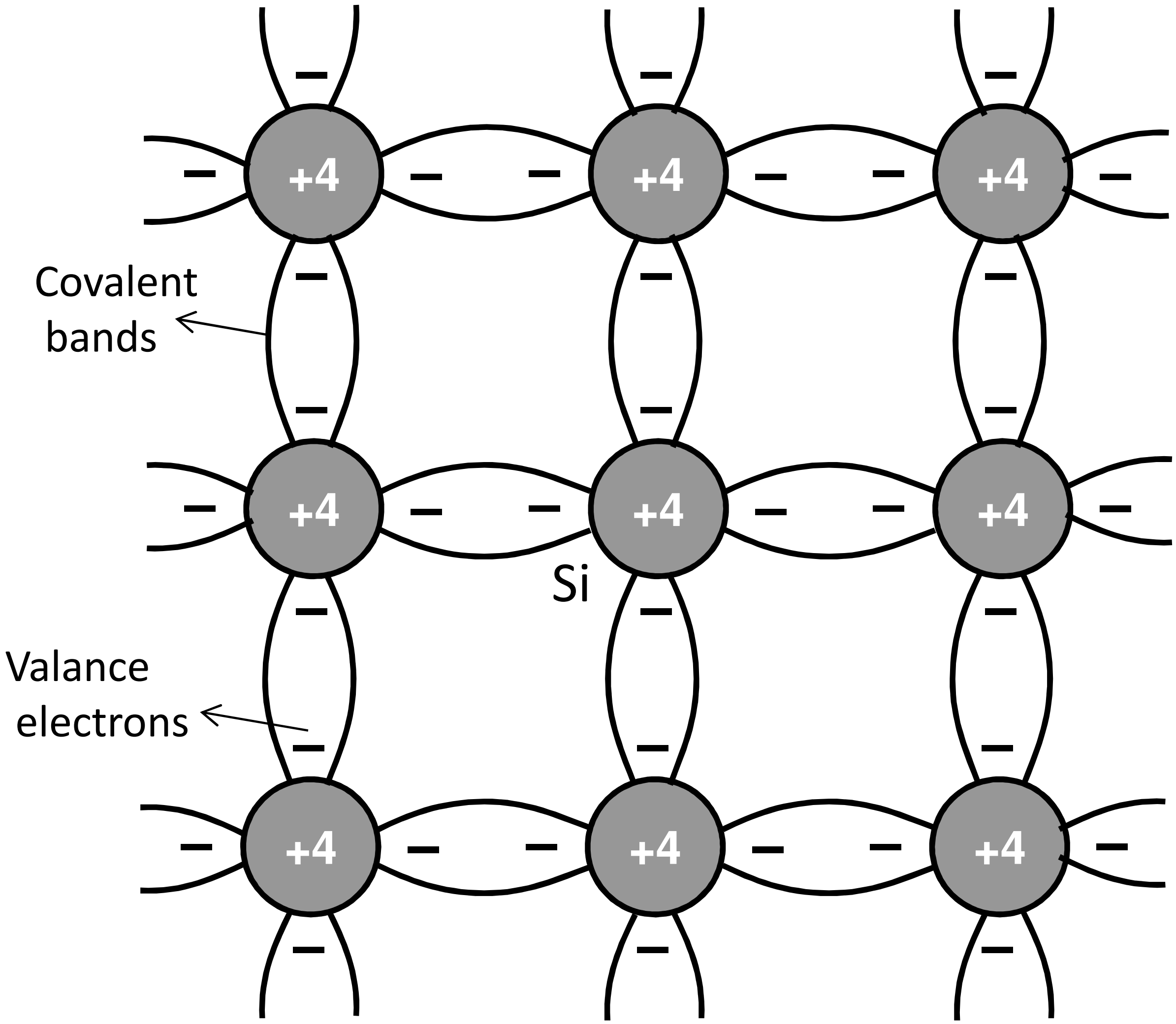


E_G (Si) = 1.1eV
E_G (Ge) = 0.7eV

E_G (SiO_2) = 9eV

empty seat / filled seat

Intrinsic Semiconductor



Intrinsic Semiconductor

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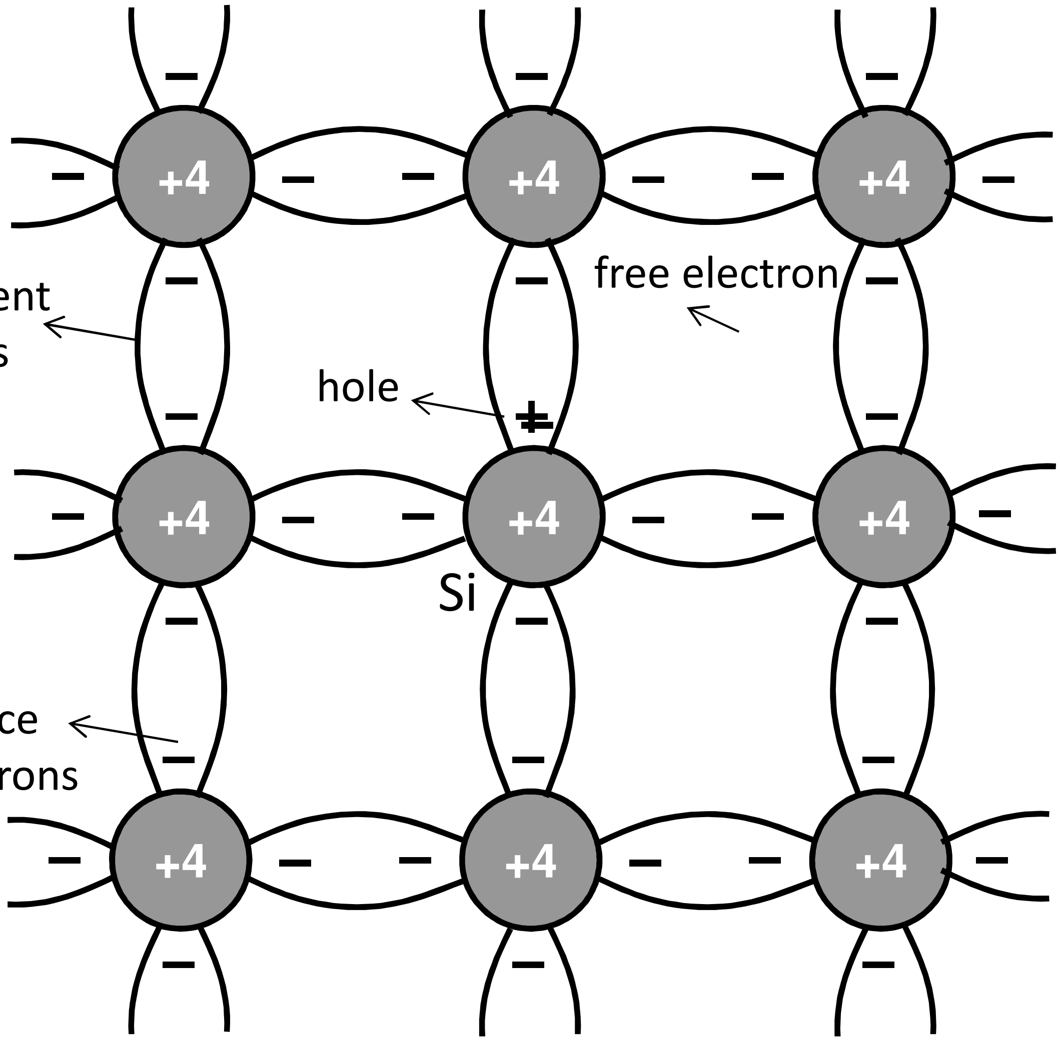


n_0 electron density

p_0 hole density

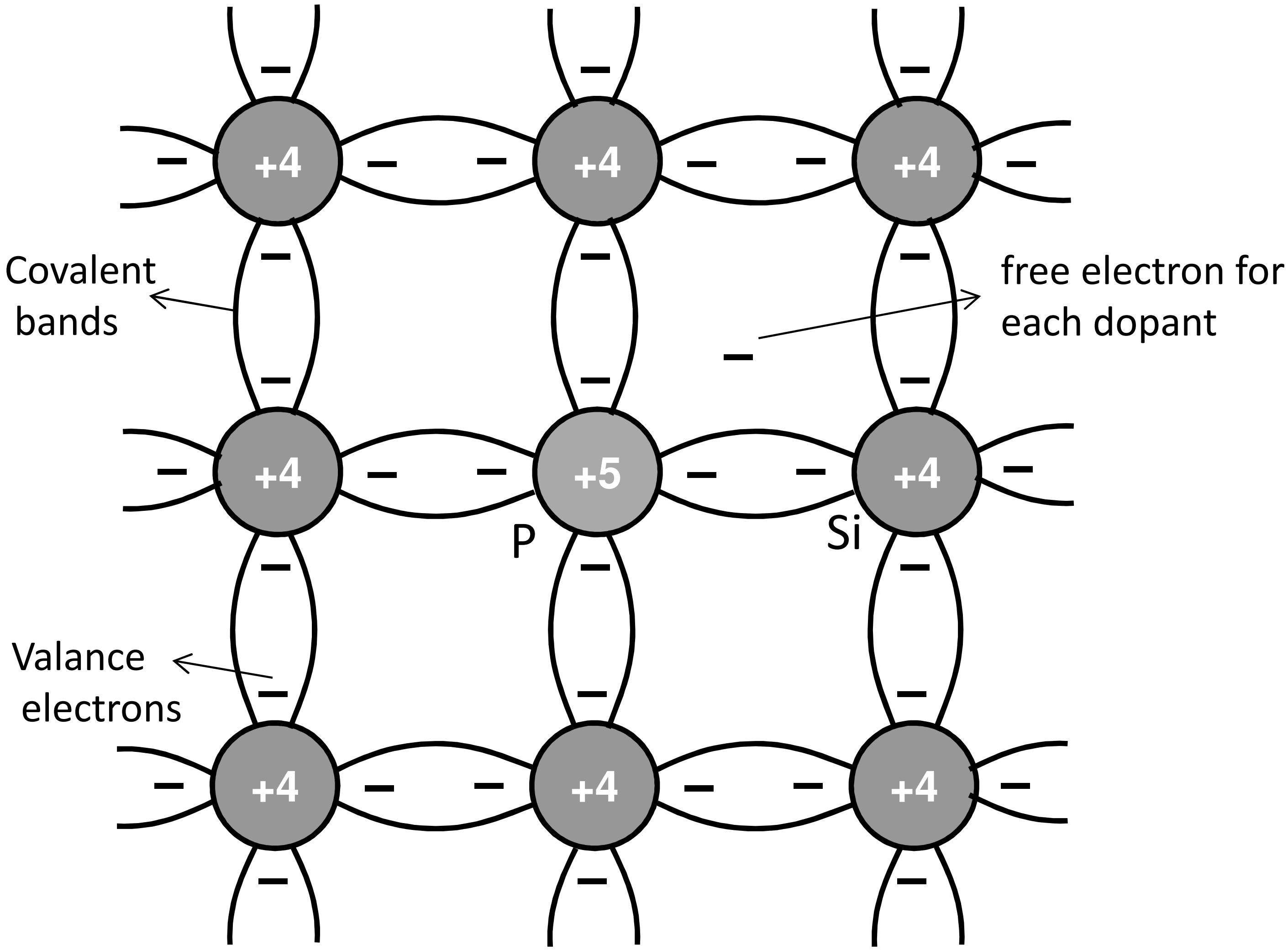
$$n_0 = p_0 = n_i$$

☹ useless!!

$$n_i\Big|_{T=300K} = 10^{10} cm^{-3} \ll \quad n(Si) = 2 \times 10^{23} cm^{-3}$$

n-type Semiconductor

Donor: P , As , Sb



n_0 electron density

p_0 hole density

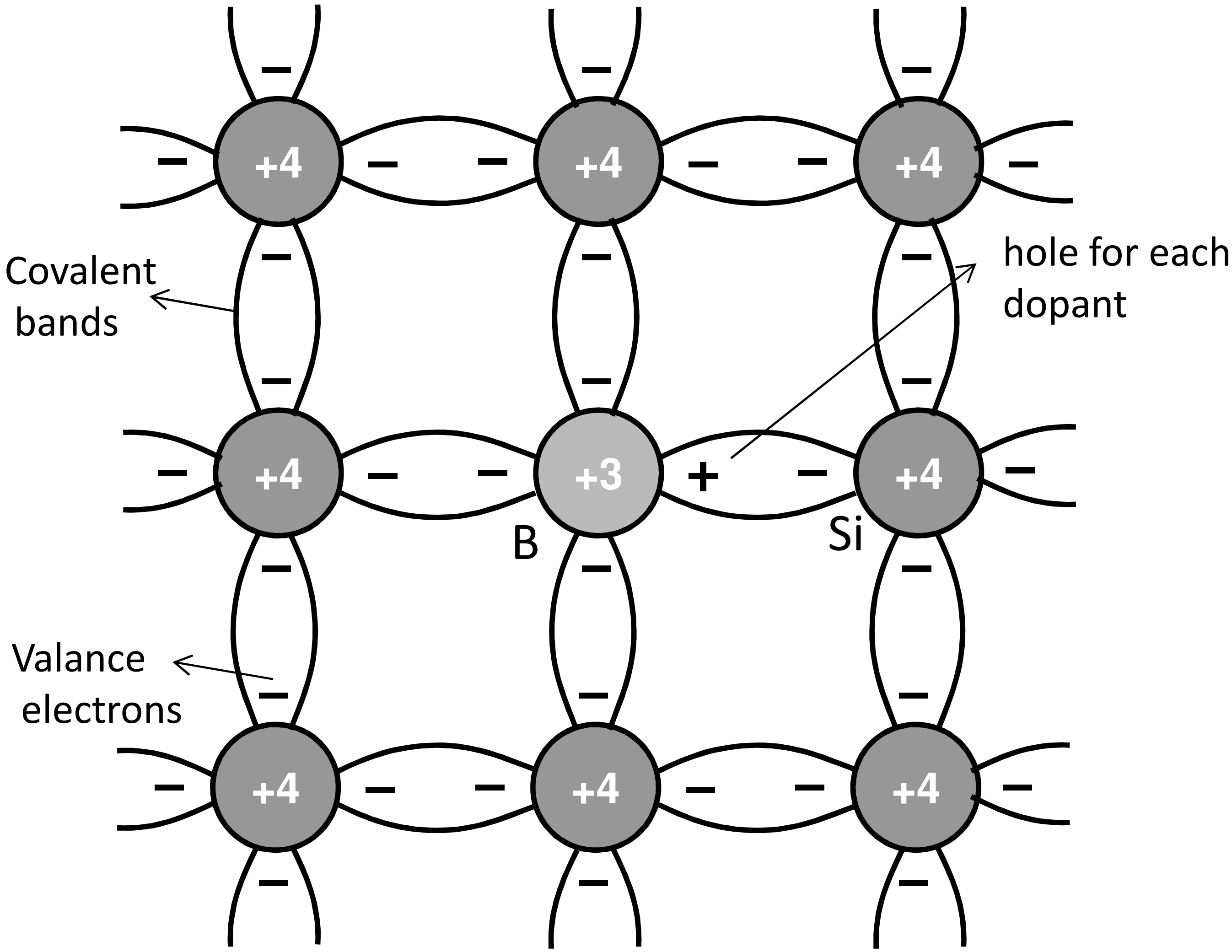
$$n_0 = N_D$$

$$n_0 p_0 = n_i^2$$

N_D $up\ to\ 10^{19}\ cm^{-3}$

☺ $n(Si) = 2 \times 10^{23} cm^{-3}$

p-type Semiconductor



Acceptor: B , Ga , In

n_0 electron density

p_0 hole density

$$n_0 = N_A$$

$$n_0 p_0 = n_i^2$$

N_A *up to* $10^{19}\ cm^{-3}$

☺ $n(Si) = 2 \times 10^{23} cm^{-3}$

Energy Diagrams

1. □□□□□□□□
2. □□□□□□□□□□
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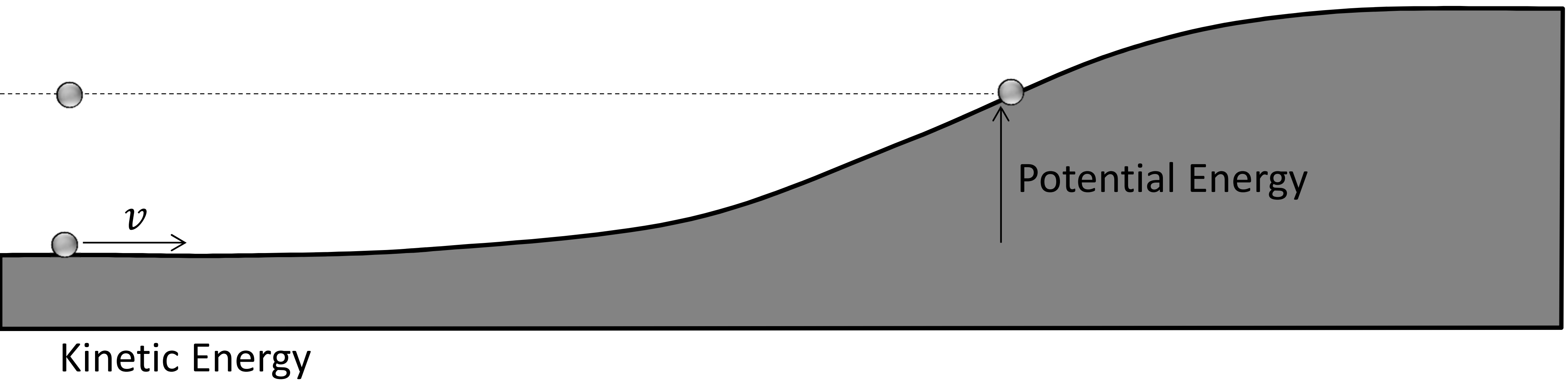

Energy Diagrams

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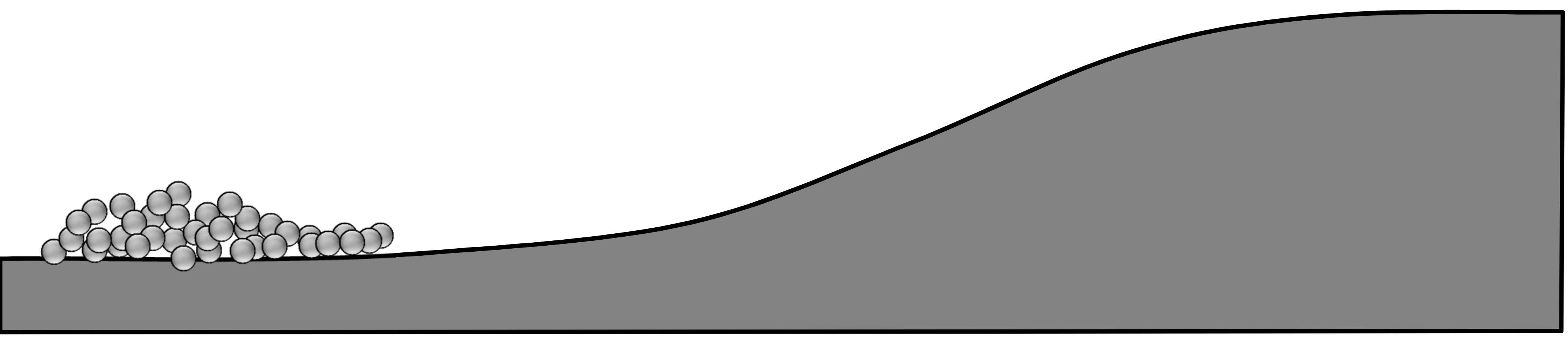

Energy Diagrams

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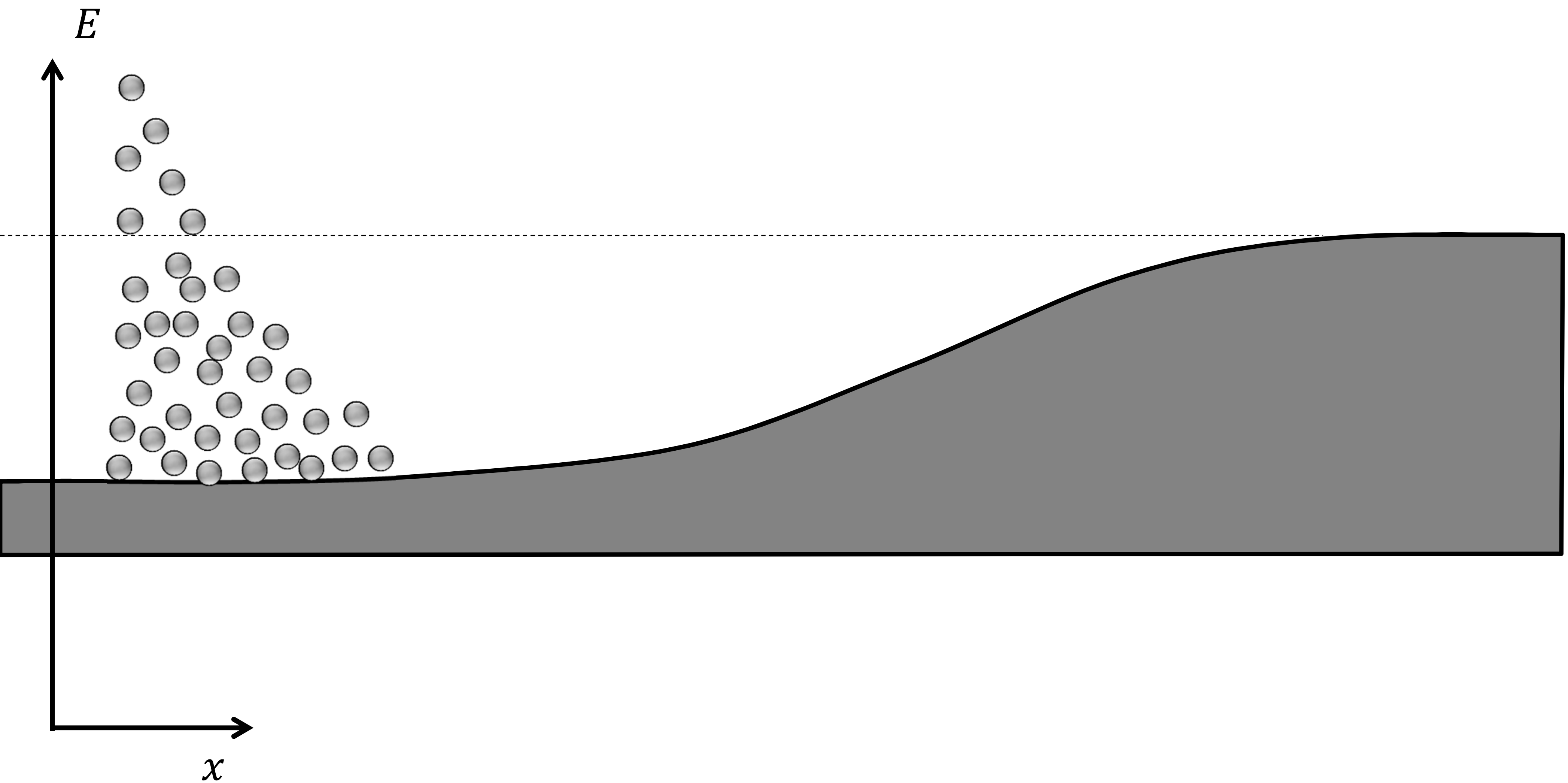

Density of States

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Azadi stadium

Boxing stadium

In Stadium: Number of available seats could be a function of distance from the center so ….

N: number of available states for the electrons could be function of “Energy” : $N(E)$

Seats are not the same for fans so empty states for electrons!

Fermi Function
Probability of Electron Distribution

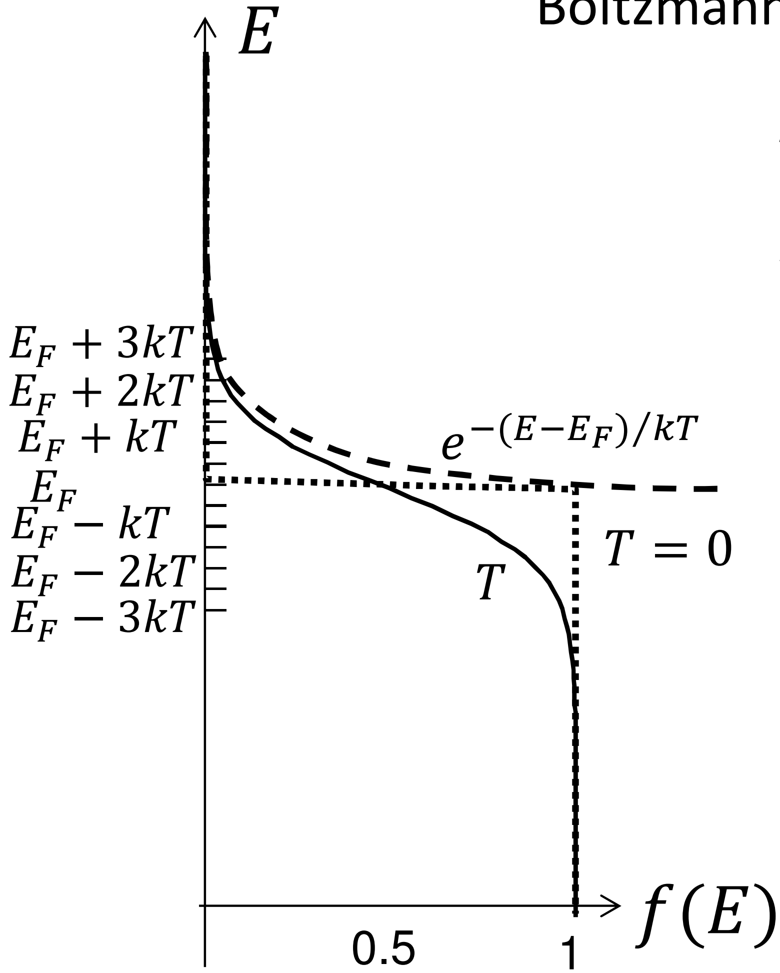
$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

E_f is called the Fermi energy or the Fermi level.

If we are 3kT away from the Fermi energy then we might use Boltzmann approximation:

$$f(E) \approx e^{-(E-E_F)/kT} \qquad \text{if} \qquad E - E_F \gg kT$$

$$f(E) \approx 1 - e^{-(E_f-E)/kT} \qquad \text{if} \qquad E - E_F \ll -kT$$



$N(E)\, f(E) =$ # of electrons at energy E

$N(E)\big(1 - f(E)\big) =$ # of holes at energy E

Materials

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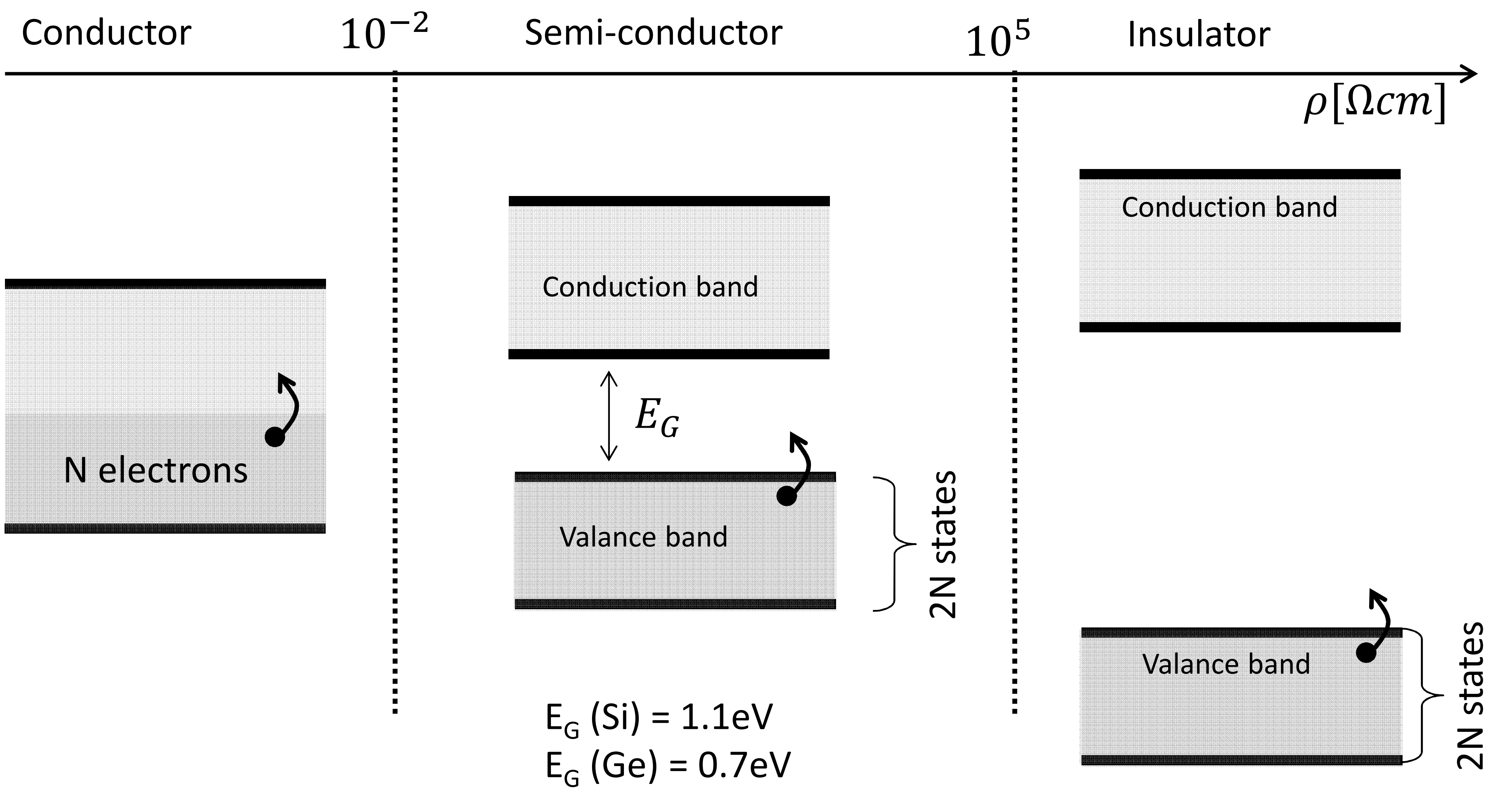


E_G (Si) = 1.1eV
E_G (Ge) = 0.7eV

E_G (SiO_2) = 9eV

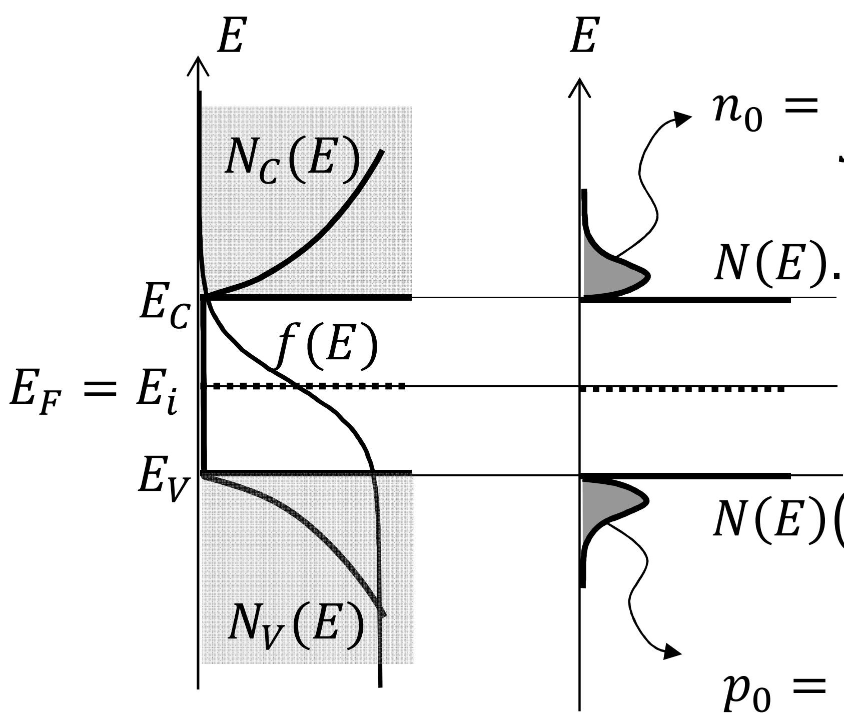
empty seat / filled seat

Electron / Holes : Intrinsic

intrinsic

$N(E)\, f(E) =$ # of electrons at energy E

$N(E)\big(1 - f(E)\big) =$ # of holes at energy E



$$n_0 = \int_{E_C}^{\infty} N(E).\, f(E)dE$$

$N(E).\, f(E)$

$N(E)\big(1 - f(E)\big)$

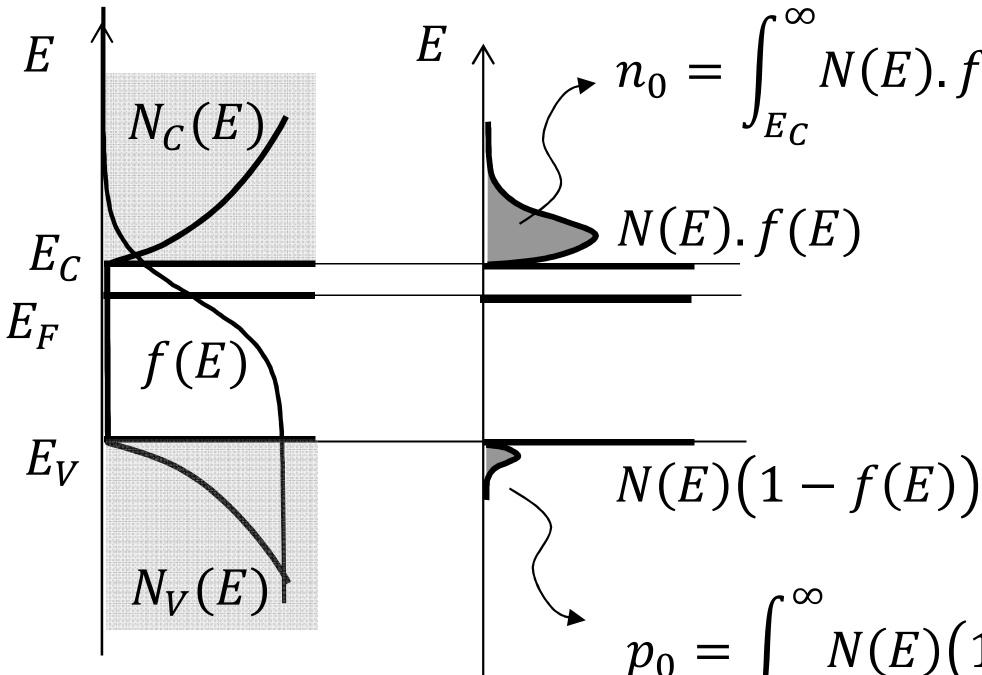
$$p_0 = \int_{E_C}^{\infty} N(E)\big(1 - f(E)\big)dE$$

Electron / Holes : n-type

n-type

$N(E)\,f(E) =$ # of electrons at energy E

$N(E)\big(1 - f(E)\big) =$ # of holes at energy E



$$n_0 = \int_{E_C}^{\infty} N(E).\,f(E)dE$$

$$p_0 = \int_{E_C}^{\infty} N(E)\big(1 - f(E)\big)dE$$

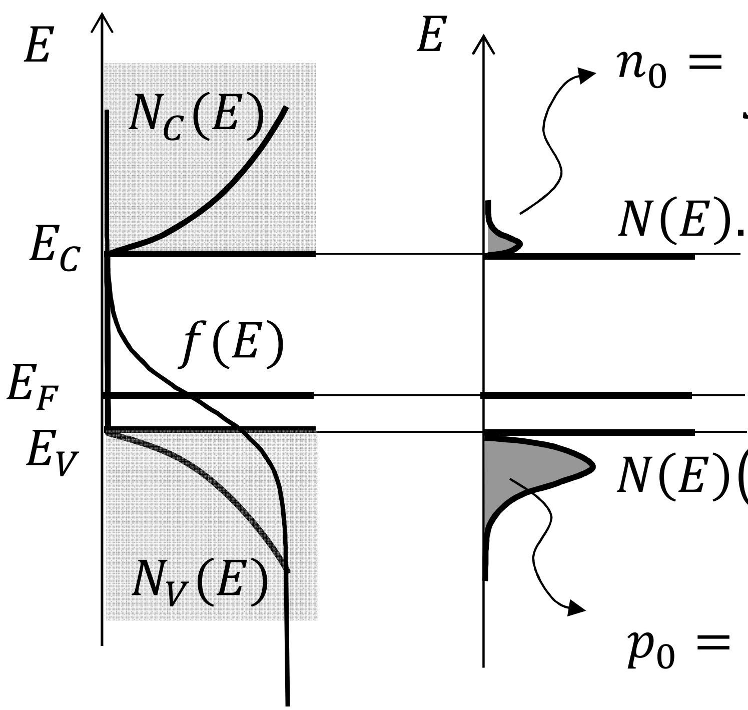
$n_0 \gg p_0$

Electron / Holes : p-type

p-type

$N(E)\,f(E) =$ # of electrons at energy E

$N(E)\big(1 - f(E)\big) =$ # of holes at energy E

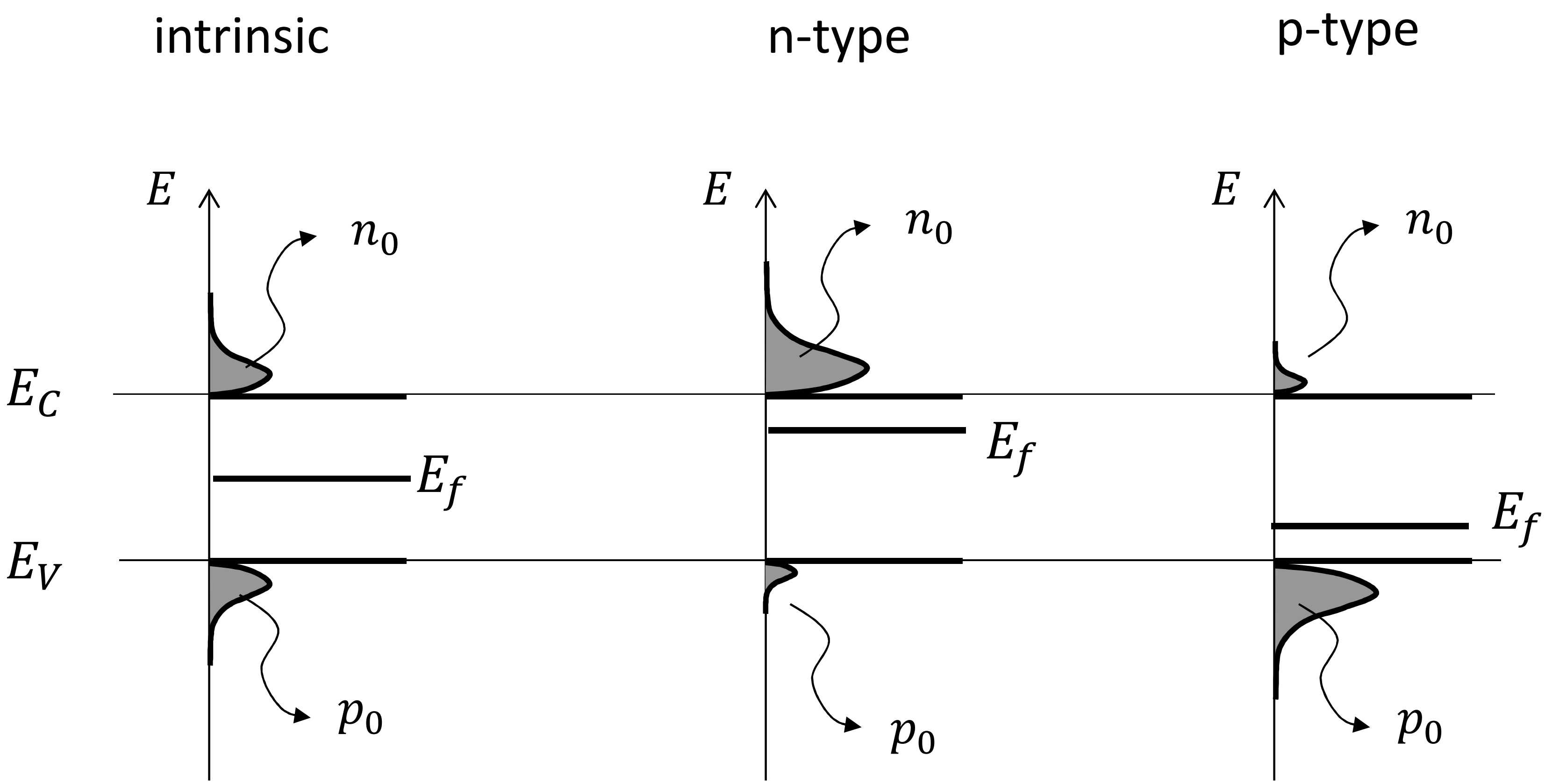


$$n_0 = \int_{E_C}^{\infty} N(E).f(E)dE$$

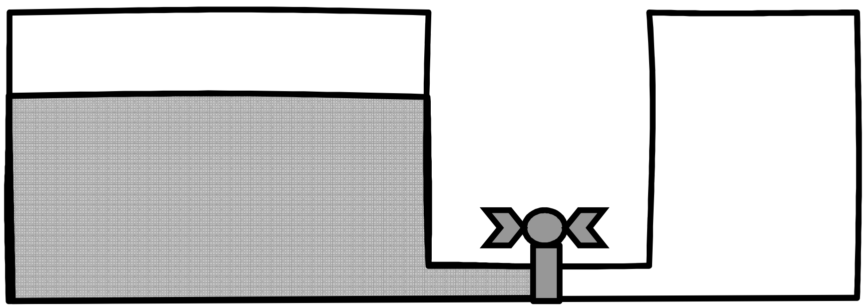
$$p_0 = \int_{E_C}^{\infty} N(E)\big(1 - f(E)\big)dE$$

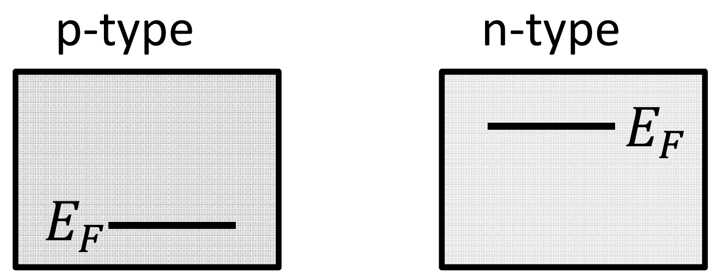
$p_0 \gg n_0$

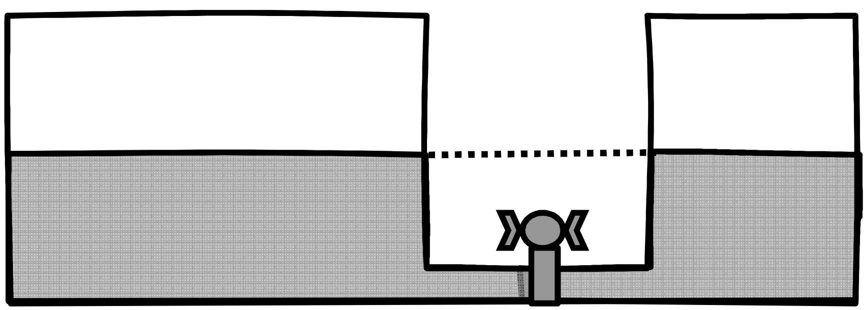
Fermi Energy

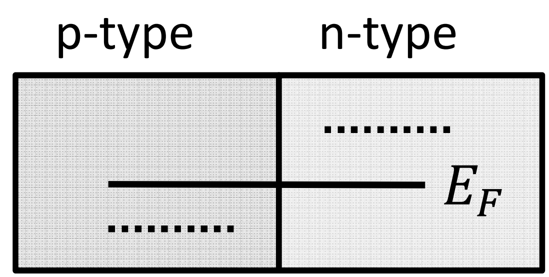


Fermi Energy



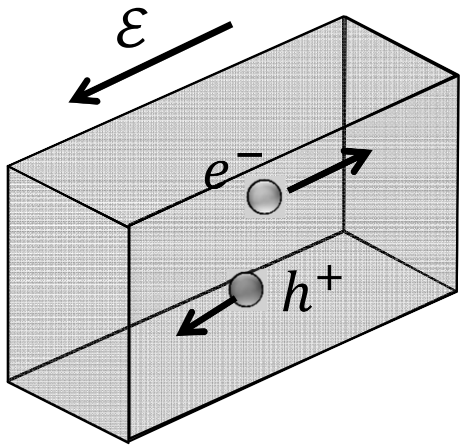


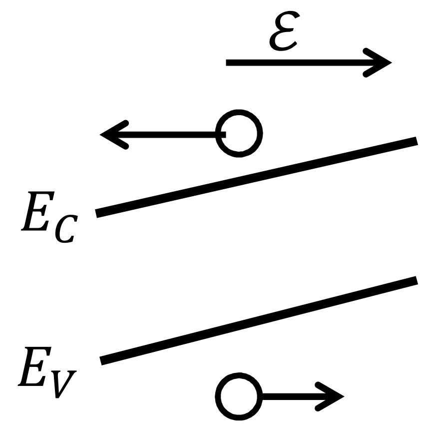


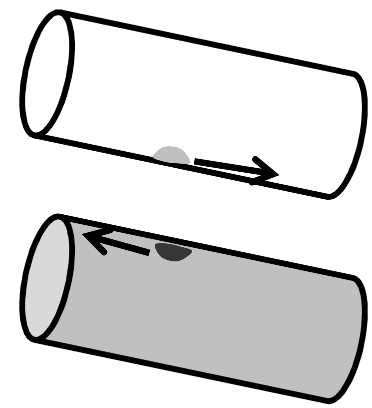


Flow of Charge

Drift Electric field → gravitational field

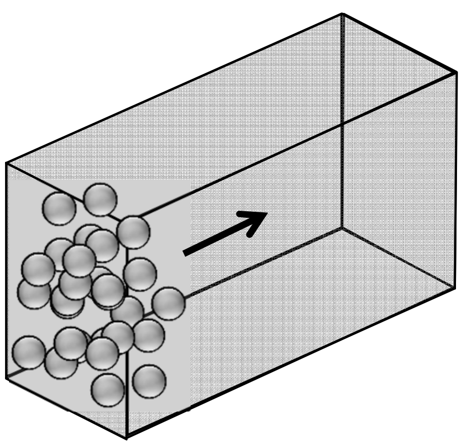






$$J_n = qn\mu\mathcal{E}$$

Diffusion

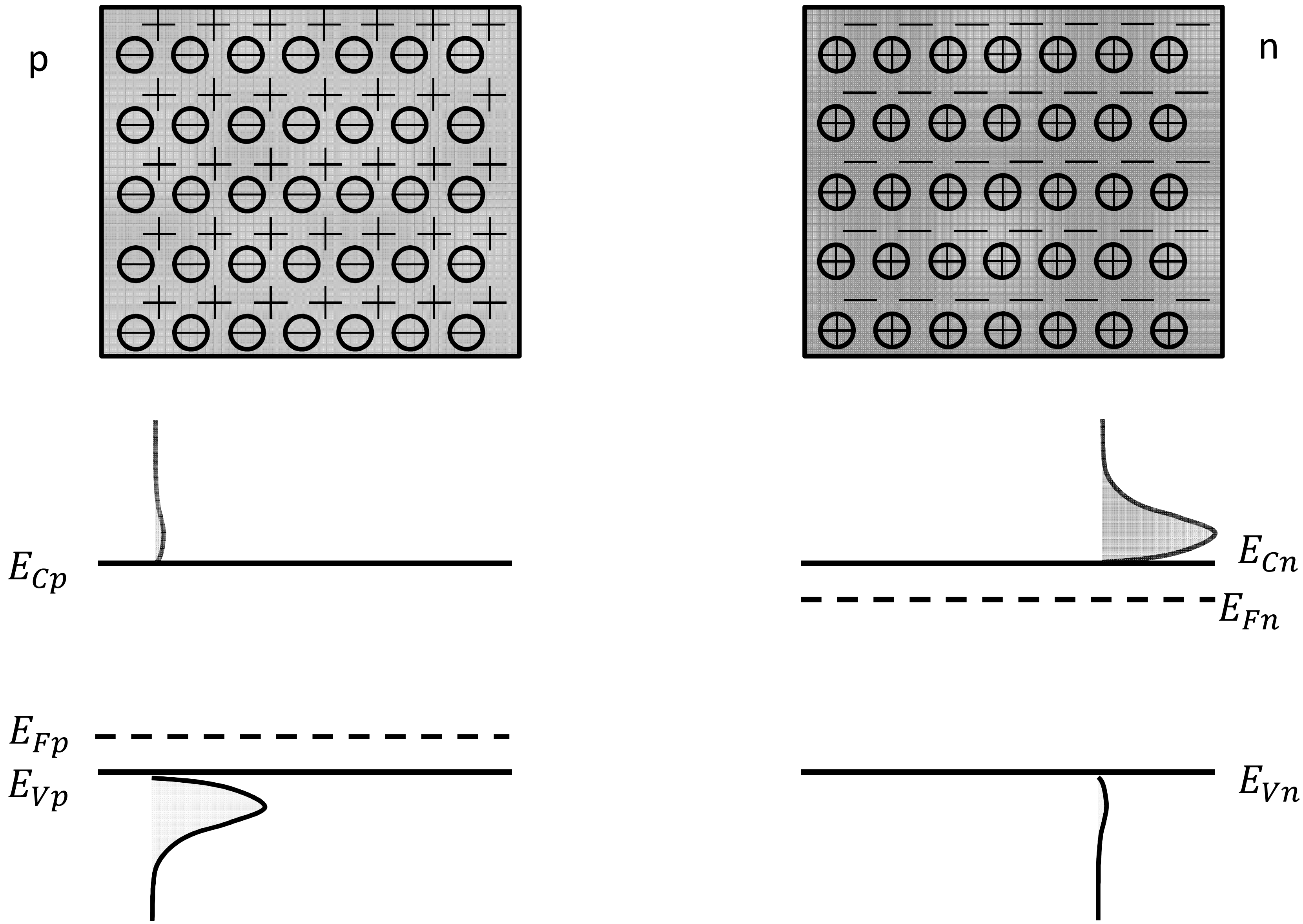


Charges move to be evenly distributed throughout space. Similar to perfume in room or heat in a solid

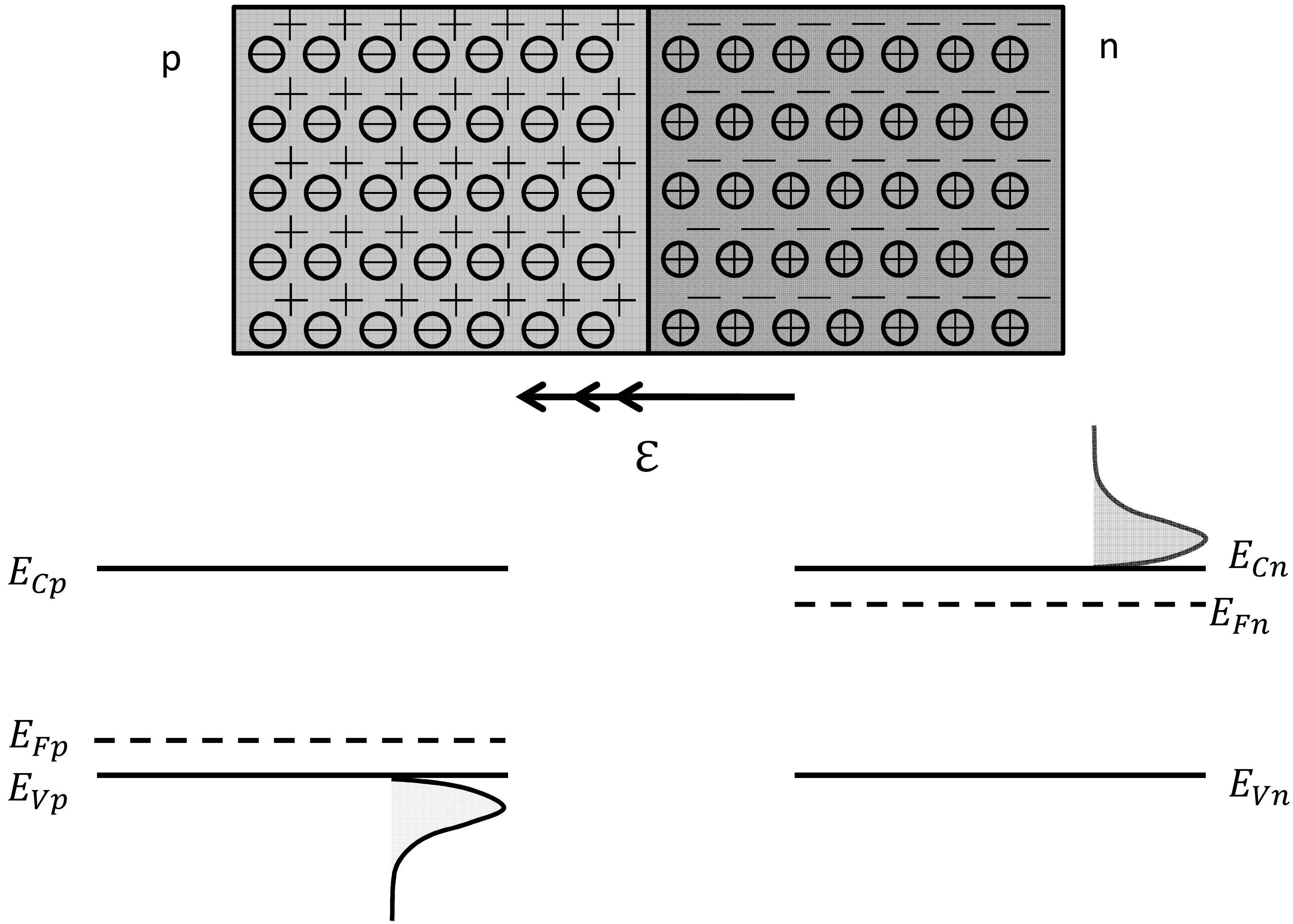
$$J_n = qD_n \frac{dn}{dx}$$

$$J_p = -qD_p \frac{dp}{dx}$$

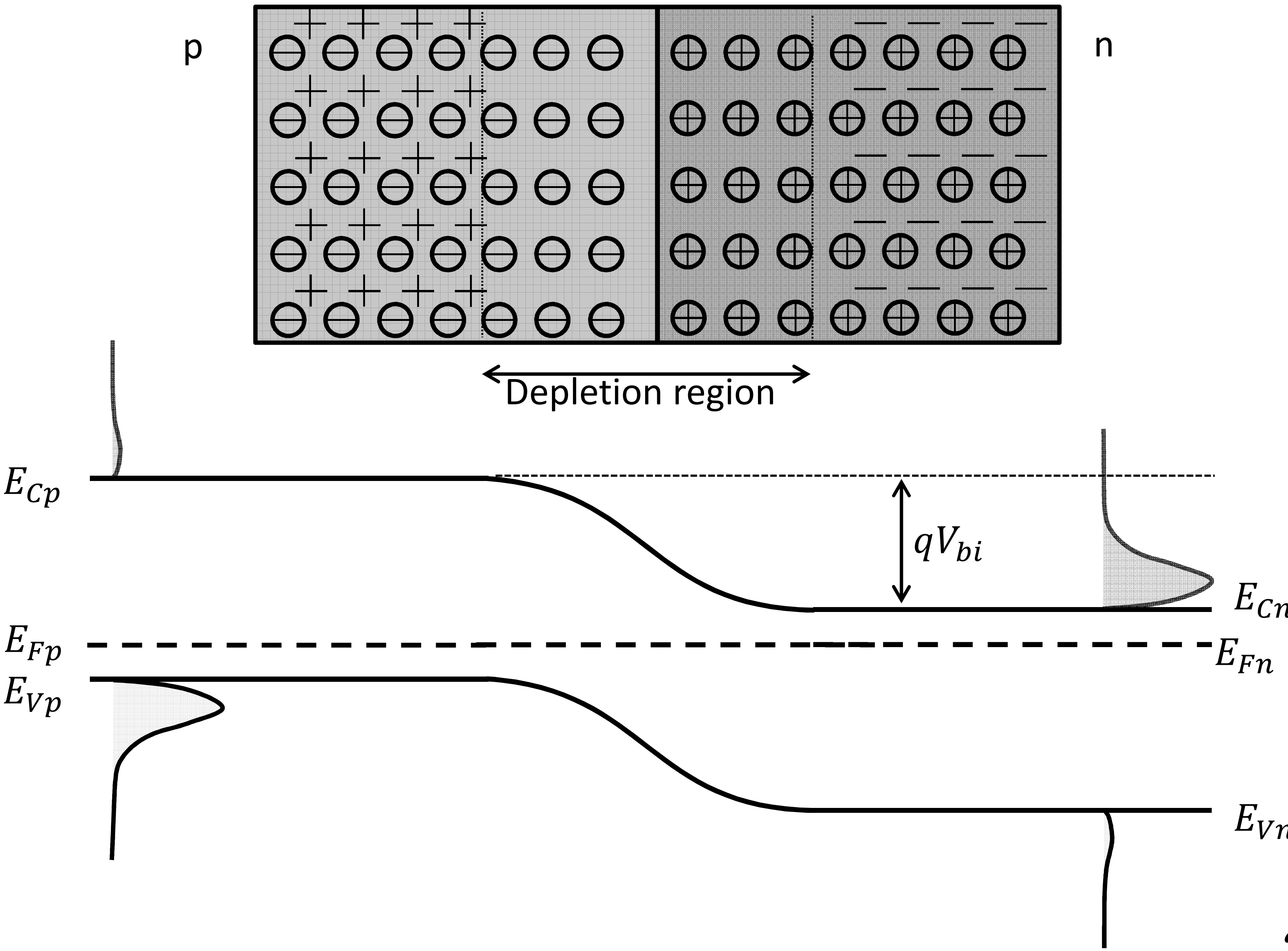
PN Junction



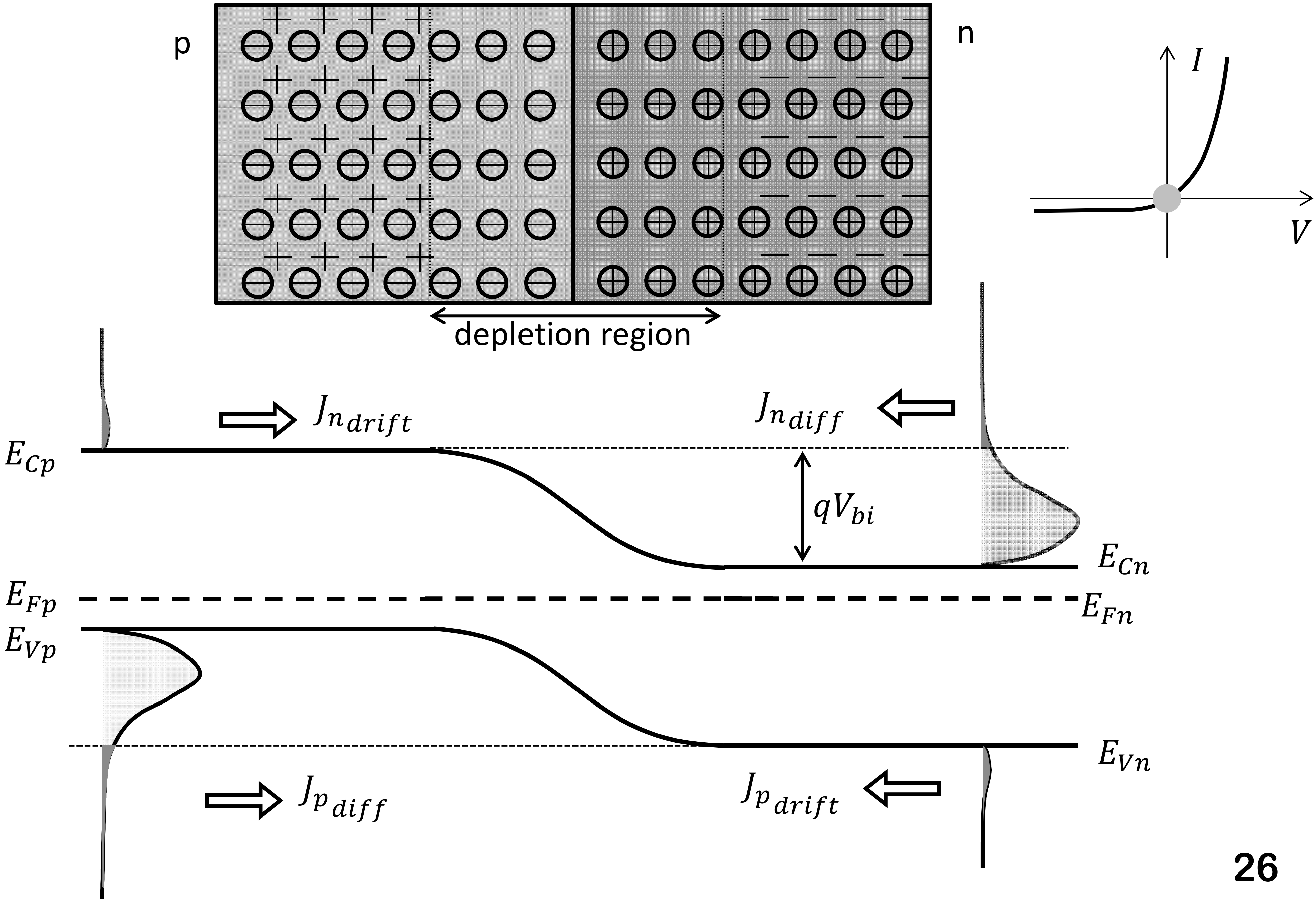
PN junctions



PN junctions



PN junctions



PN junctions , Reverse Biased

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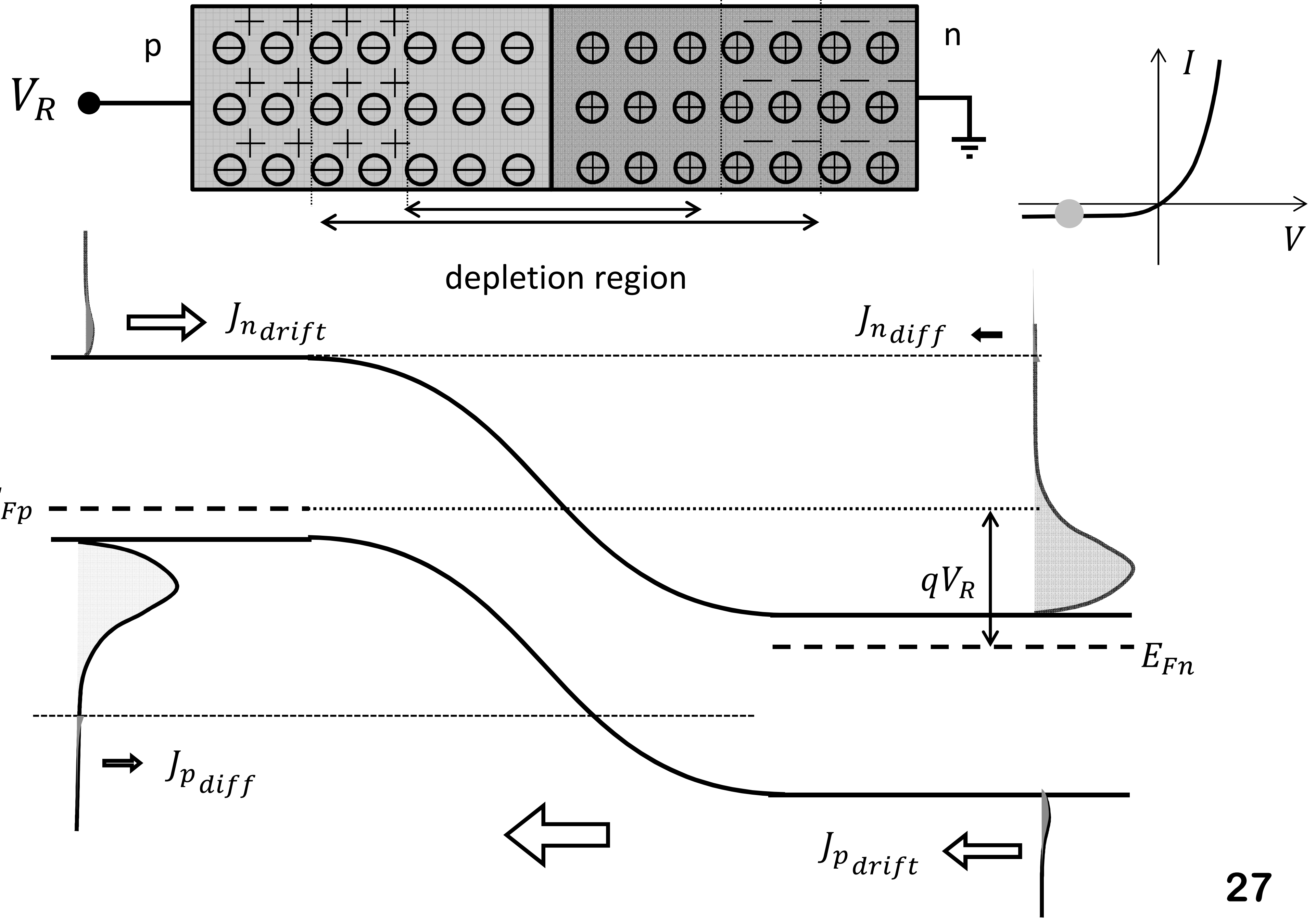
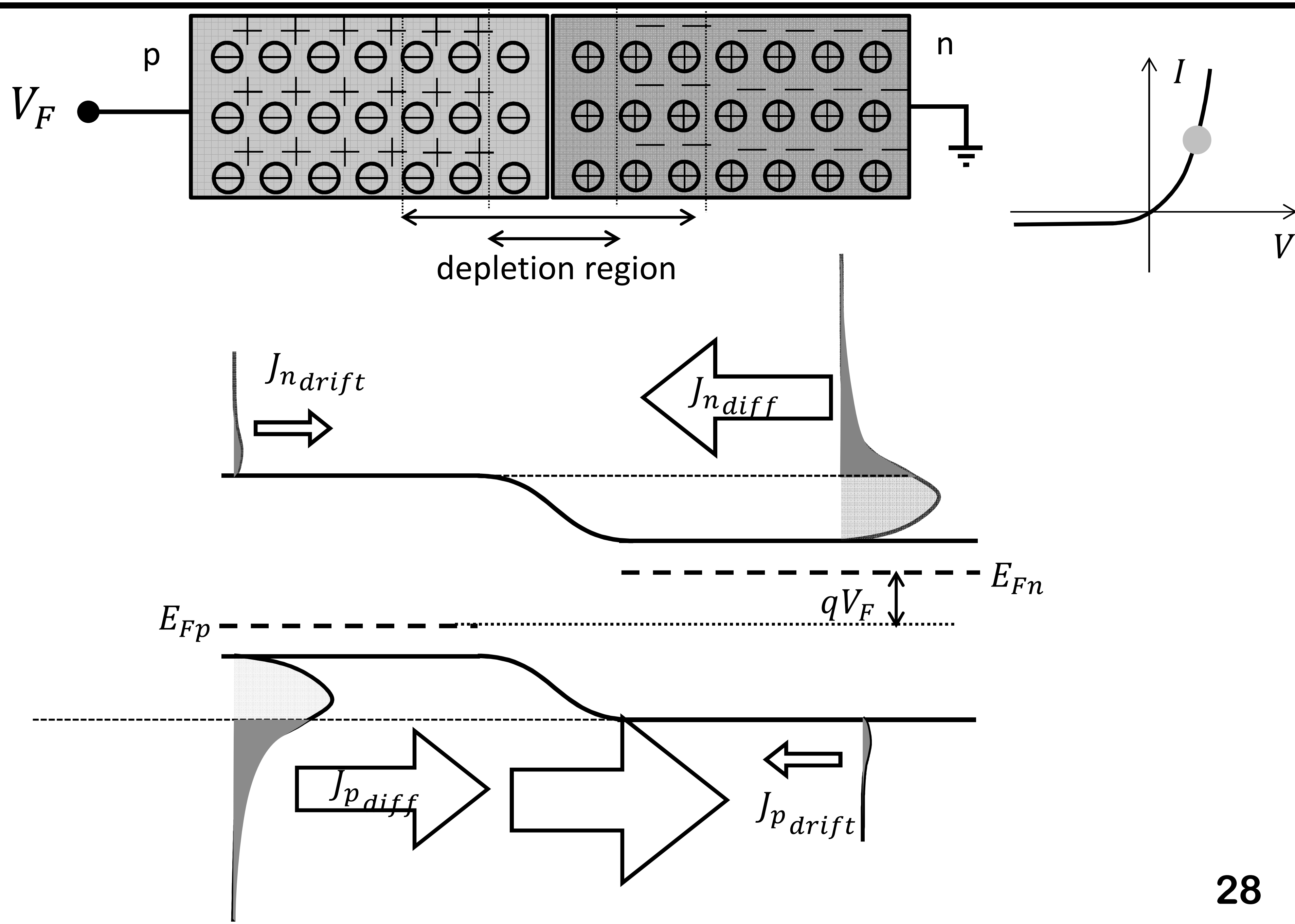


PN junctions , Forward Biased



BJT Electrostatics

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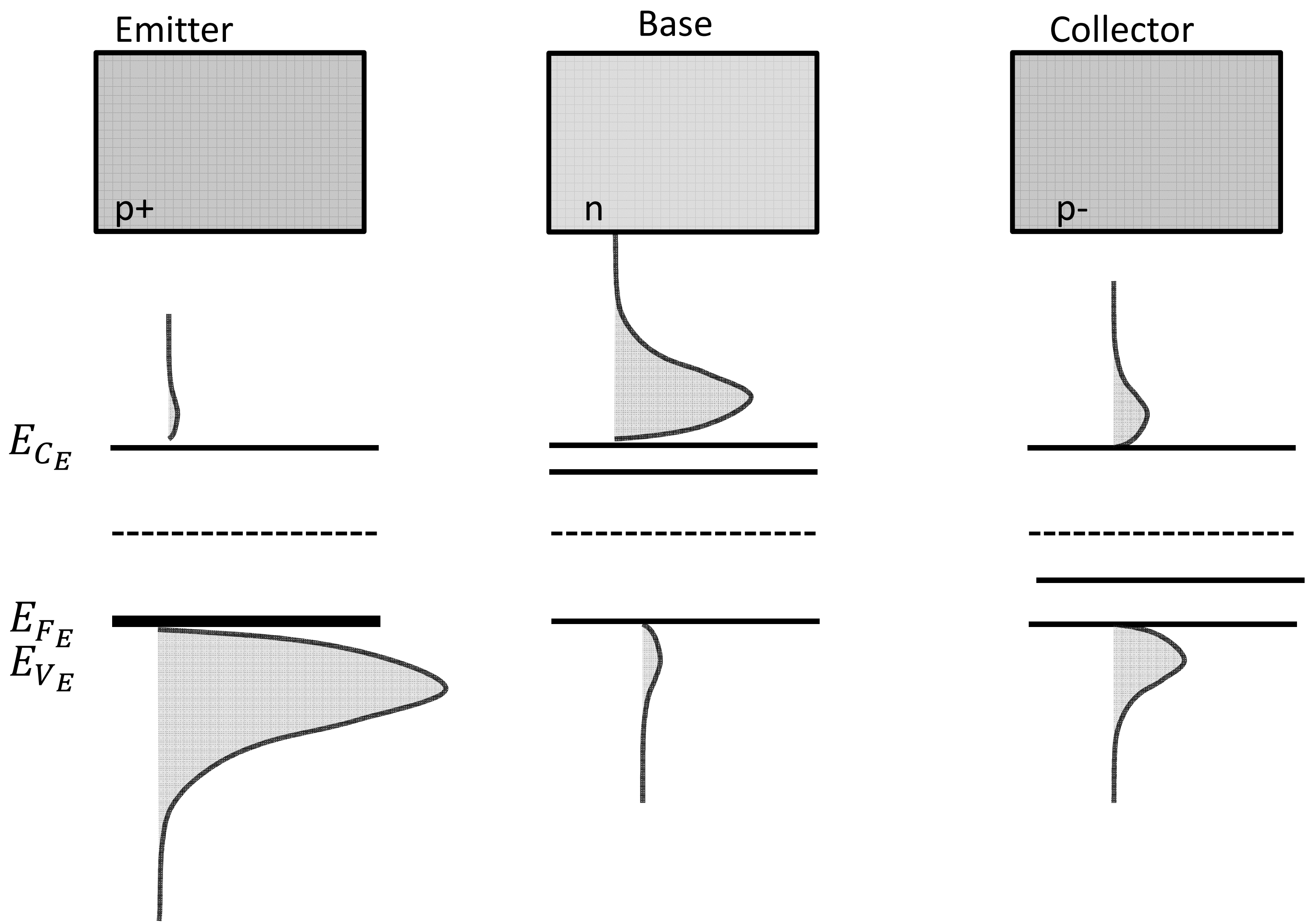
pnp



Under normal operating conditions, the BJT may be viewed electrostatically as two independent pn junctions

BJT Electrostatics

1.
2.
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4.
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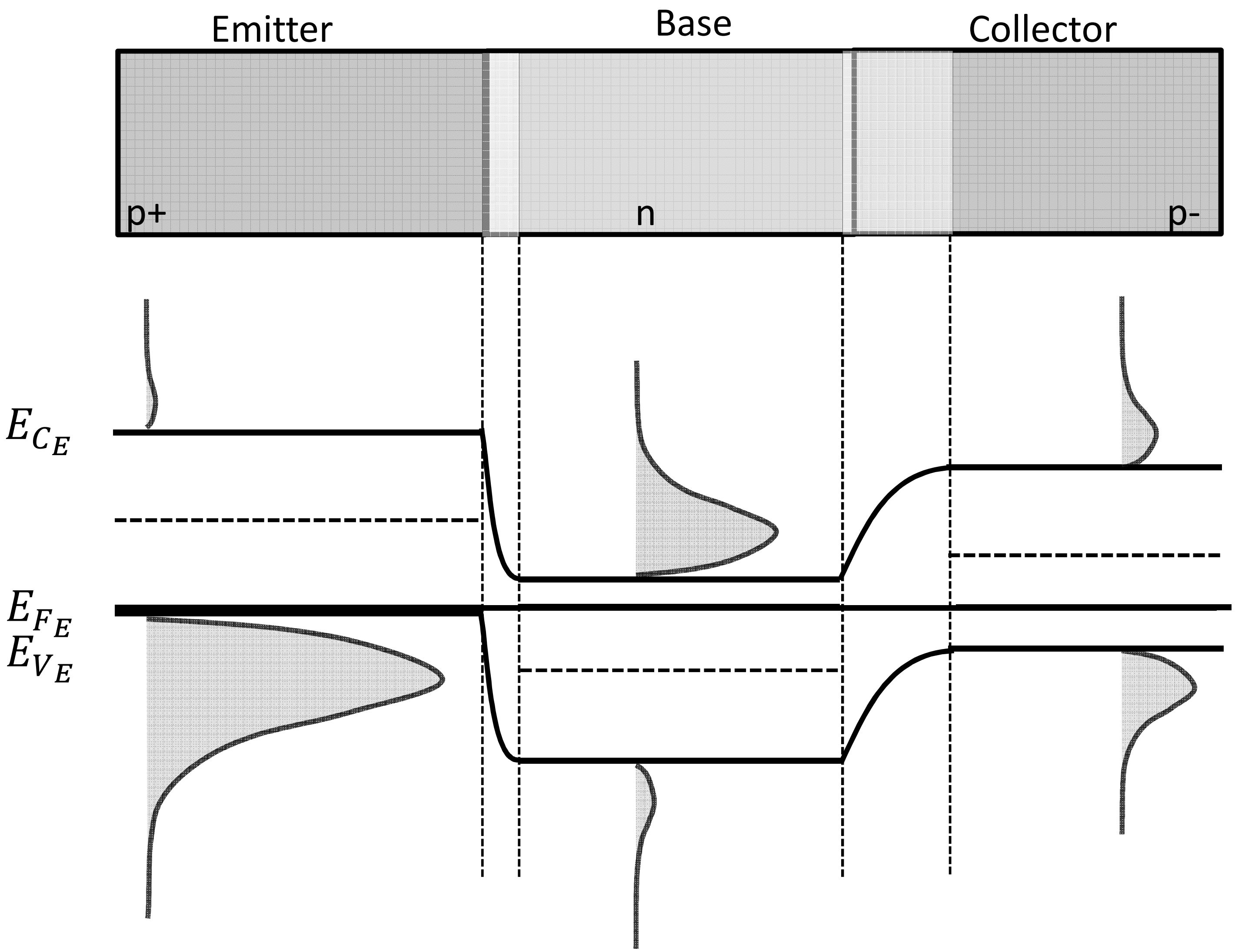
pnp



Under normal operating conditions, the BJT may be viewed electrostatically as two independent pn junctions

BJT Electrostatics

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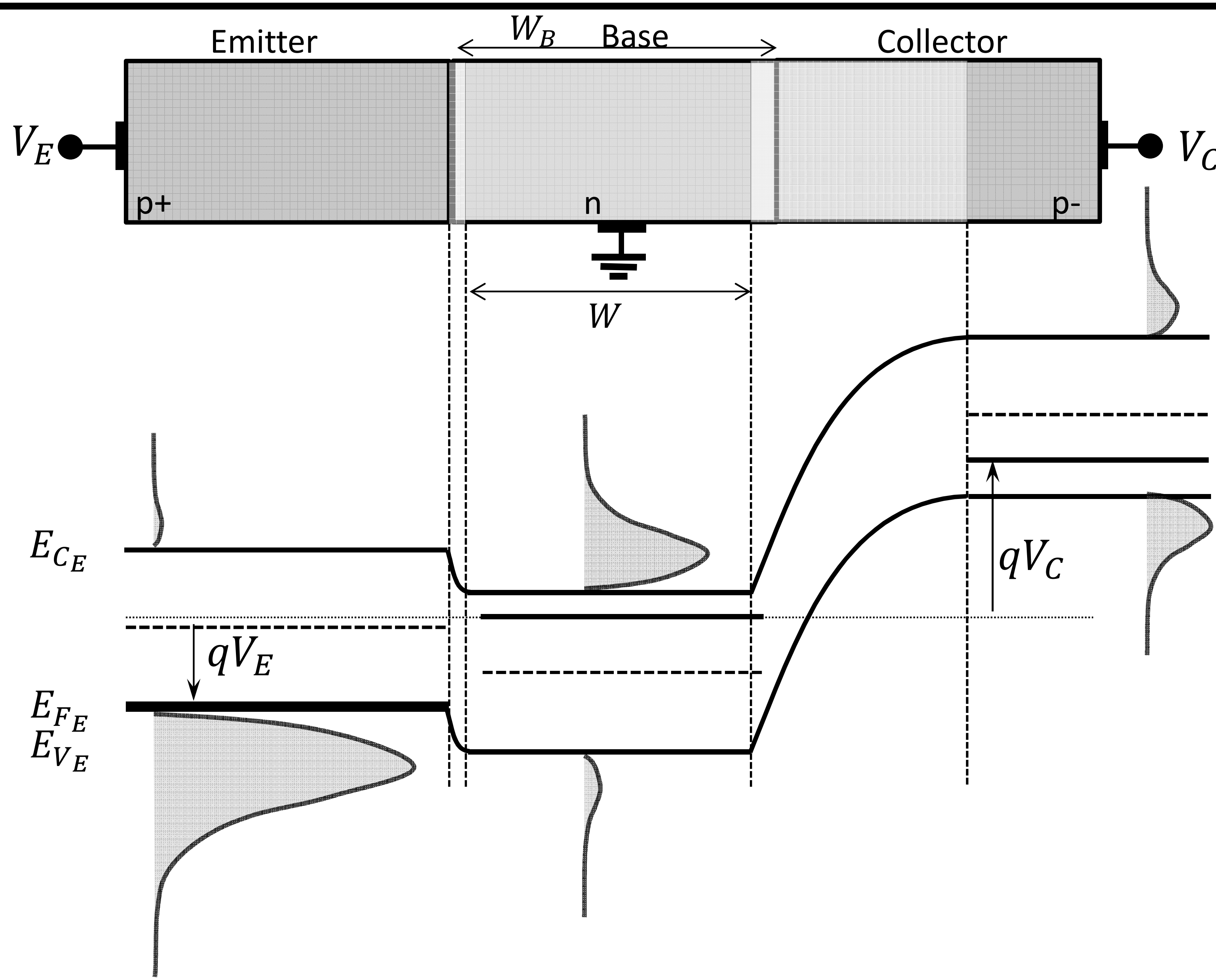
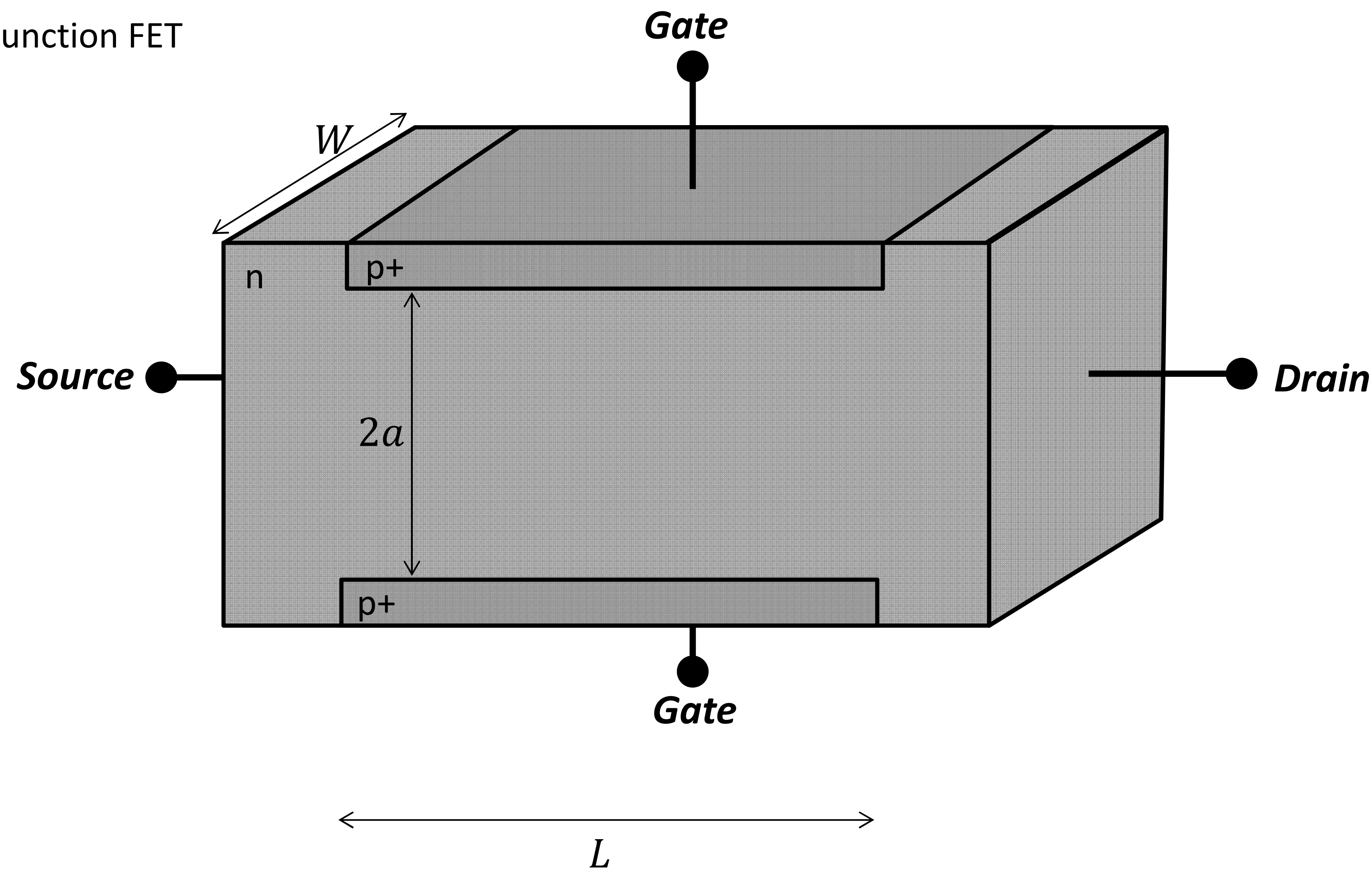
pnp



JFET

Junction FET



JFET

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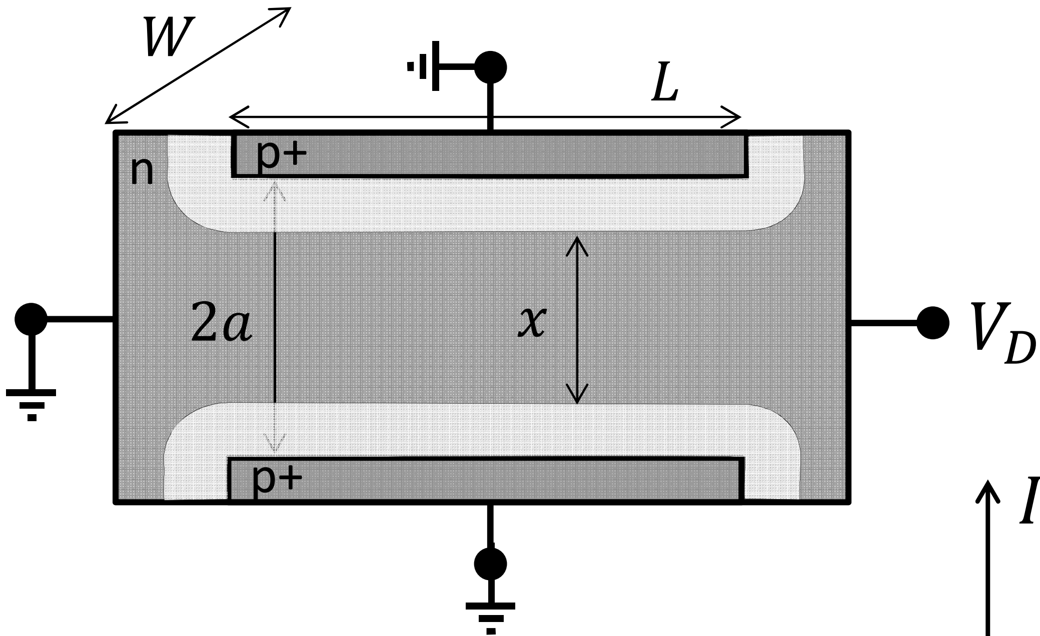
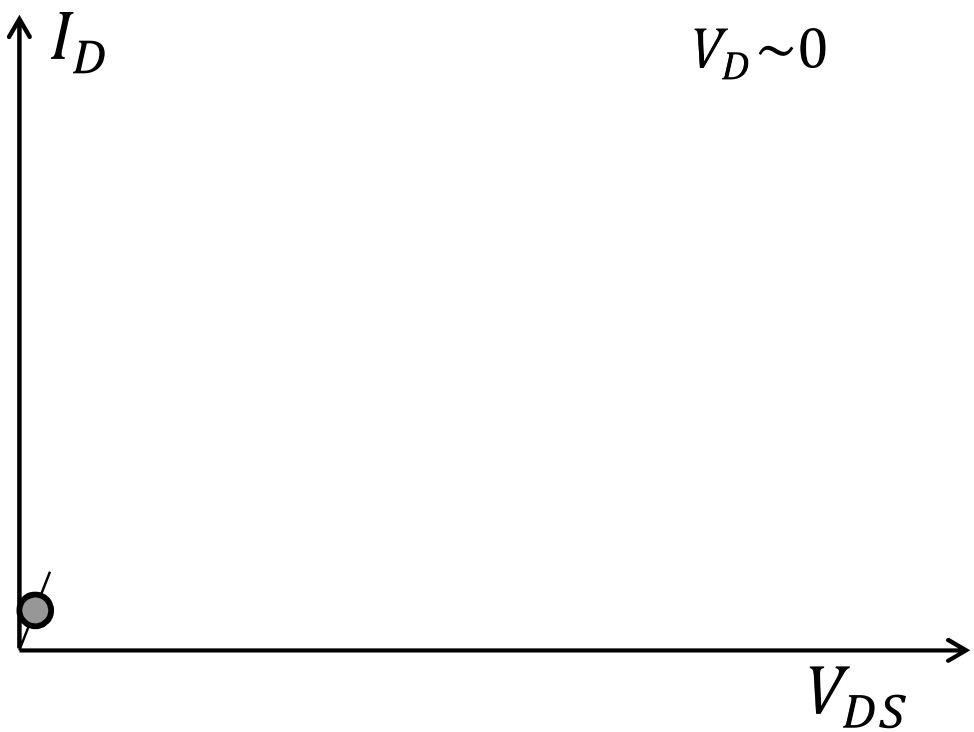


$$R = \rho \frac{L}{Wx}$$



JFET

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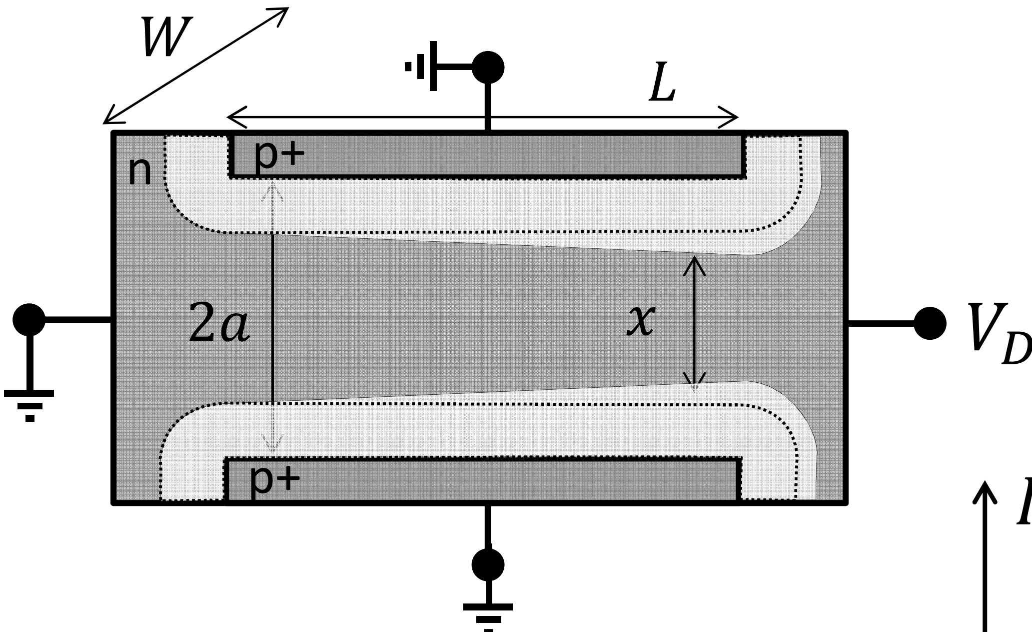
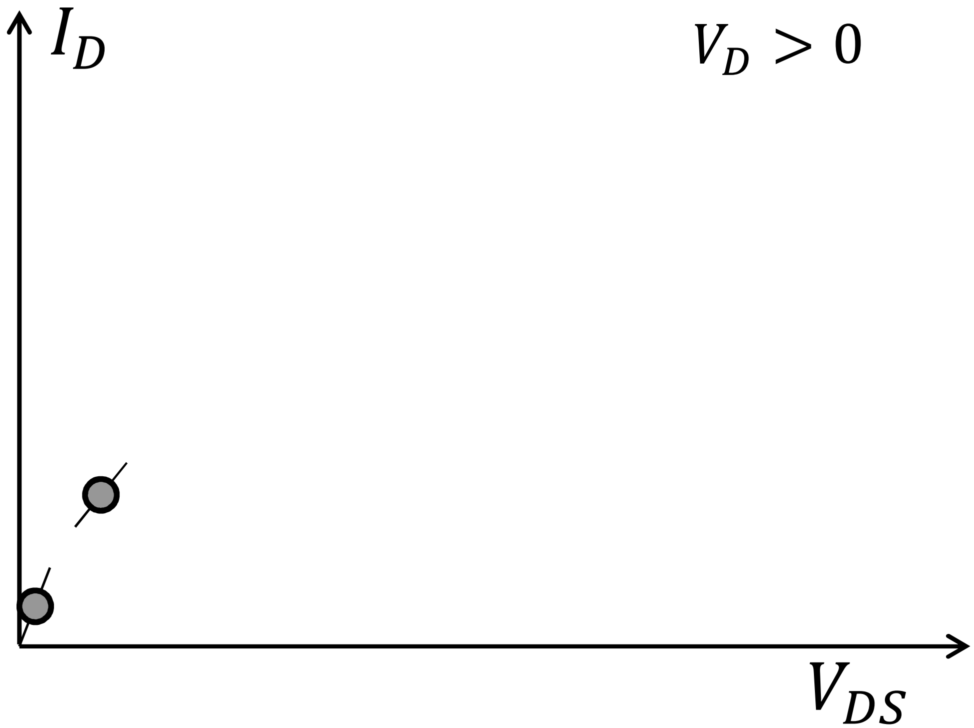
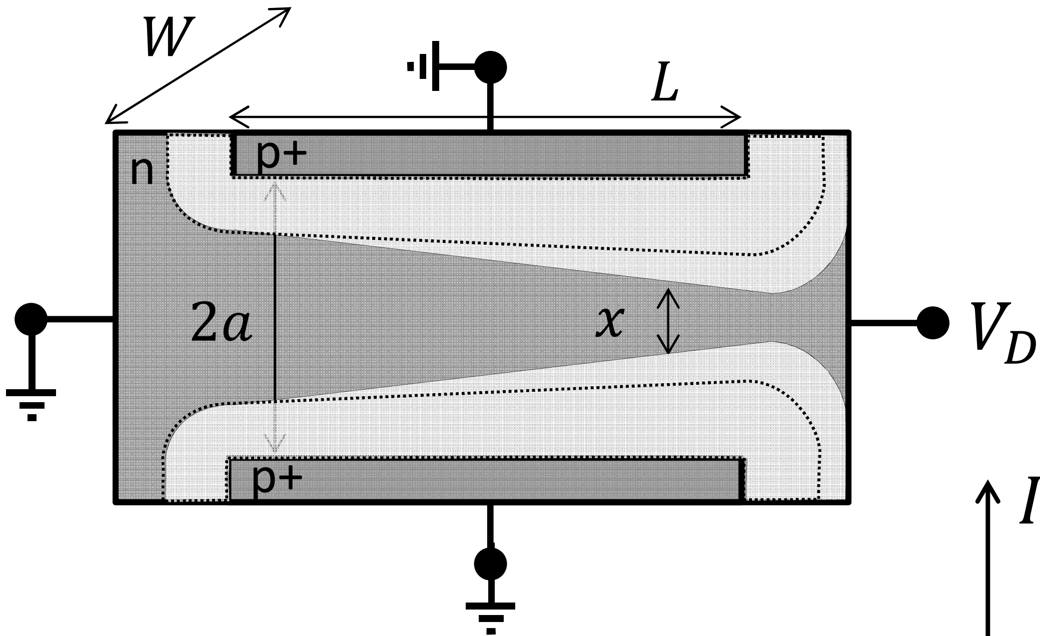


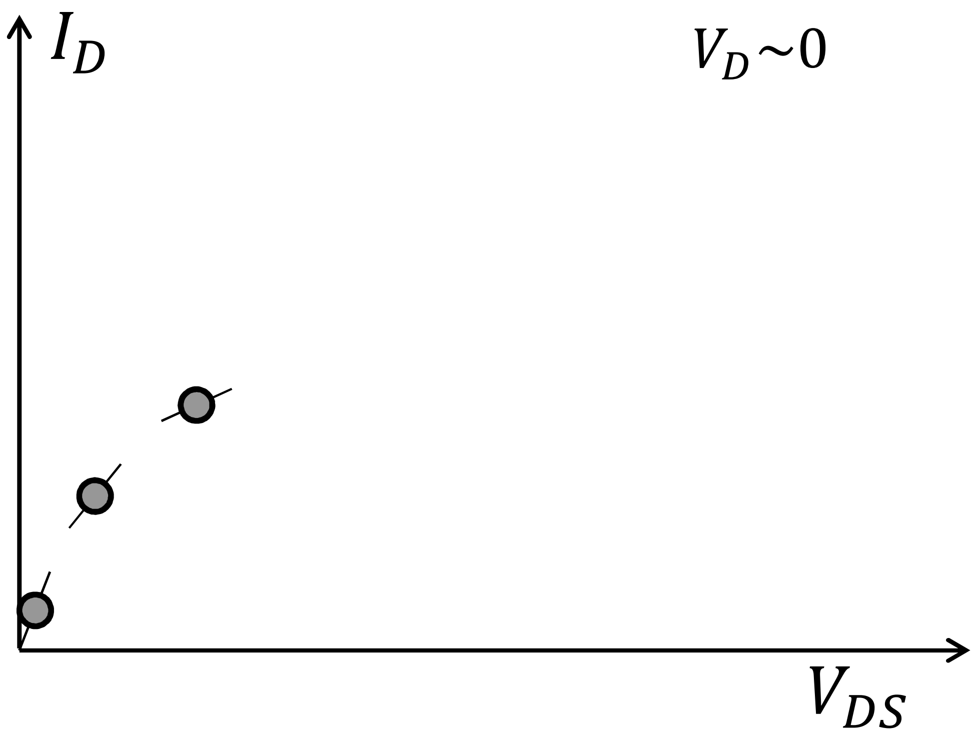
$$R = \rho \frac{L}{Wx}$$

$V_D > 0$



JFET





$$R = \rho \frac{L}{Wx}$$

JFET

1. □□□□□□□□
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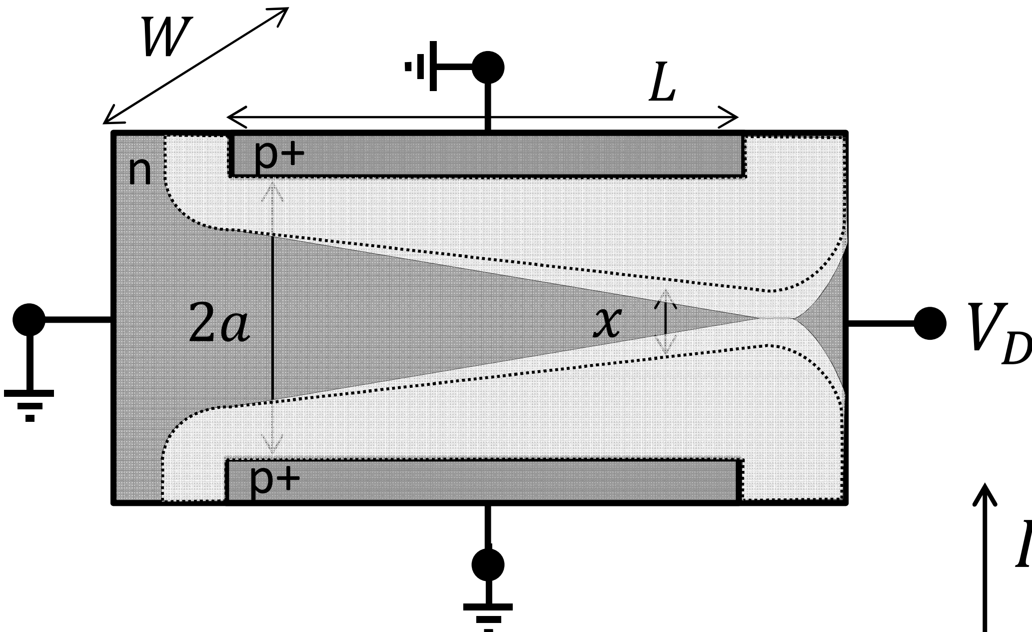
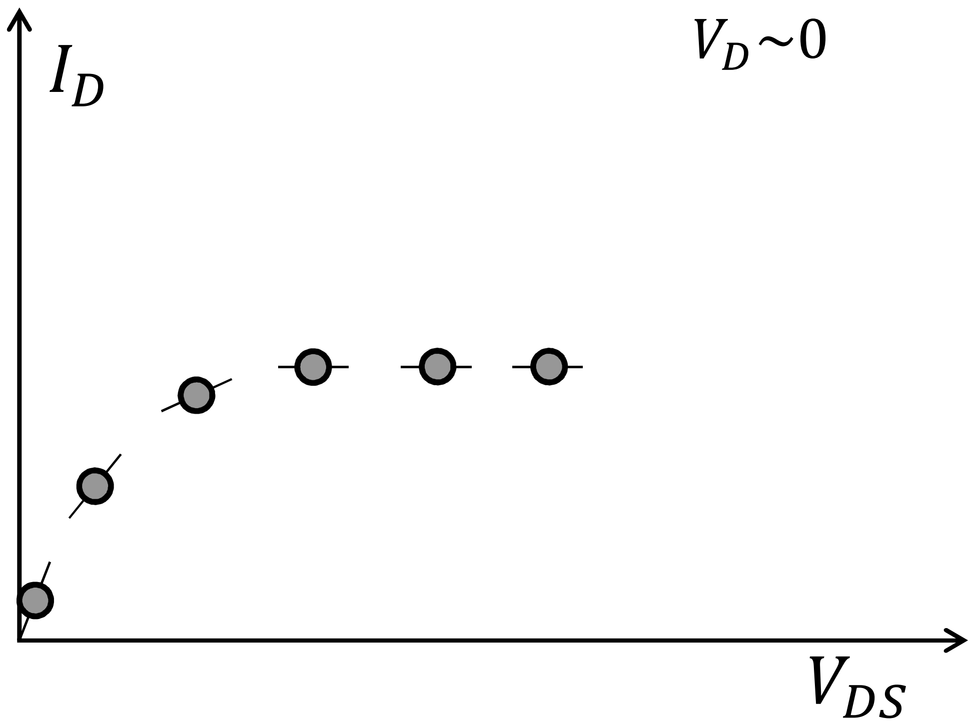


$$R = \rho \frac{L}{Wx}$$



JFET

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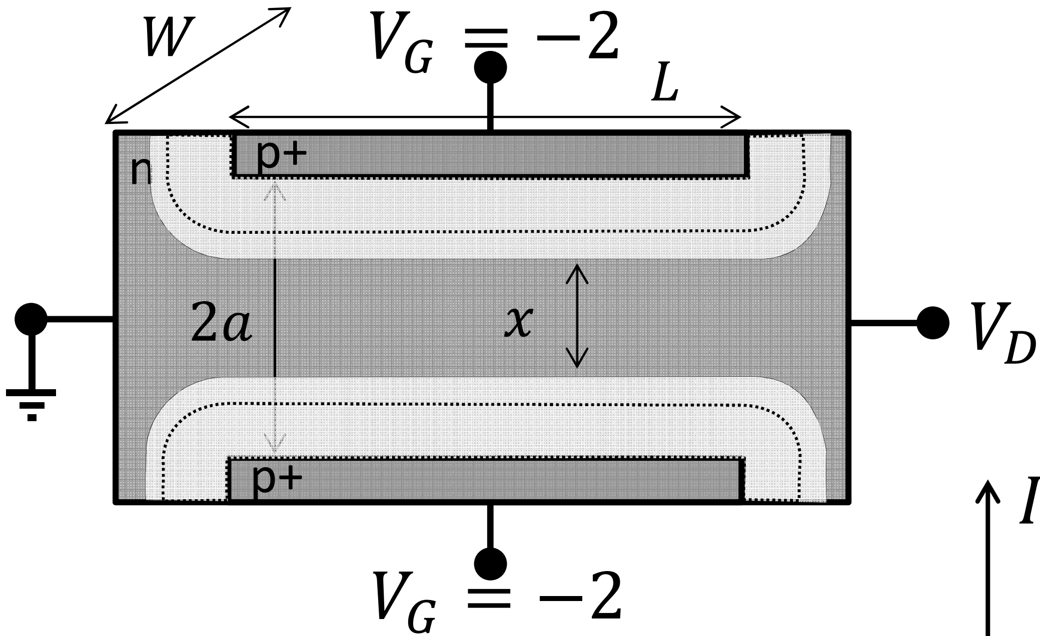
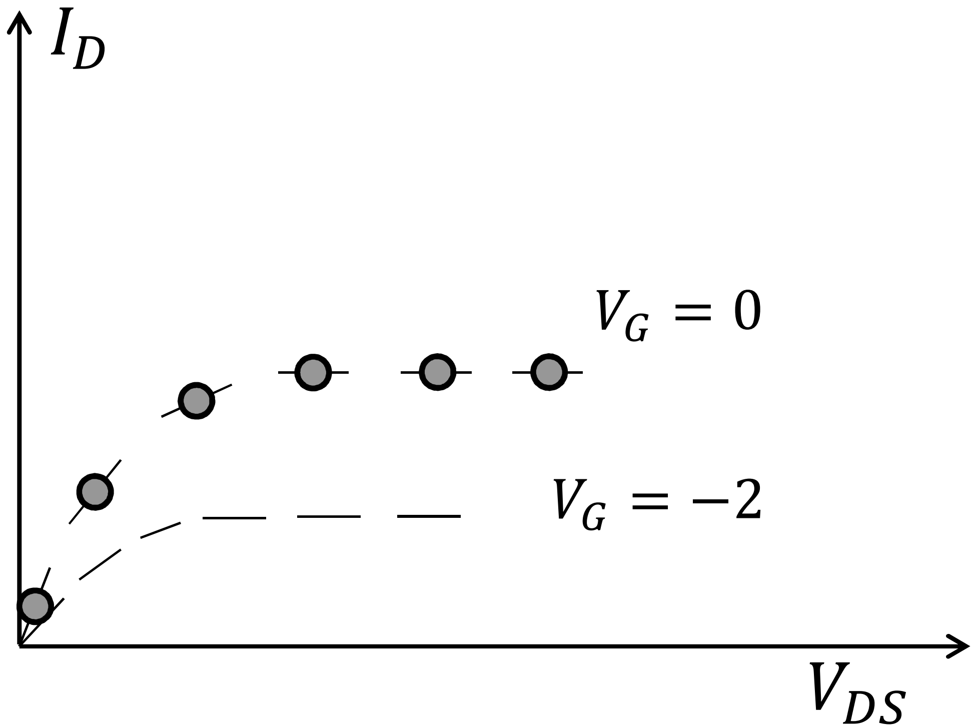
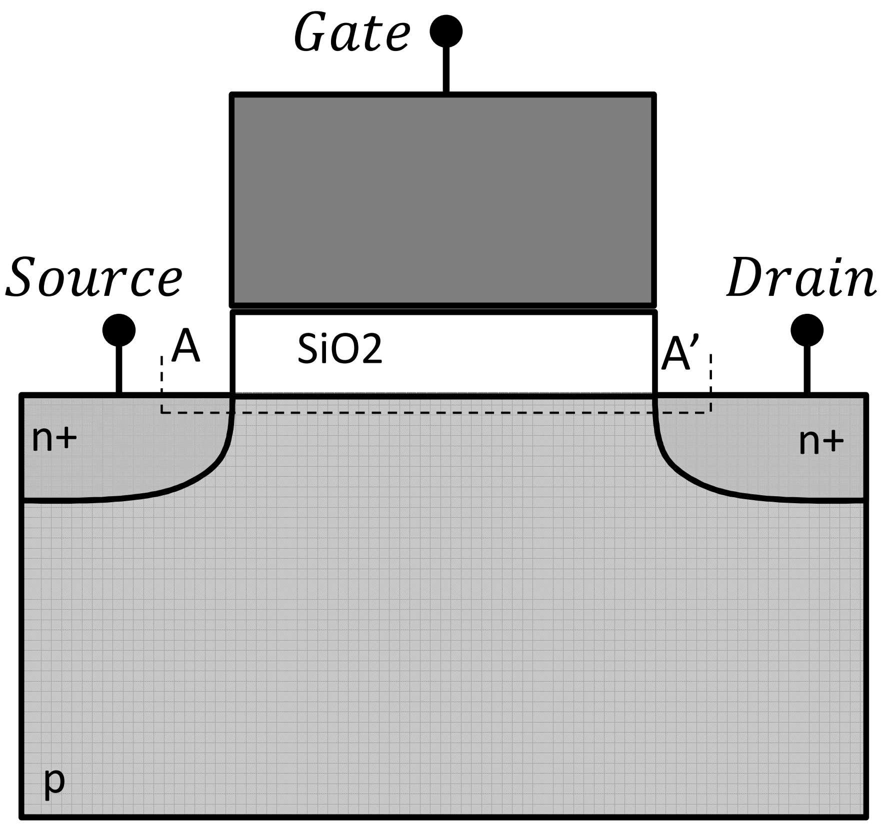
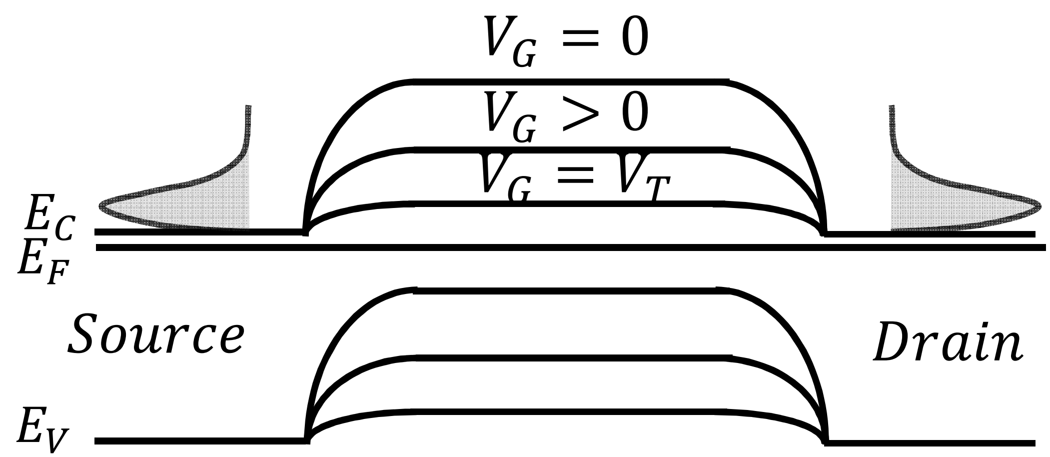


$$R = \rho \frac{L}{Wx}$$



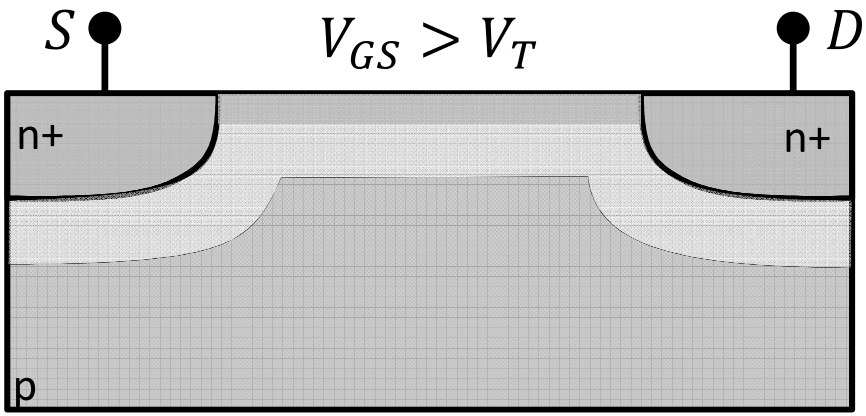
Qualitative Theory of the NMOSFET



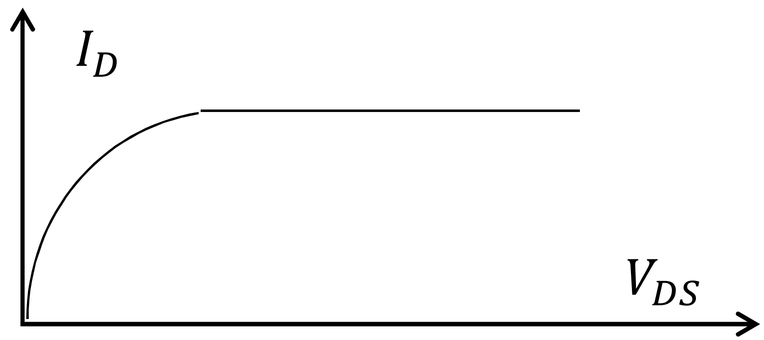


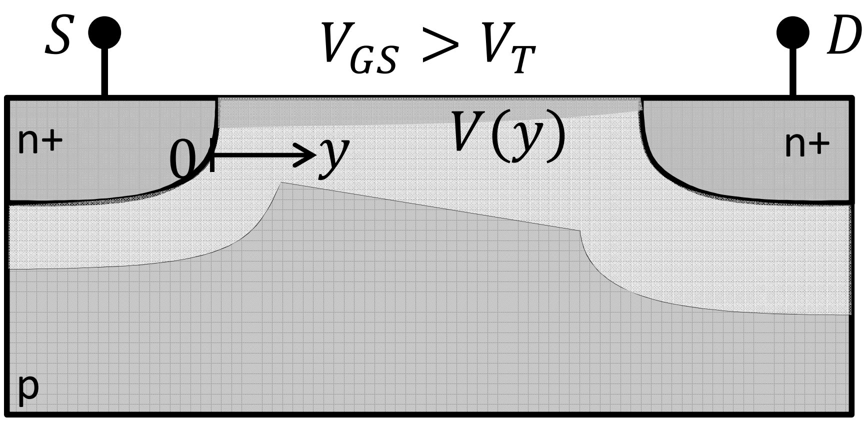
The potential barrier to electron flow from the source into the channel region is lowered by applying $V_{GS} > V_T$

Qualitative Theory of the NMOSFET

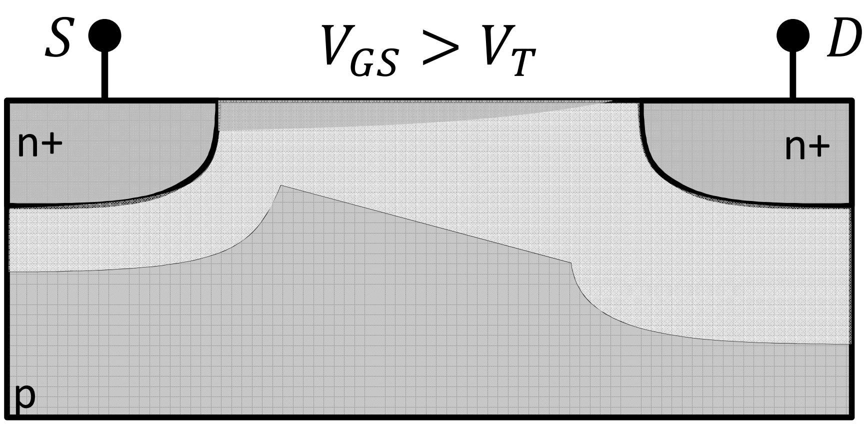


$V_{DS} \approx 0$





$V_{DS} > 0$

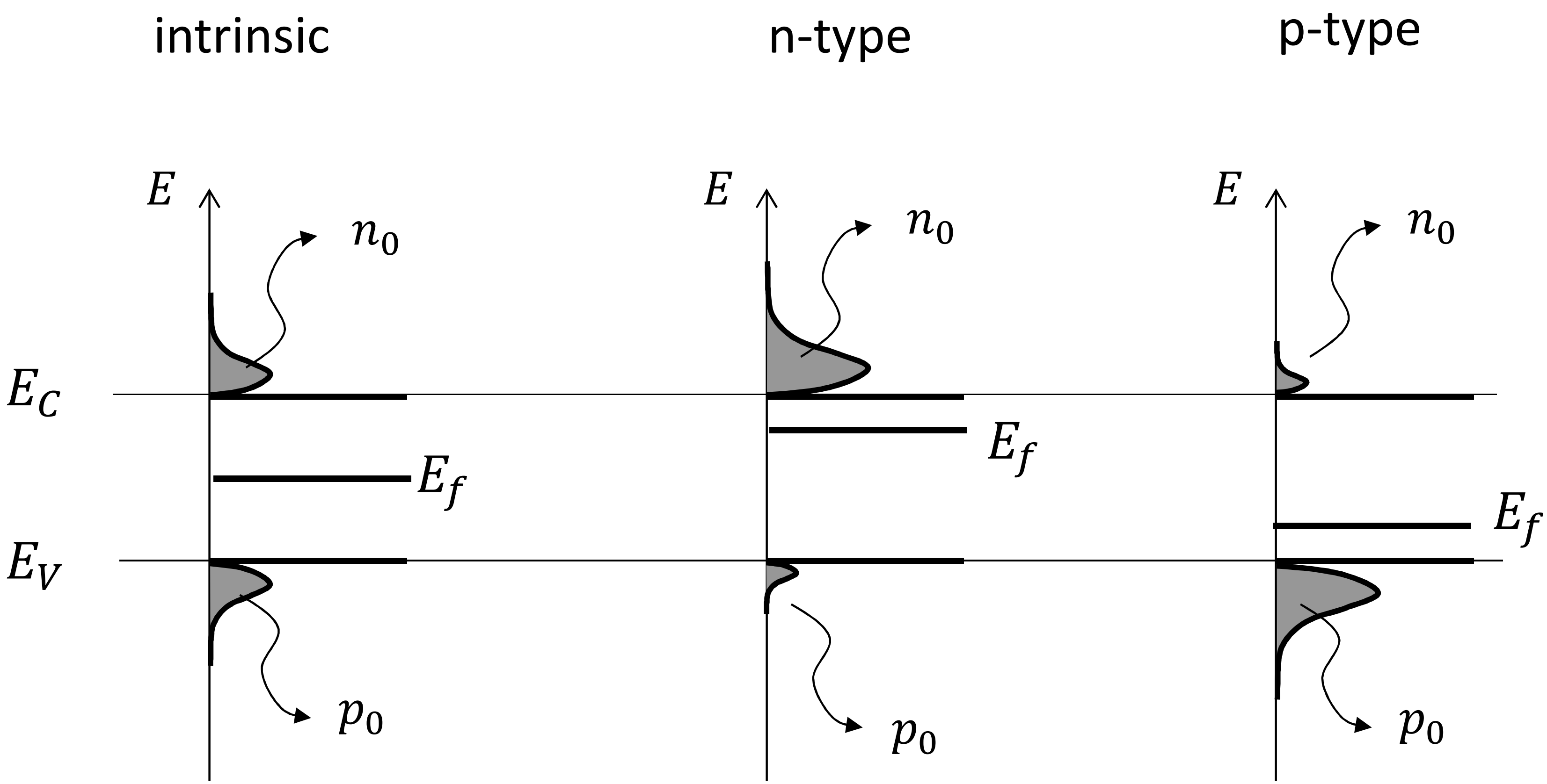


$V_{DS} > V_{GS} - V_T$

$$V_{DS_{sat}} = V_{GS} - V_T$$

Fermi Energy

1. □□□□□□□□
2. □□□□□□□□□□
3. □□□□□□□
4. □□□□
5. □□□□□



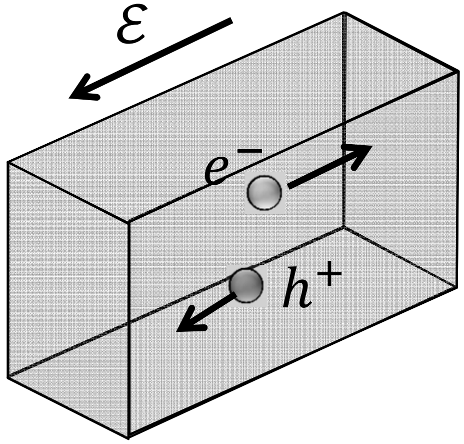
$$n_0 p_0 = n_i^2 \qquad n_{i\_Si} = 5.2 \times 10^{15}\ T^{3/2}\ e^{-E_G/2kT}$$

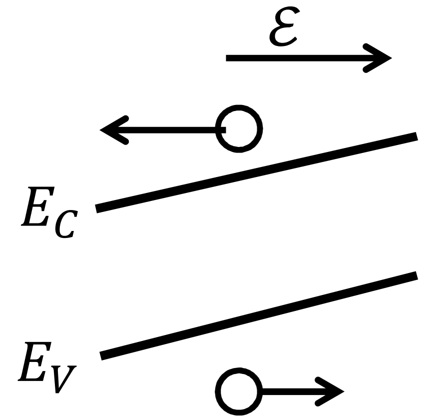
$$n_{i\_Si}(T = 300K) = 1.08 \times 10^{10}\ \ cm^{-3}$$

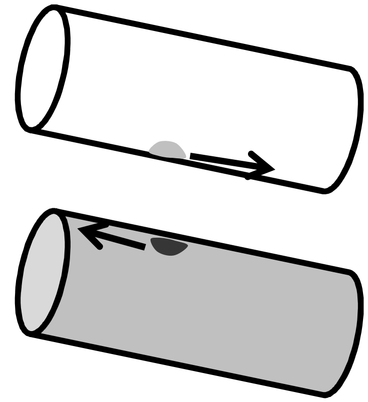
$$n_{i\_Si}(T = 600K) = 1.54 \times 10^{15}\ \ cm^{-3}$$

Drift

Electric field → gravitational field







$$v_e = -\mu_n \mathcal{E}$$

$$v_h = \mu_p \mathcal{E}$$

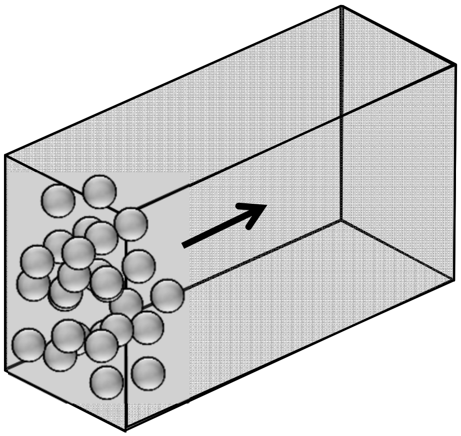
$$J = qn\mu_n\mathcal{E} + qp\mu_p\mathcal{E}$$

$$\mu_n = 1350\ cm^2/Vs$$

$$\mu_p = 480\ cm^2/Vs$$

Velocity Saturation at high fields!

Diffusion



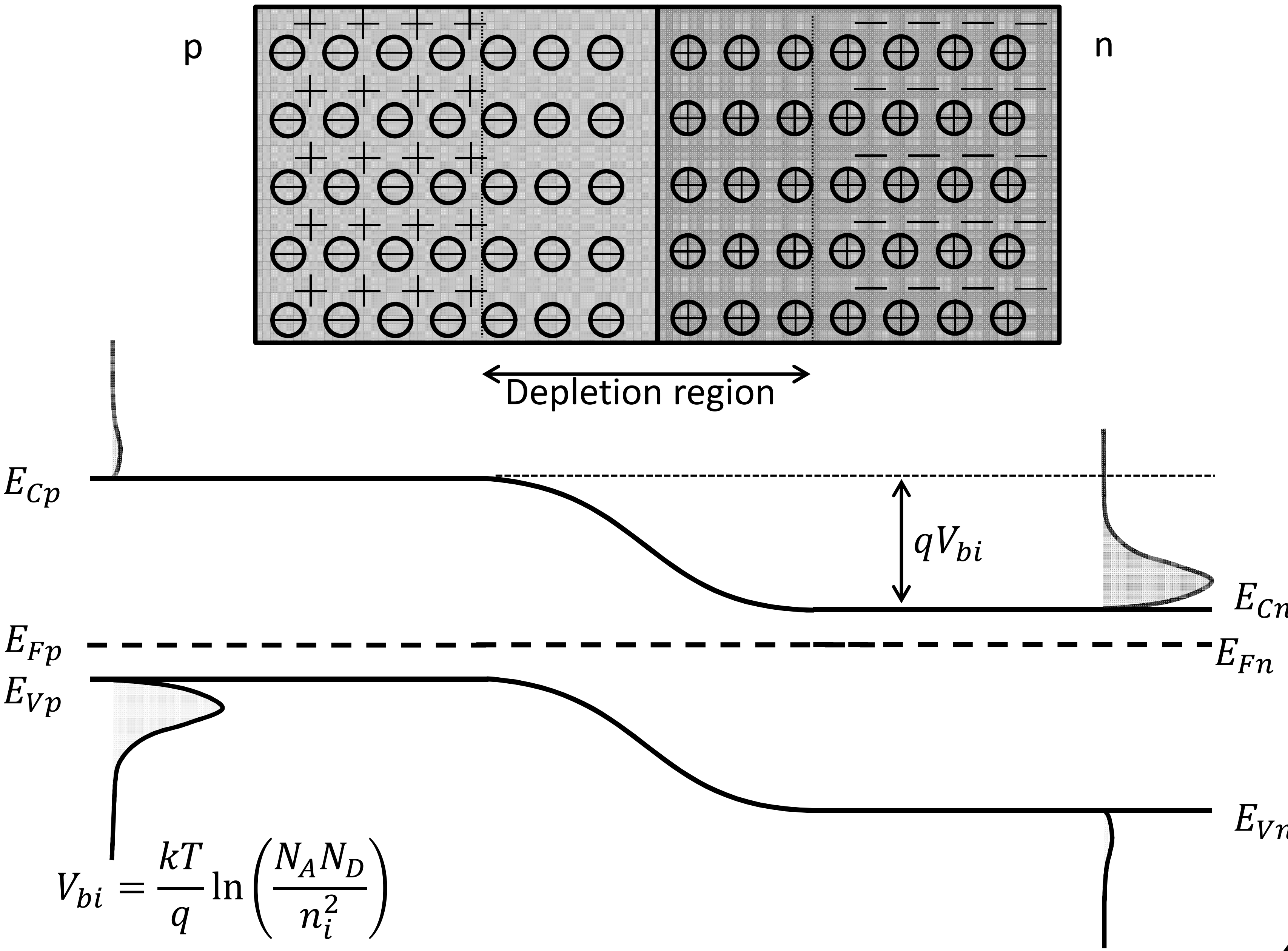
Charges move to be evenly distributed throughout space. Similar to perfume in room or heat in a solid

$$J_n = qD_n \frac{dn}{dx}$$

$$J_p = -qD_p \frac{dp}{dx}$$

Einstein Relation $\frac{D}{\mu} = \frac{kT}{q}$

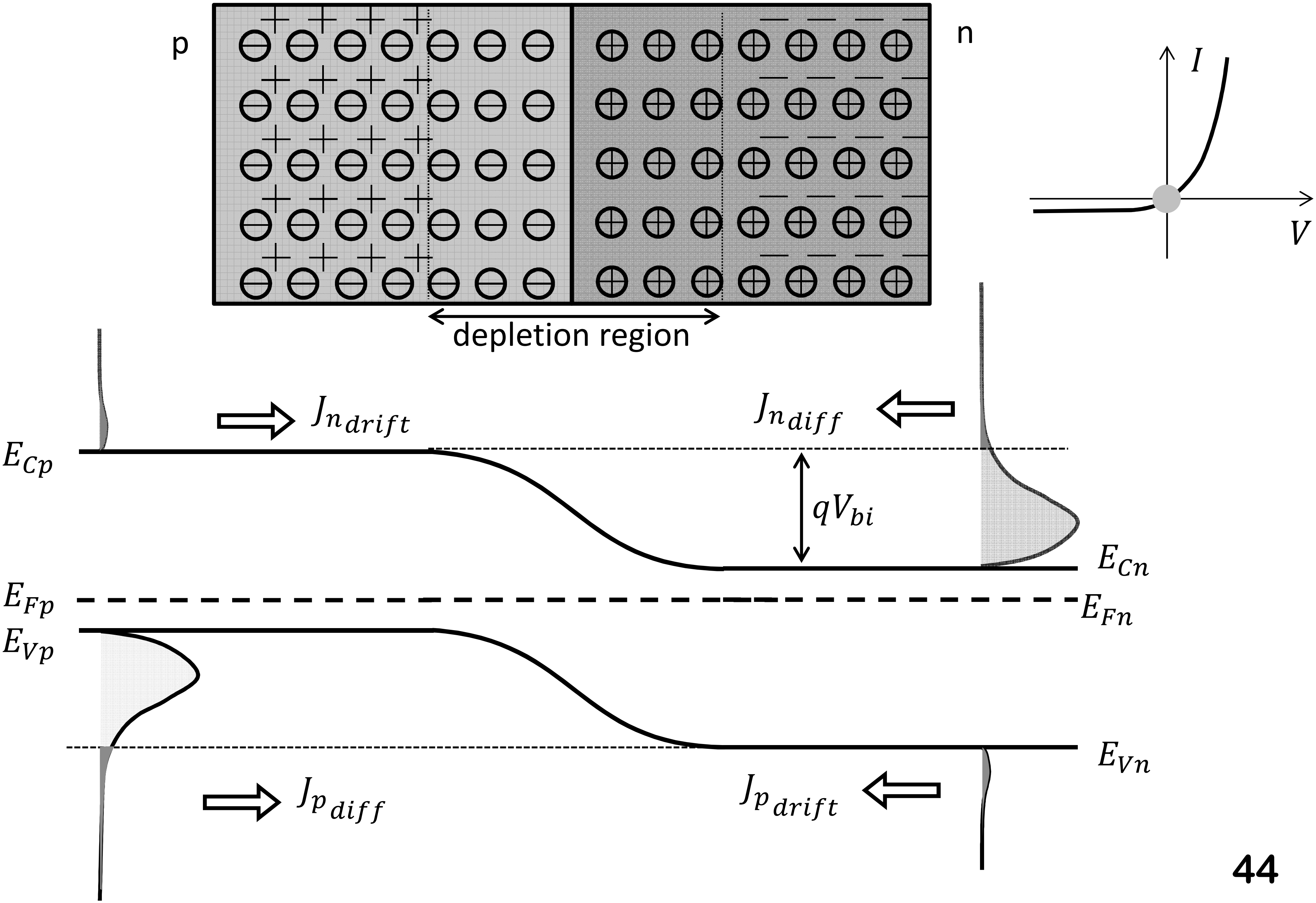
PN junctions



$$V_{bi} = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

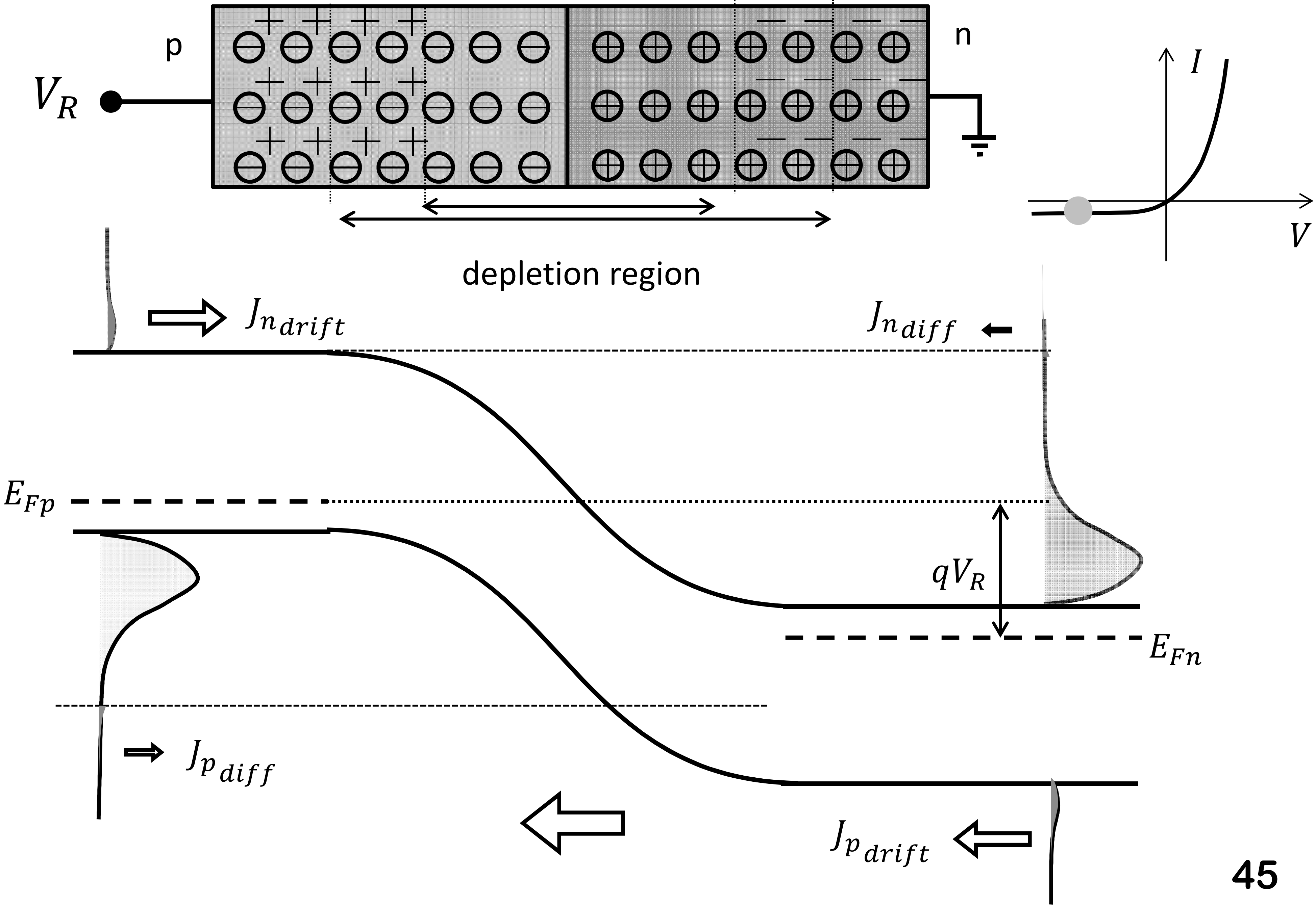
PN junctions

1.
2.
3.
4.
5.



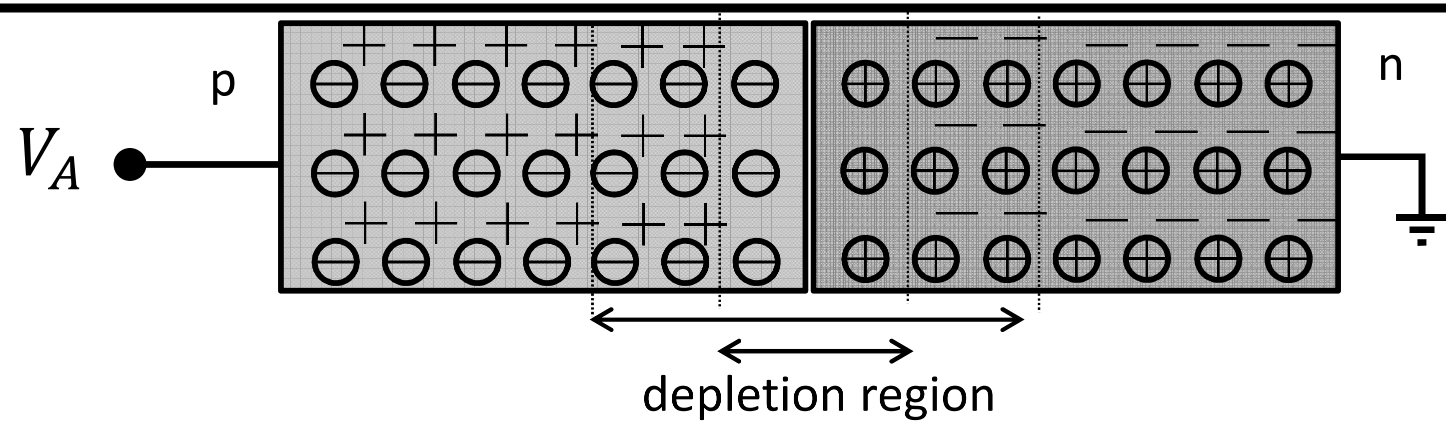
PN junctions , Reverse Biased

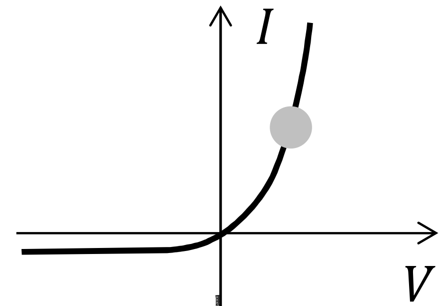
1.
2.
3.
4.
5.



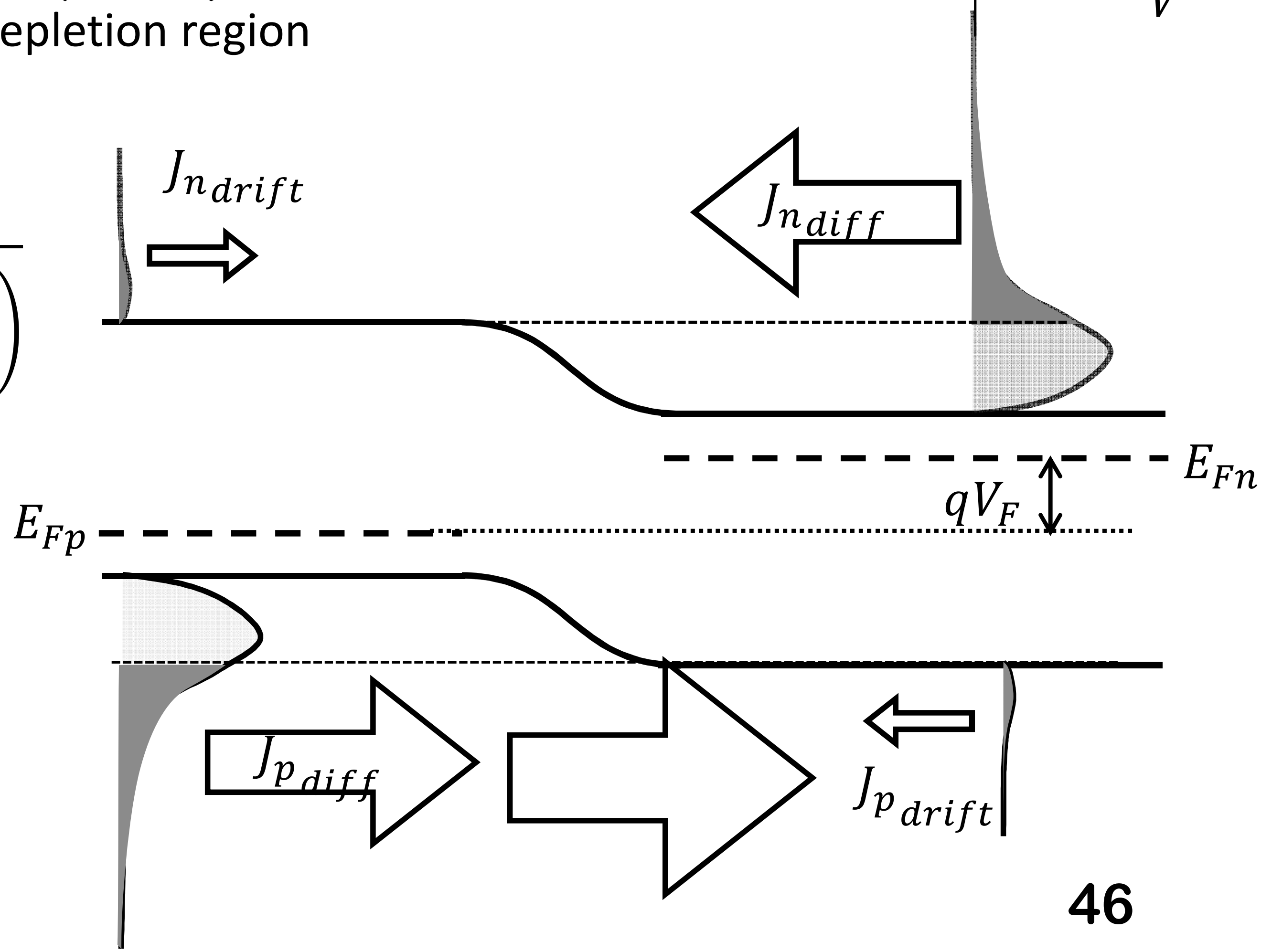
PN junctions , Forward Biased

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]



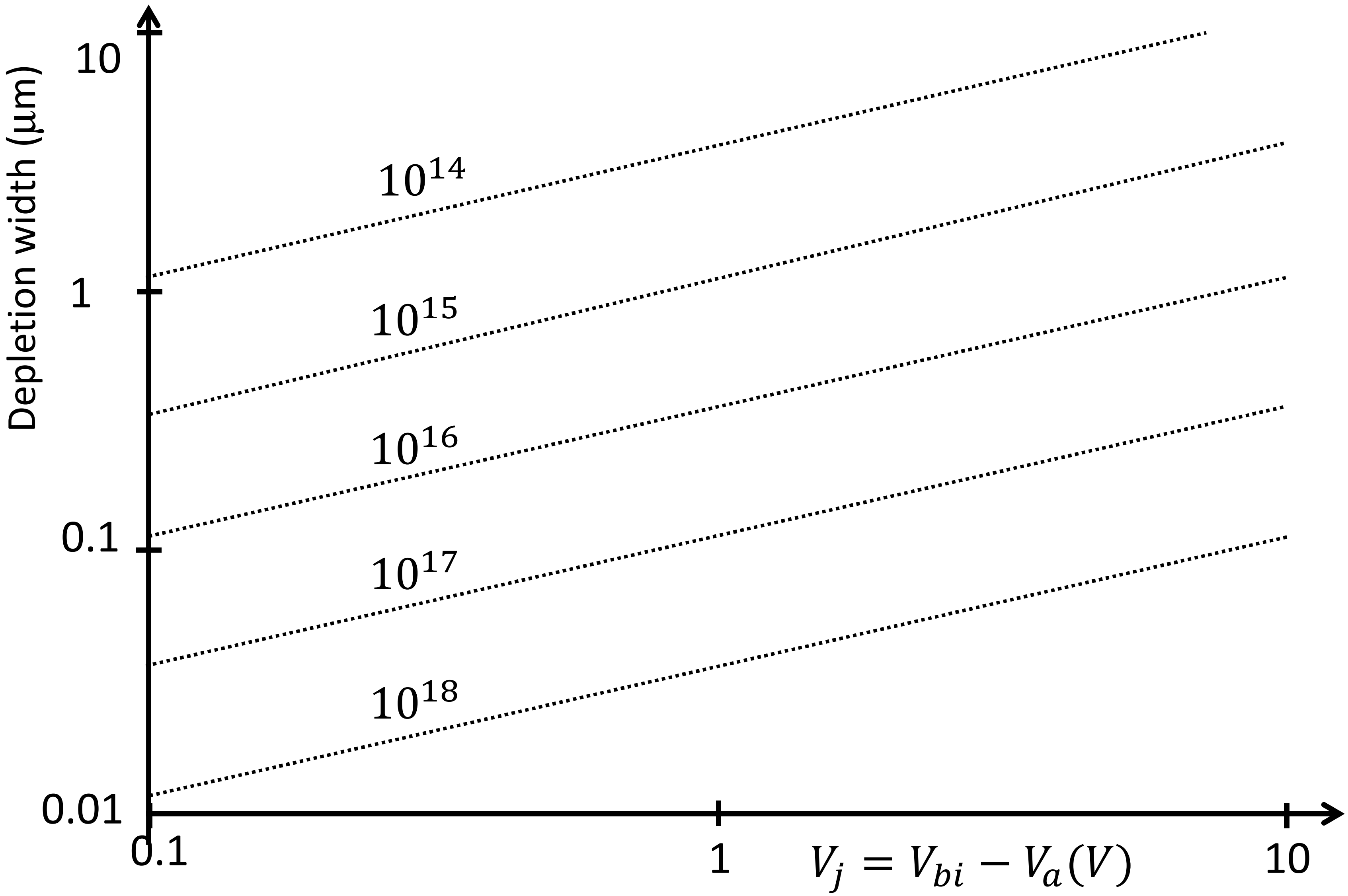


$$W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_A)}{q}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)}$$



W vs. Va

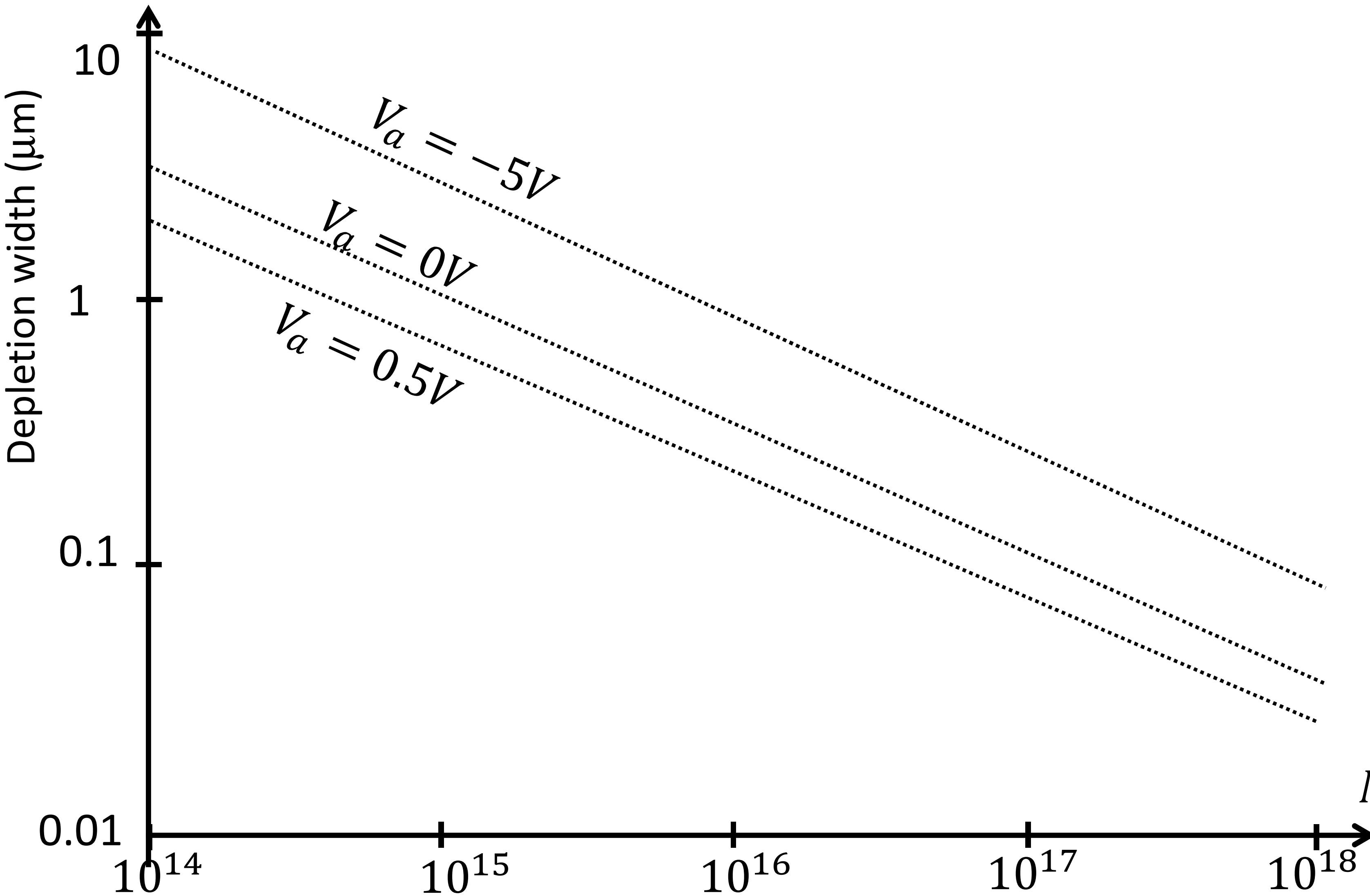
The junction width for one-sided step junctions in silicon as a function of junction voltage with the doping on the lightly doped side as a parameter.



W vs. Na

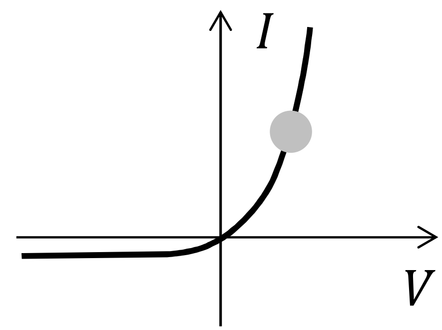
1.
2.
3.
4.
5.

Junction width for a one-sided junction is plotted as a function of doping on the lightly doped side for three different operating voltages.



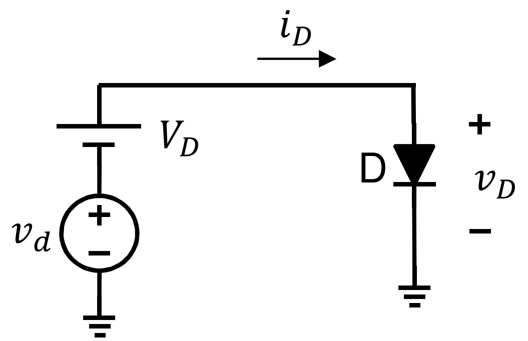
pn Junction: I-V Characteristic

$$I = qA\left(\frac{D_n}{L_n}\frac{n_i^2}{N_A} + \frac{D_p}{L_p}\frac{n_i^2}{N_D}\right)\left(e^{qV/kT} - 1\right)$$
$$= I_s\left(e^{qV/kT} - 1\right)$$

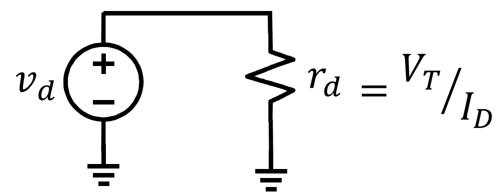


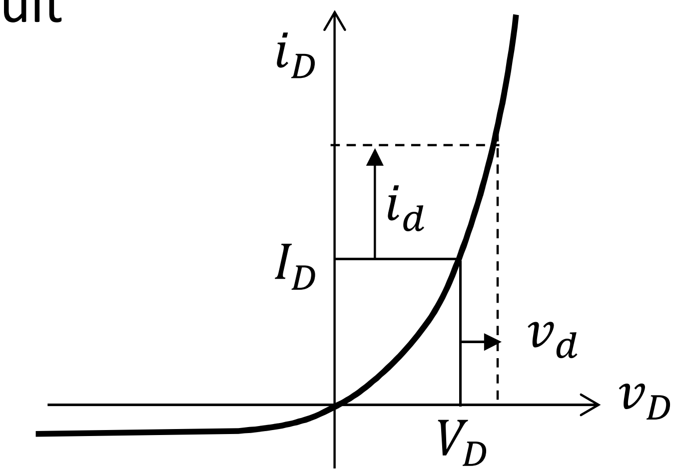
$$\left.\begin{array}{r} T = 300K \\ A = 100\ \mu m^2 \\ Lp = 30\ \mu m. Ln \\ = 20\ \mu m, \end{array}\right\} I_S = 1.77 \times 10^{-17}\ A$$

Small signal model



small signal equivalent circuit





$$i_D = I_s\left(e^{qv_D/kT} - 1\right)$$

$$\cong I_s e^{v_D/V_T}$$

$$= I_s e^{v_D/V_T}\ e^{\hat{v}_i \sin \omega t/V_T}$$

$$= I_s e^{v_D/V_T}\left(1 + \frac{\hat{v}_i \sin \omega t}{V_T} + \cdots\right)$$

$$\cong I_s e^{v_D/V_T}\left(1 + \frac{\hat{v}_i \sin \omega t}{V_T}\right)$$

$$I_D + i_d \quad = I_s e^{v_D/V_T}\left(1 + \frac{v_d}{V_T}\right)$$

$$i_d = I_D \frac{v_d}{V_T}$$

$$kT/q = V_T$$

$$i_D = I_D + i_d$$

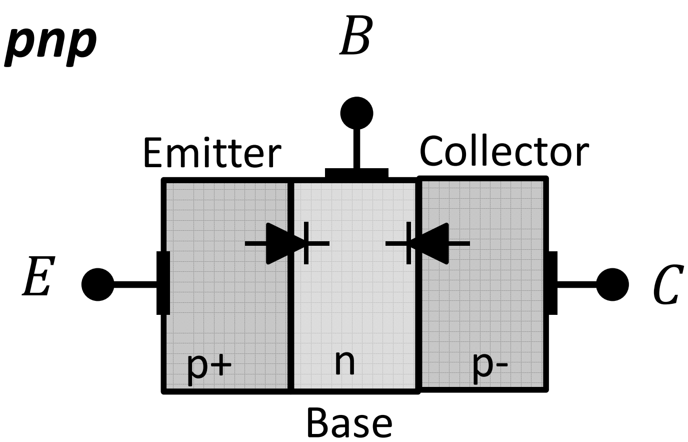
$$v_D = V_D + v_d \quad = V_D + \hat{v}_i \sin \omega t$$

BIG IFFFF! $\hat{v}_i \ll V_T$

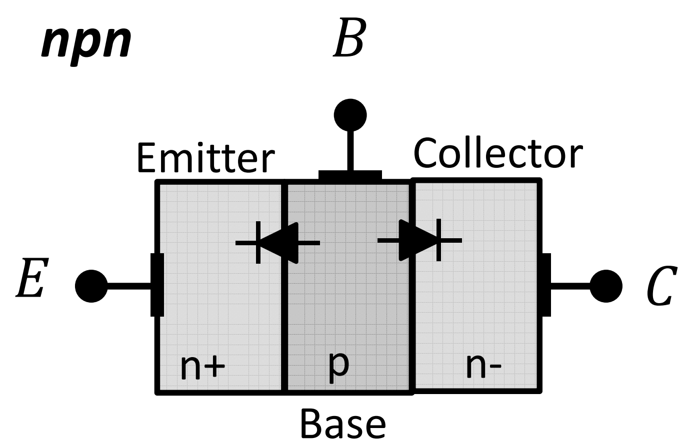
BJT Types and Definitions

The BJT is a 3-terminal device, with two types: PNP and NPN

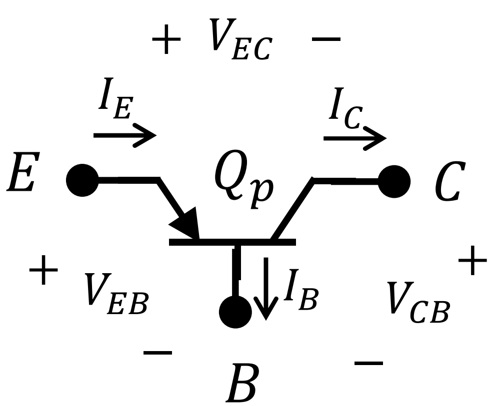
pnp

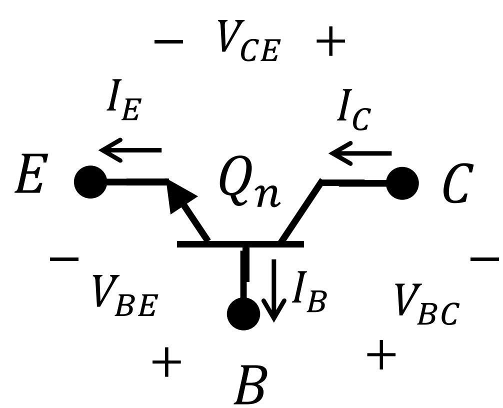


npn



Asymmetric!

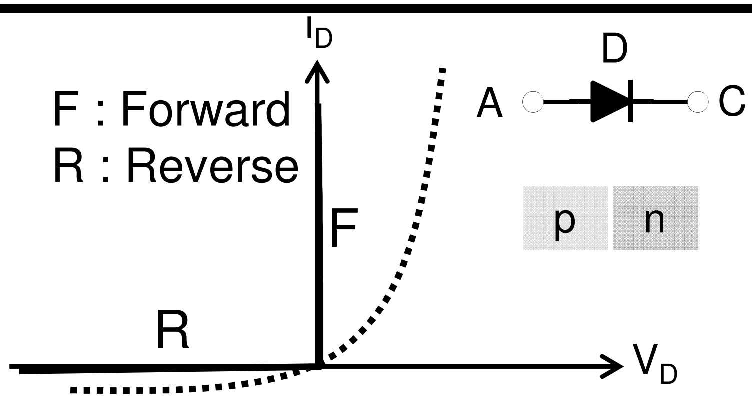


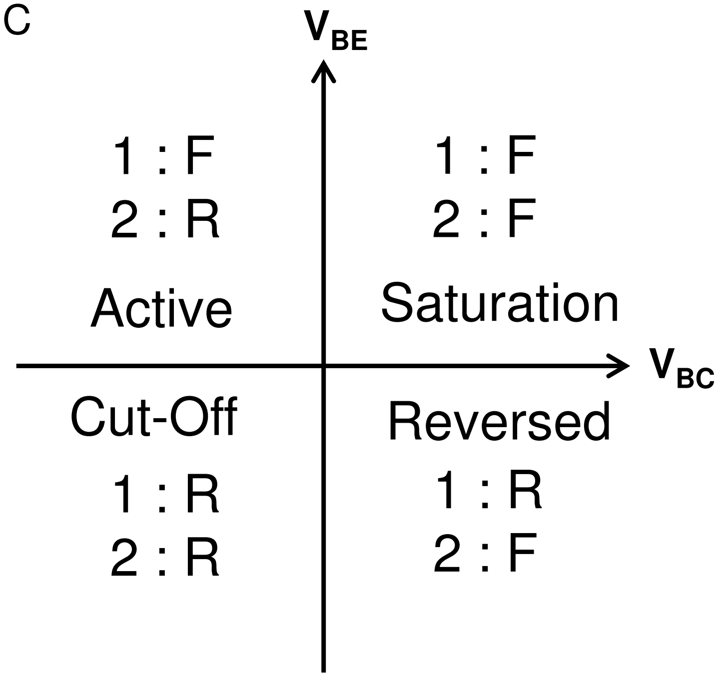


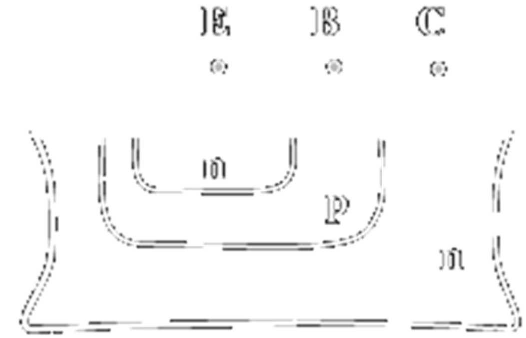
BJT: Bipolar Junction Transistor

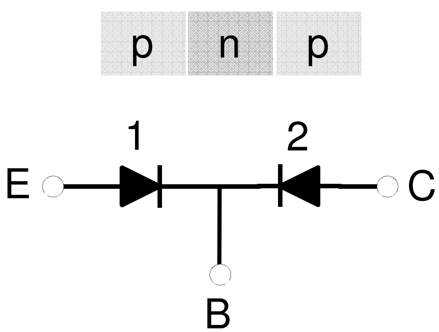
1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]

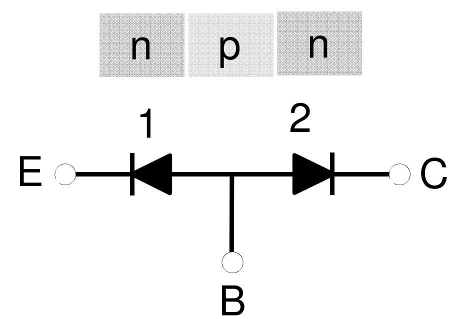
F : Forward
R : Reverse

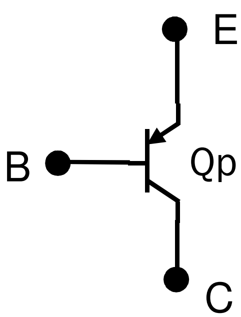


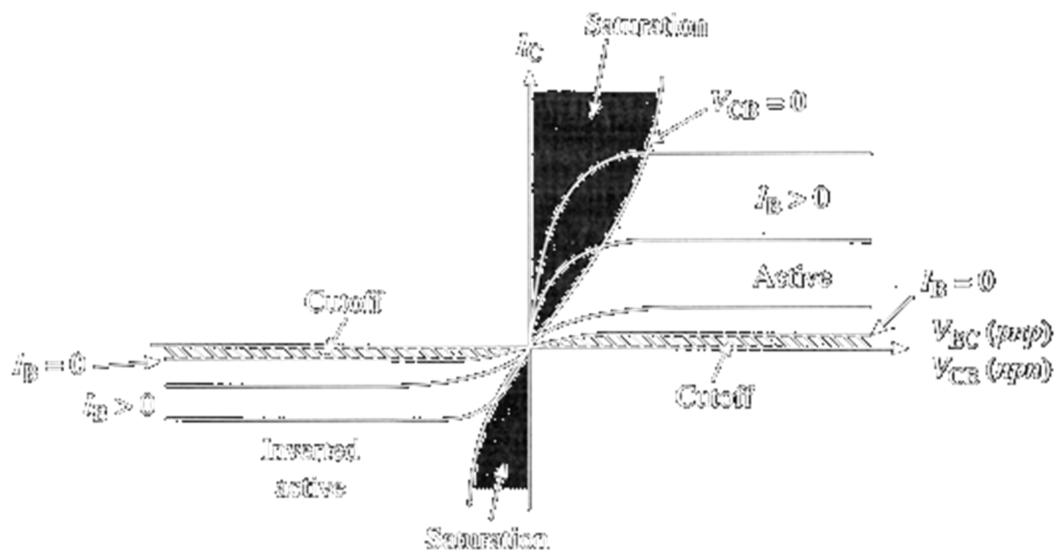


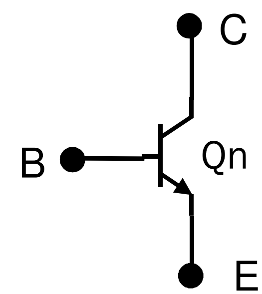






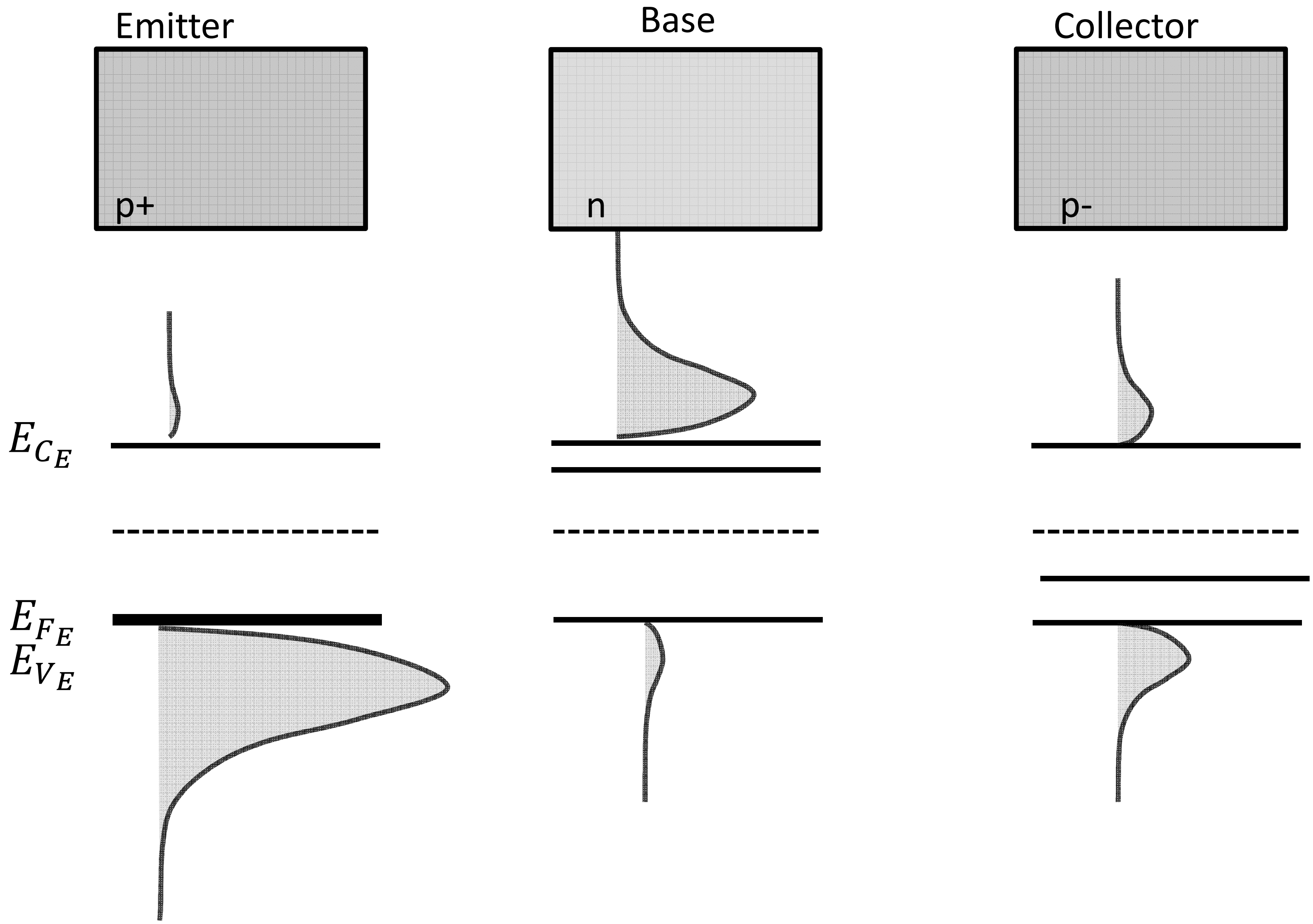






BJT Electrostatics

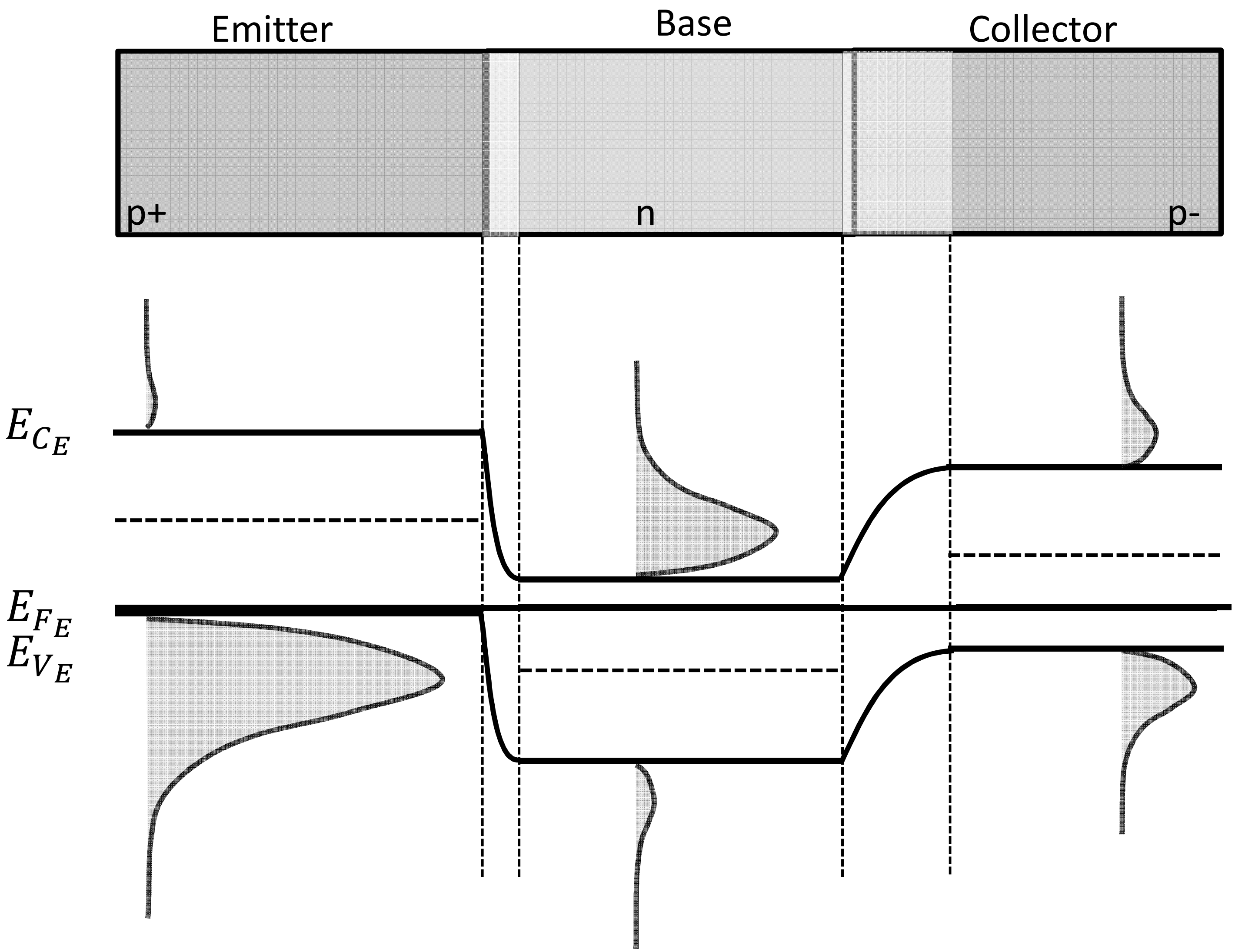
pnp



Under normal operating conditions, the BJT may be viewed electrostatically as two independent pn junctions

BJT Electrostatics

pnp

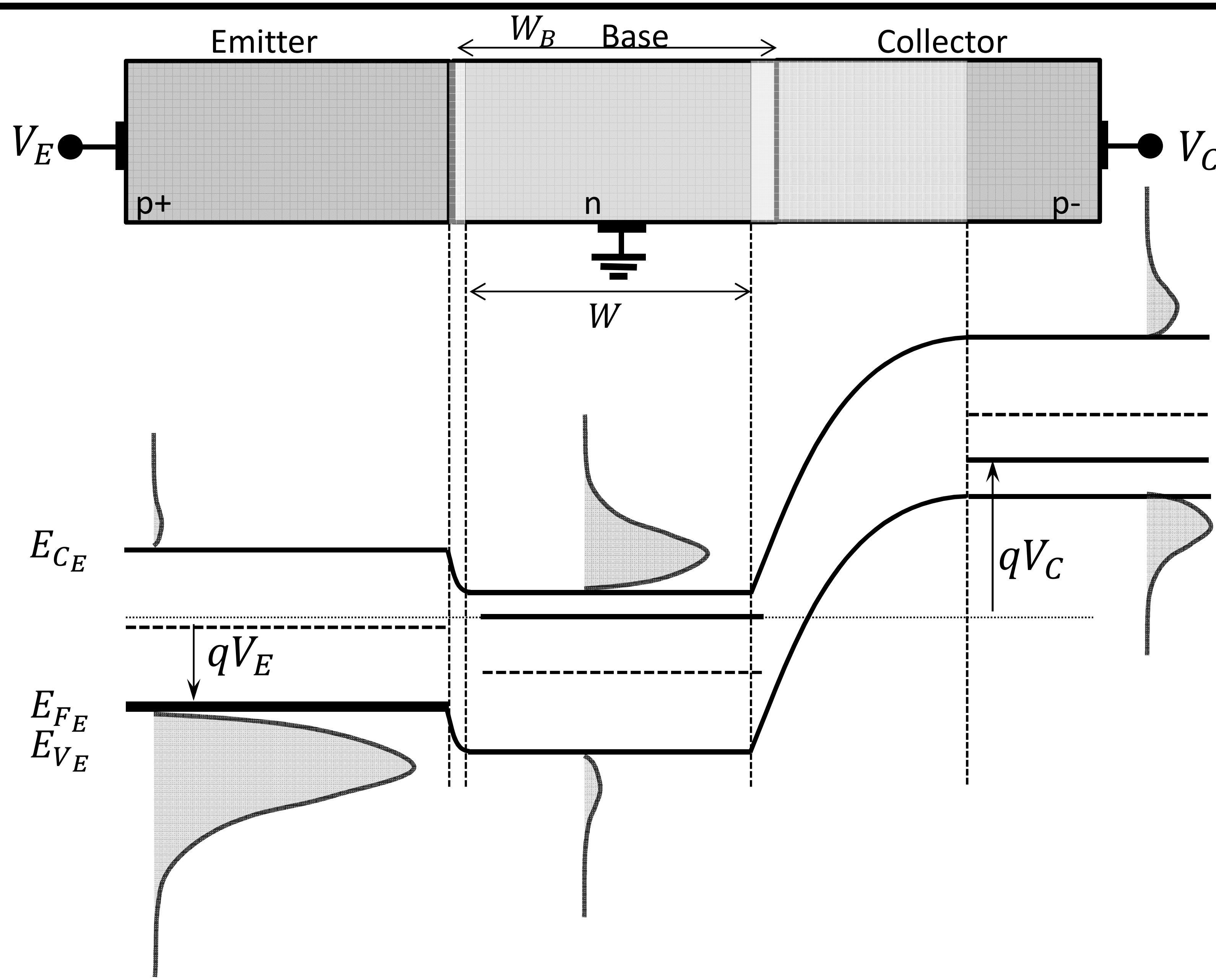


Under normal operating conditions, the BJT may be viewed electrostatically as two independent pn junctions

BJT Electrostatics

1.
2.
3.
4.
5.

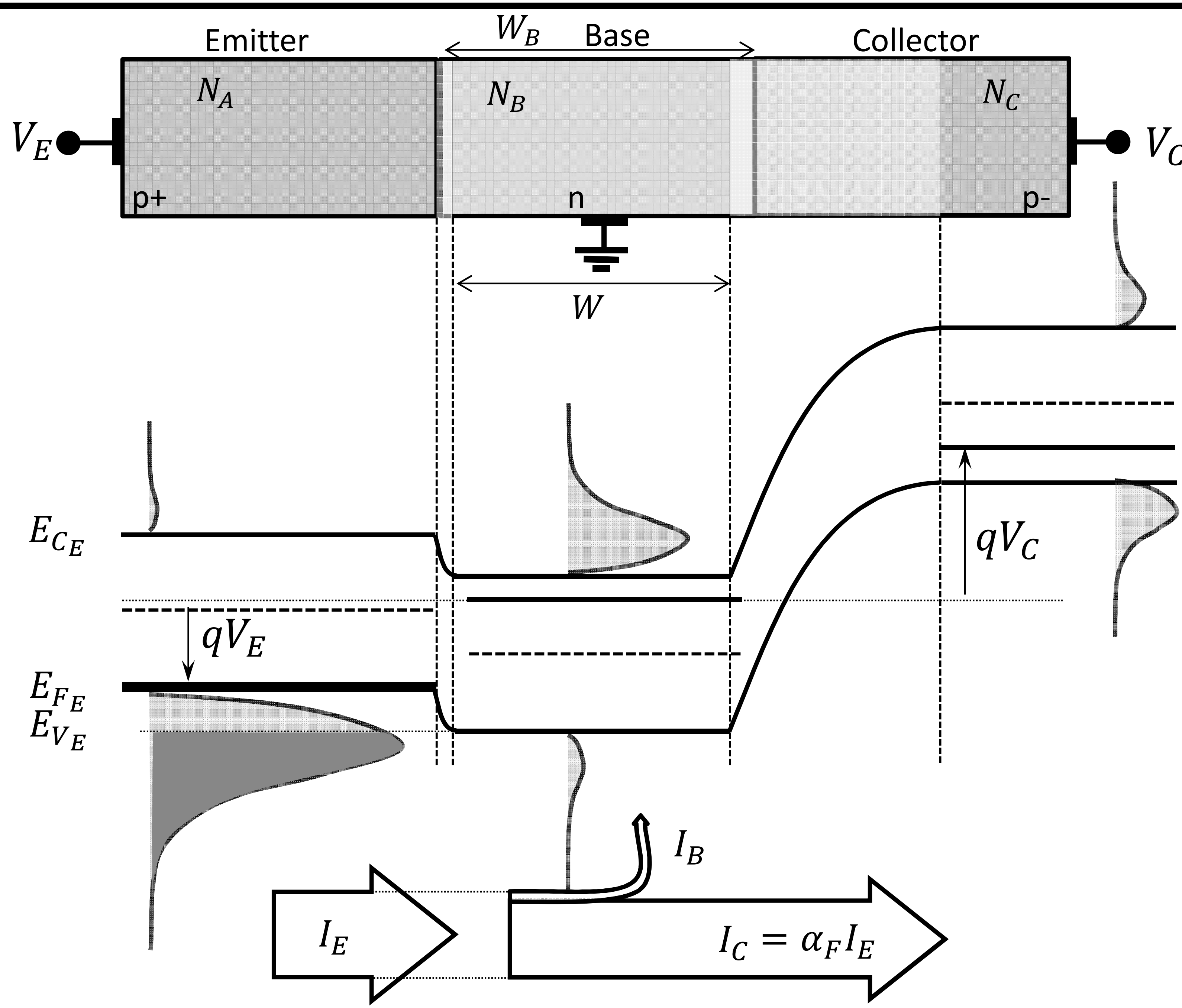
pnp



BJT Electrostatics

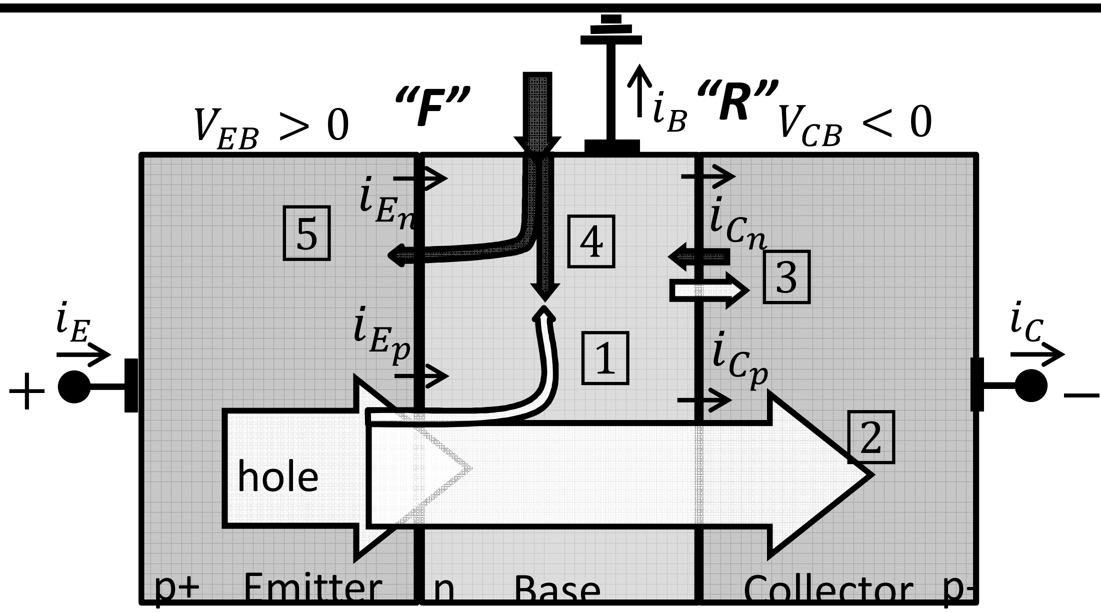
1.
2.
3.
4.
5.

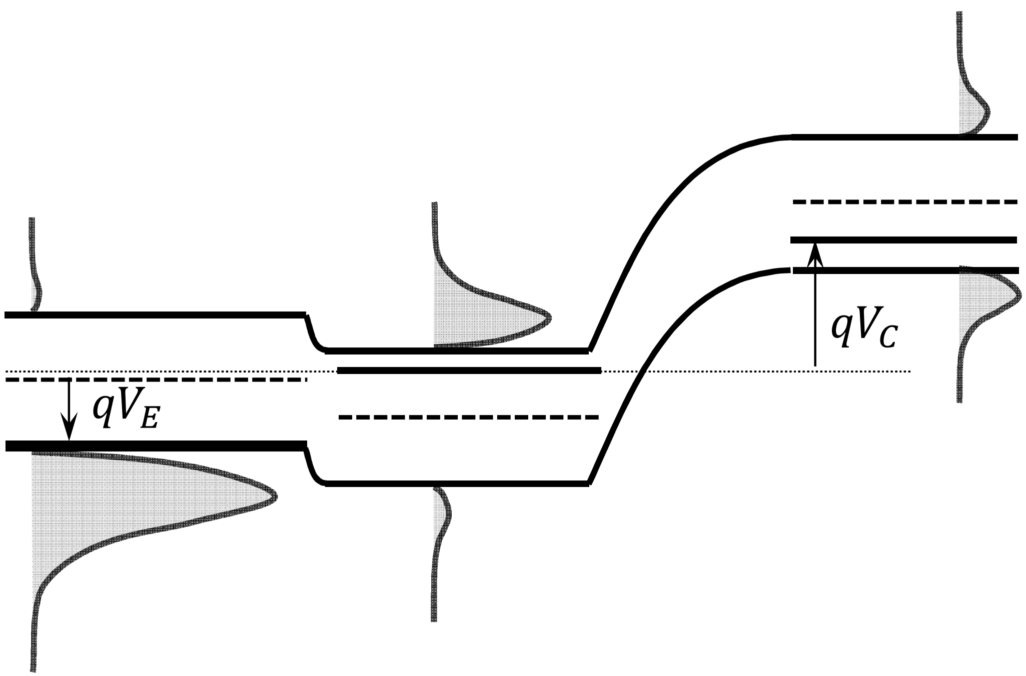
pnp



Collector Current (PNP)

1. □□□□□□□□
2. □□□□□□□□□□
3. □□□□□□□
4. □□□□
5. □□□□□





The collector current is comprised of:

2 Holes injected from emitter, which do not recombine in the base

3 Reverse saturation current of collector junction

$$I_C = \alpha_{dc} I_E + I_{CBO}$$

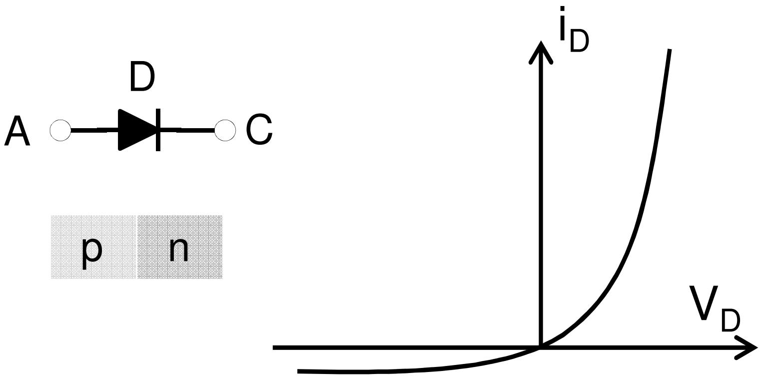
where I_{CBO} is the collector current which flows when $I_E = 0$

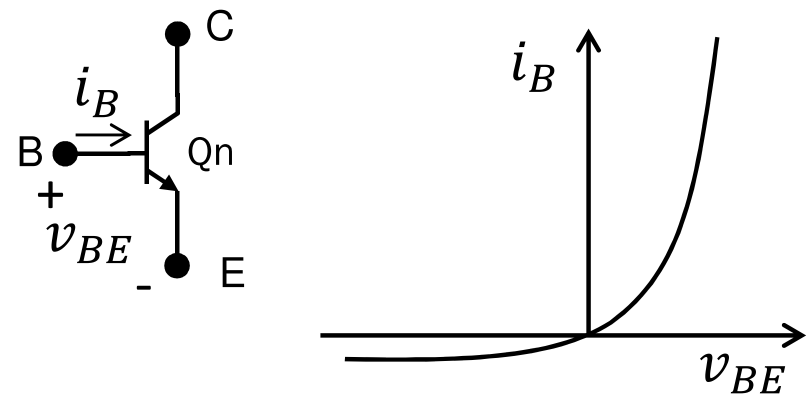
$$I_C = \alpha_{dc}(I_C + I_B) + I_{CBO}$$

$$I_C = \frac{\alpha_{dc}}{1-\alpha_{dc}} I_B + \frac{I_{CBO}}{1-\alpha_{dc}}$$

$$= \beta I_B + I_{CEO}$$

BJT: Bipolar Junction Transistor





$$i_D \propto e^{v_D/nV_T}$$

$$\frac{\Delta v_D}{\Delta T} \approx -2\frac{mV}{K}\bigg|_{i_D=cte}$$

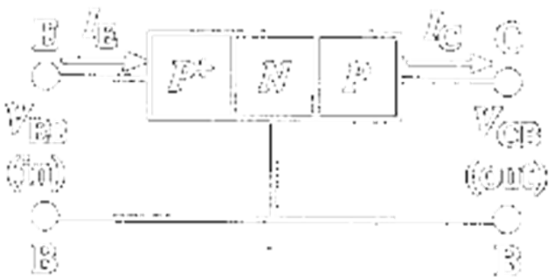
$$\frac{i_D(T_2)}{i_D(T_1)} \approx 2^{(T_2-T_1)/10K}$$

$$i_B \propto e^{v_{BE}/nV_T}$$

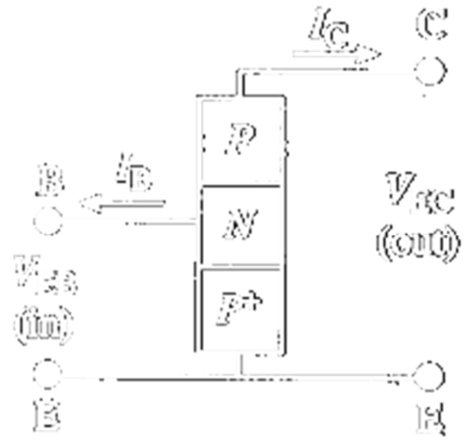
$$\frac{\Delta v_{BE}}{\Delta T} \approx -2\frac{mV}{K}\bigg|_{i_E=cte}$$

$$\frac{i_C(T_2)}{i_C(T_1)} \approx 2^{(T_2-T_1)/10K}$$

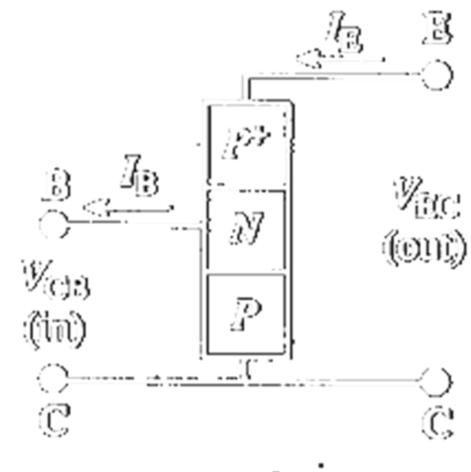
BJT Circuit Configurations



CB: Common Base

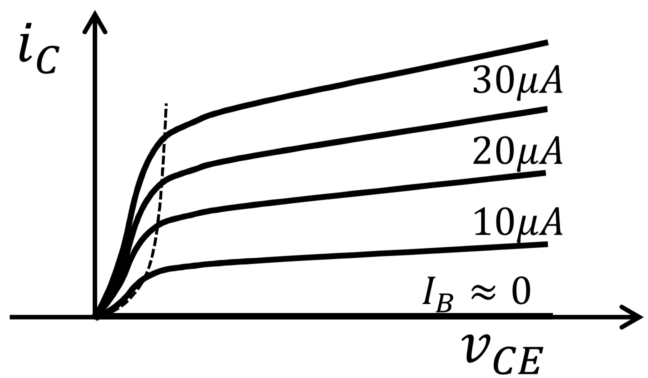


CE: Common Emitter



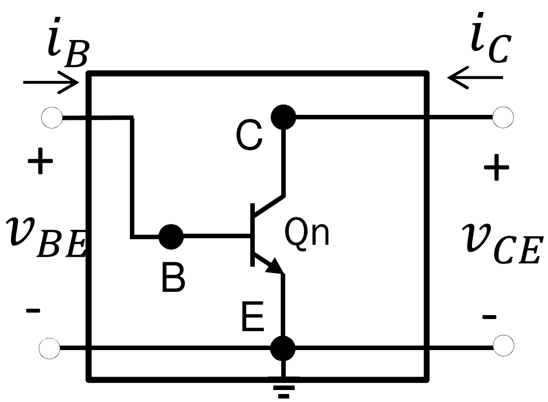
CC: Common Collector

Output Characteristics for Common-Emitter Configuration

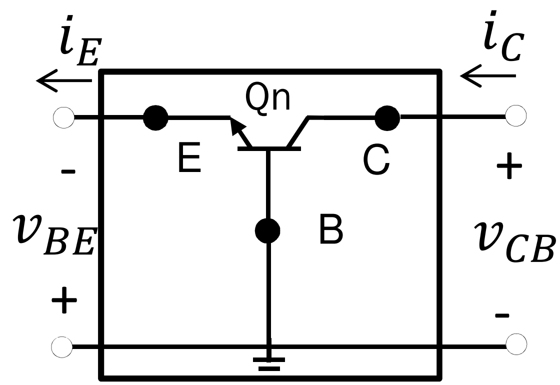


BJT Configurations

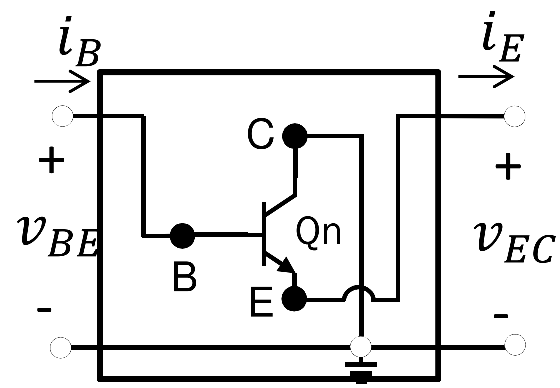
CE: Common Emitter



CB: Common Base



CC: Common Collector



Input Characteristic

i_B vs. v_{BE} for different V_{CE}

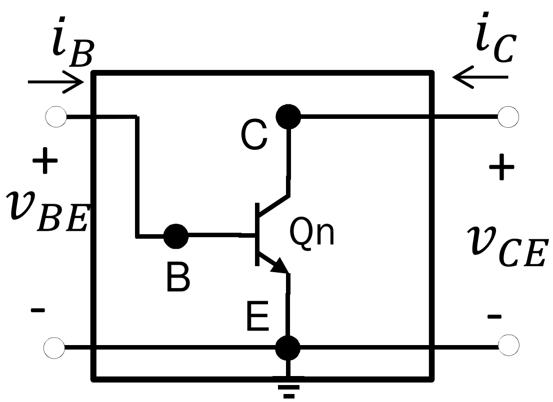
Output Characteristic

i_C vs. v_{CE} for different V_{BE} or i_B

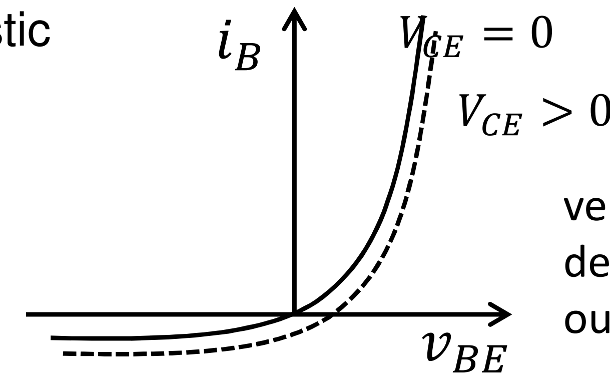
Transfer Characteristic

i_C vs. v_{BE} or i_B for different V_{CE}

CE: Common Emitter

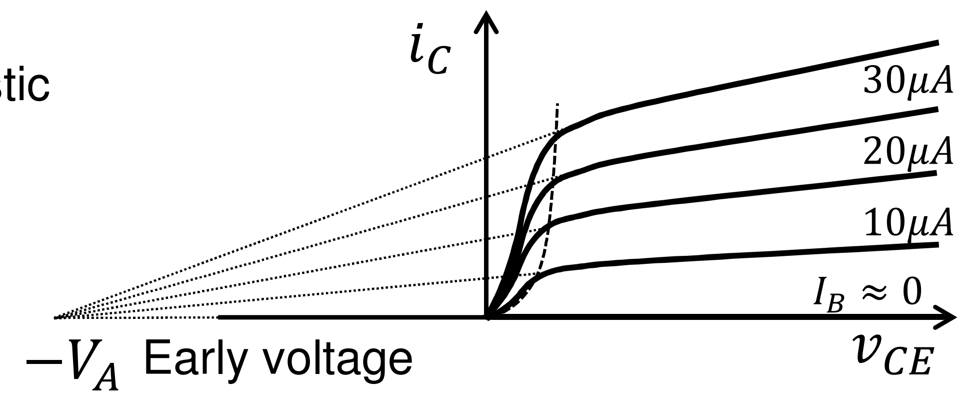


Input Characteristic



very little dependence to output voltage

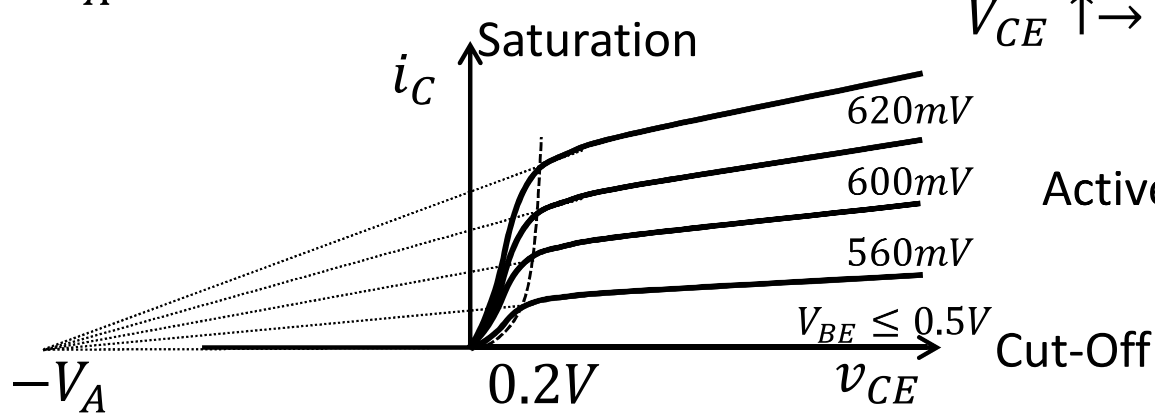
Output Characteristic



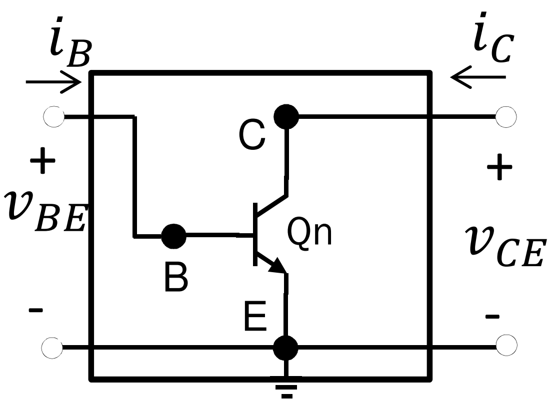
Active:

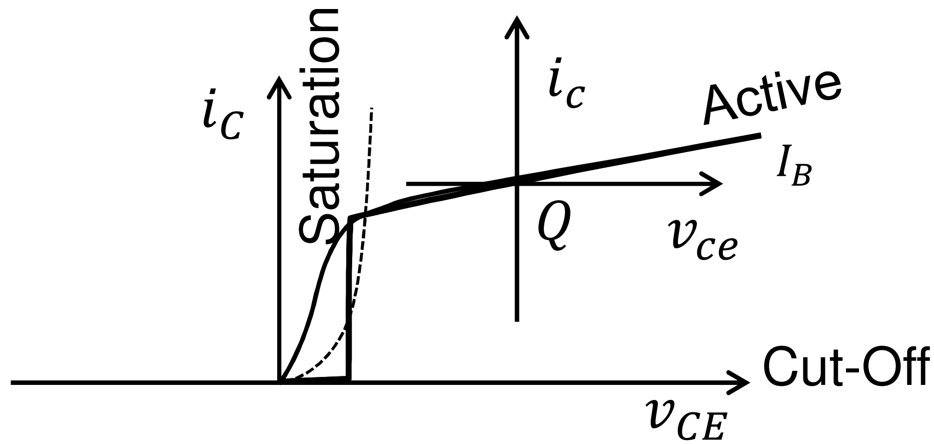
$$i_C = I_S e^{\frac{v_{BE}}{nV_T}}\left(1 + \frac{v_{CE}}{V_A}\right)$$

$$V_{CE} \uparrow \to W \uparrow \to \beta \uparrow \to I_C \uparrow$$

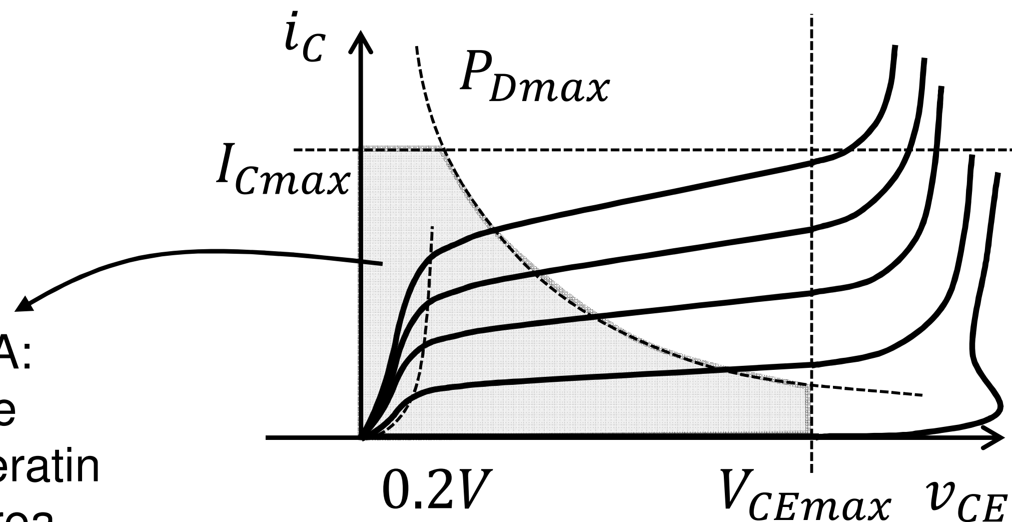


CE: Transistor Model



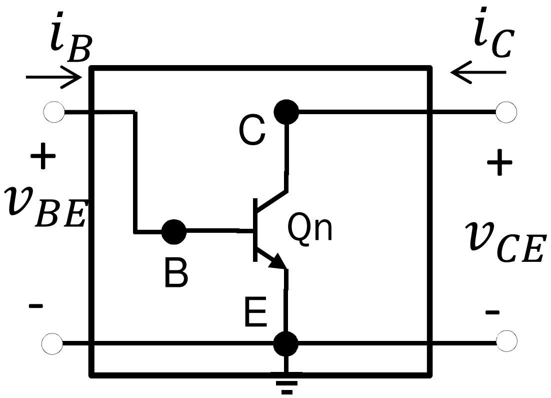


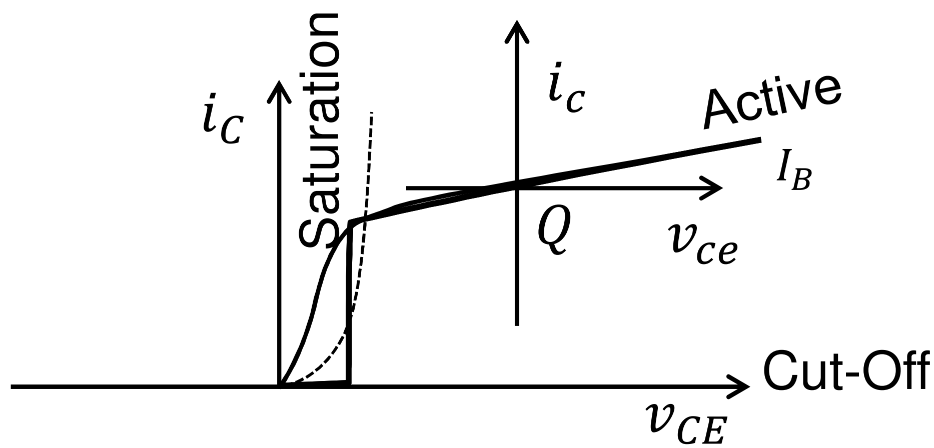
Output Characteristic



SOA:
Safe
Operatin
g Area

CE: Output Characteristic





$$i_B = I'_S e^{\frac{v_{BE}}{nV_T}}(1 + \frac{v_{CE}}{V_A})$$

$$i_C \approx I_S e^{\frac{v_{BE}}{nV_T}}(1 + \frac{v_{CE}}{V_A})$$

$$i_B = \frac{I_S}{\beta} e^{\frac{v_{BE}}{nV_T}}(1 + \frac{v_{CE}}{V_A})$$

Ideally linear: $\beta = \frac{i_C}{i_B} = \frac{I_C}{I_B} = \frac{i_c}{i_b}$

$$BDC = \beta_{DC} = \left.\frac{i_C}{i_B}\right|_{I_C=I_Q} = \beta_F(1 + \frac{v_{CE}}{V_A})$$

SPICE $\quad BF = \beta_F = \left.\frac{i_C}{i_B}\right|_{V_{CB}=0}$

$$BAC = \left.\frac{\partial i_C}{\partial i_B}\right|_{I_C=I_Q} = \frac{i_c}{i_b}$$

CE: Common Emitter

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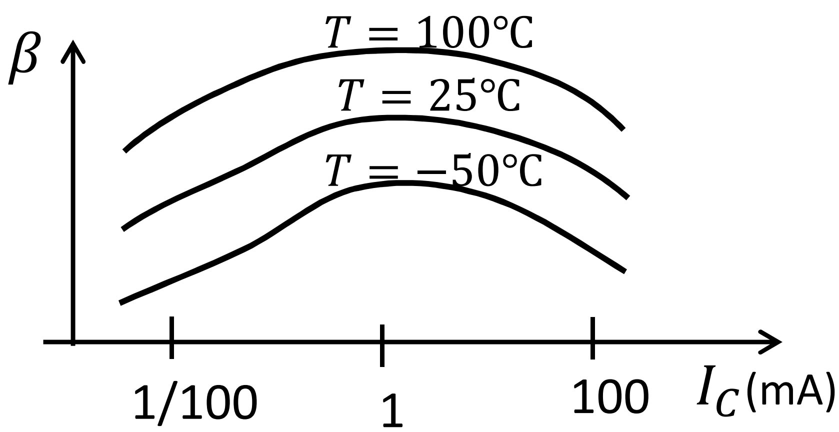
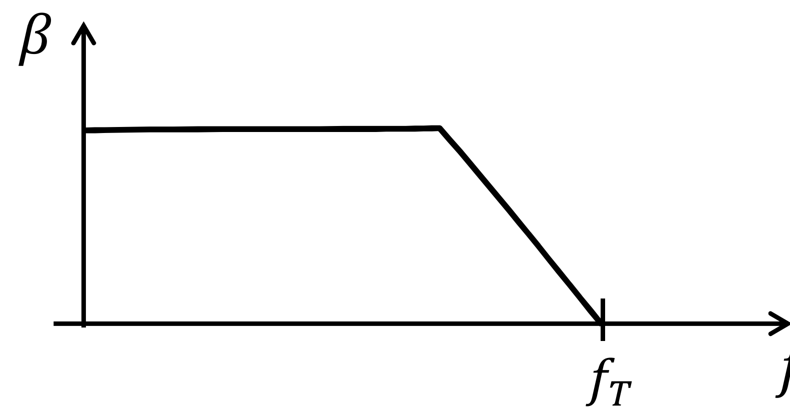
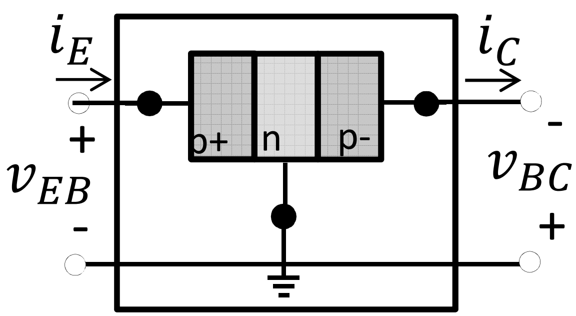
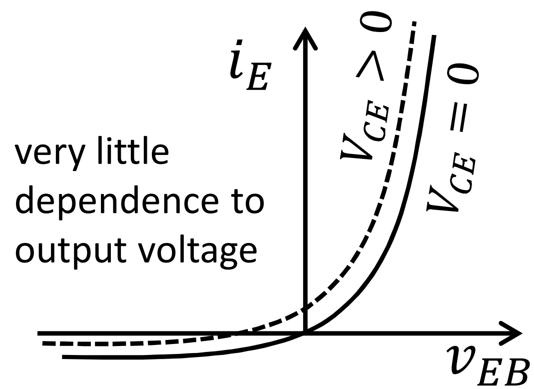




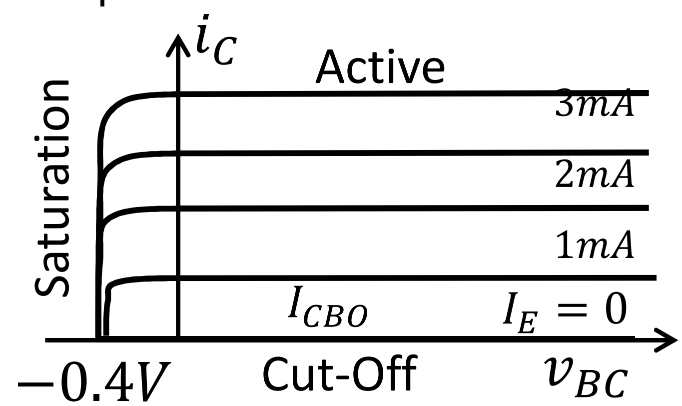
CB: Common Base



Input Characteristic



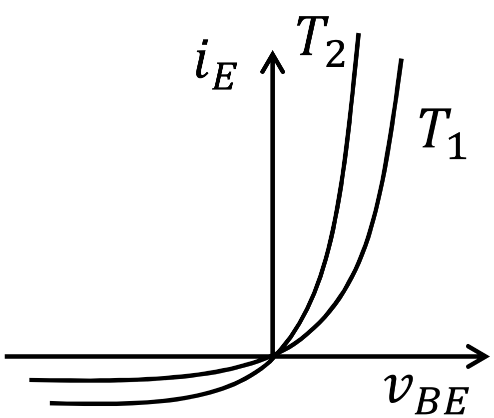
Output Characteristic



Active:

$$i_C = \alpha i_E$$

$$\alpha = \frac{\beta}{\beta + 1}$$

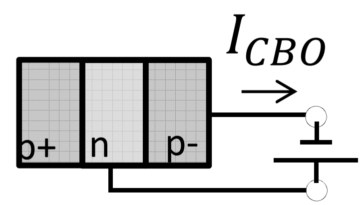


$T_2 > T_1$

$$\frac{\Delta v_{BE}}{\Delta T} \approx -2\frac{mV}{K}\bigg|_{i_E = cte}$$

I_{CBO} Leakage current of BC diode

Determined by n_{C0} as $n_{C0} \gg p_{B0}$



Large Signal Model

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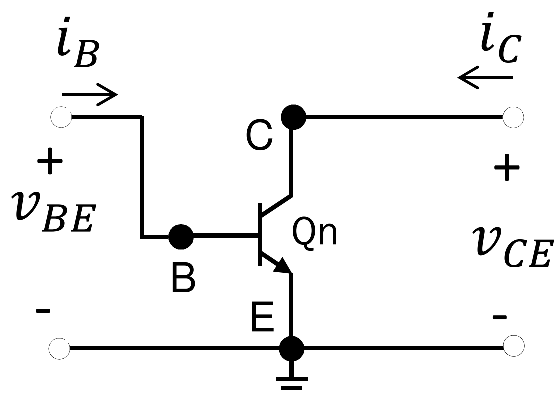
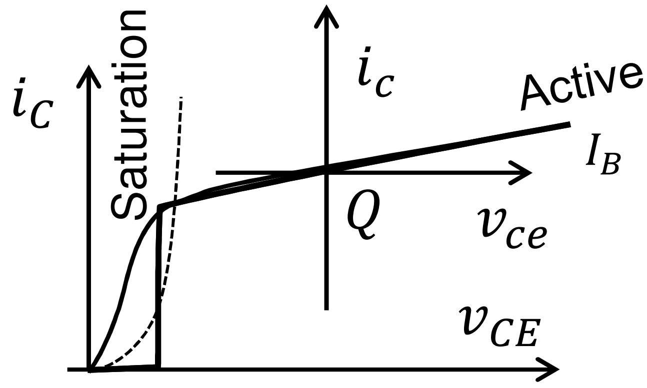
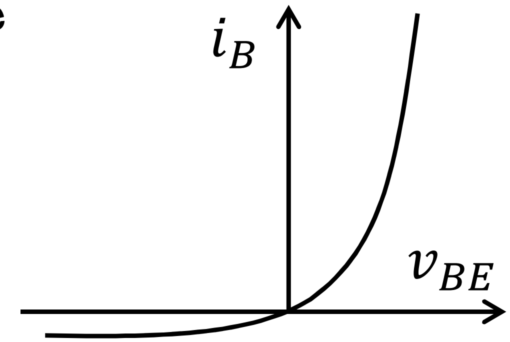
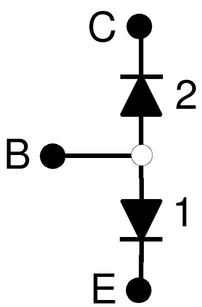
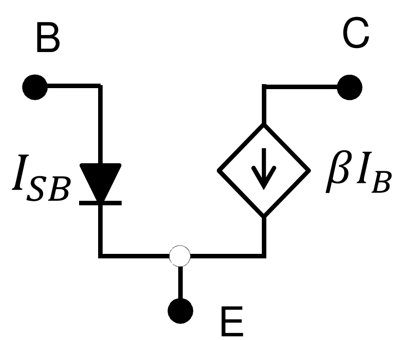




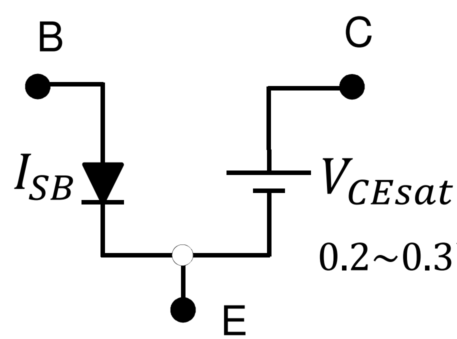


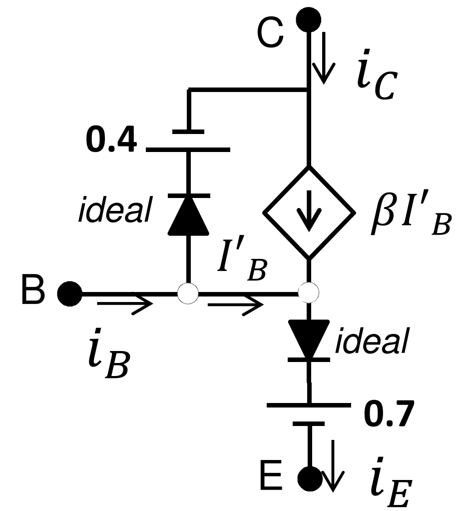


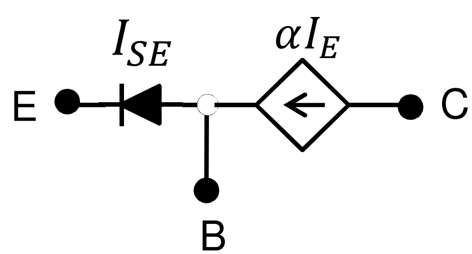
Active



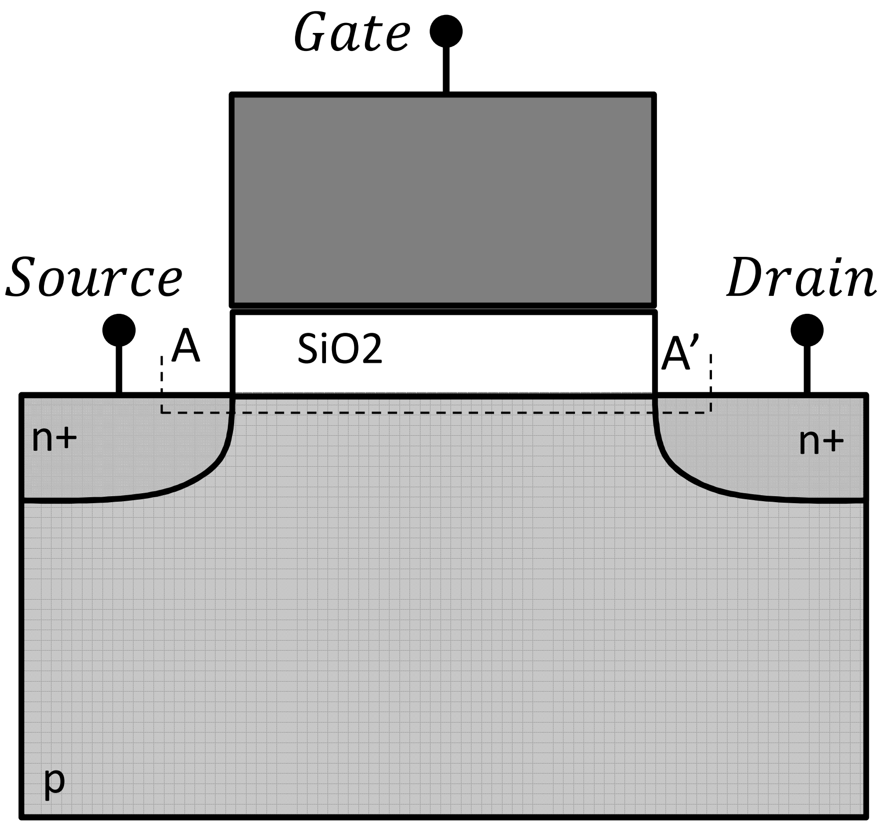
Saturation

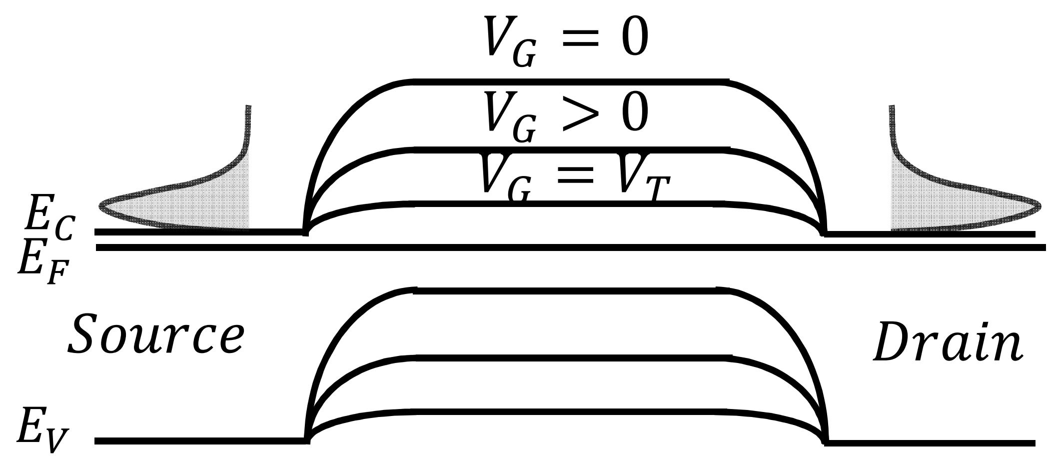






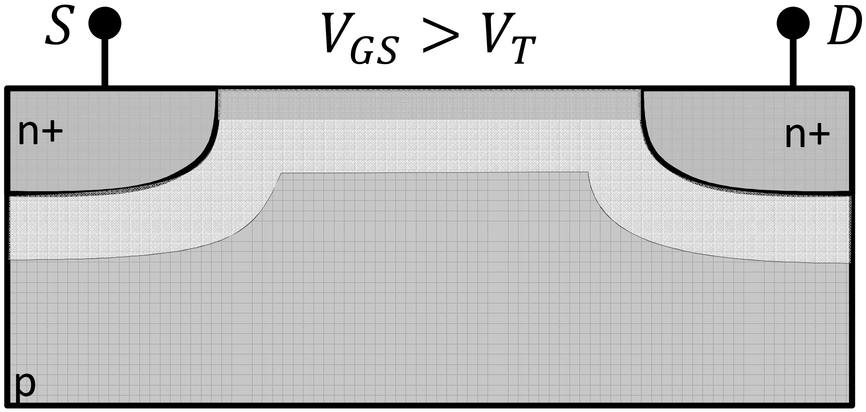
Qualitative Theory of the NMOSFET



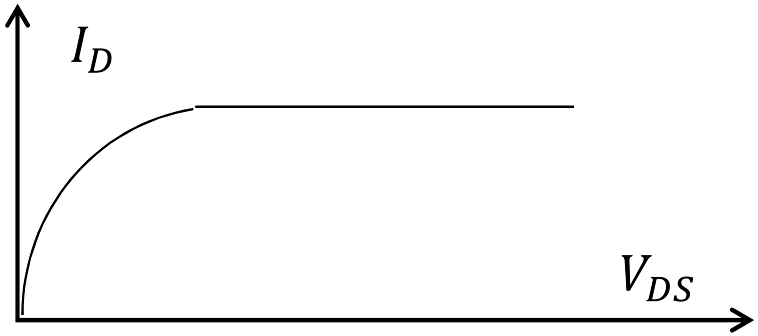


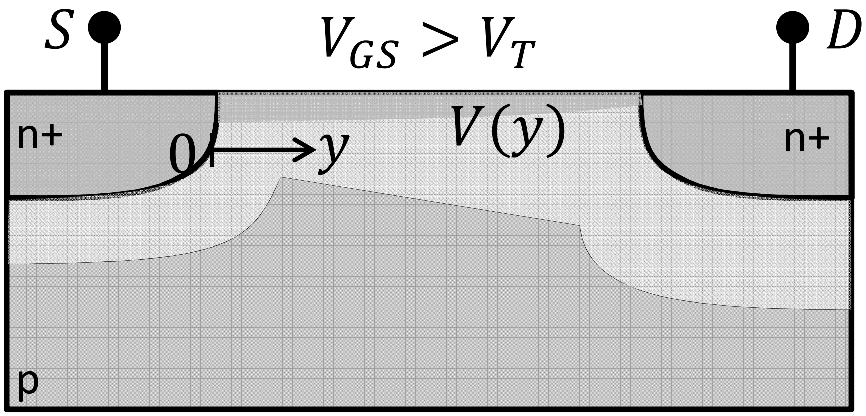
The potential barrier to electron flow from the source into the channel region is lowered by applying $V_{GS} > V_T$

Qualitative Theory of the NMOSFET

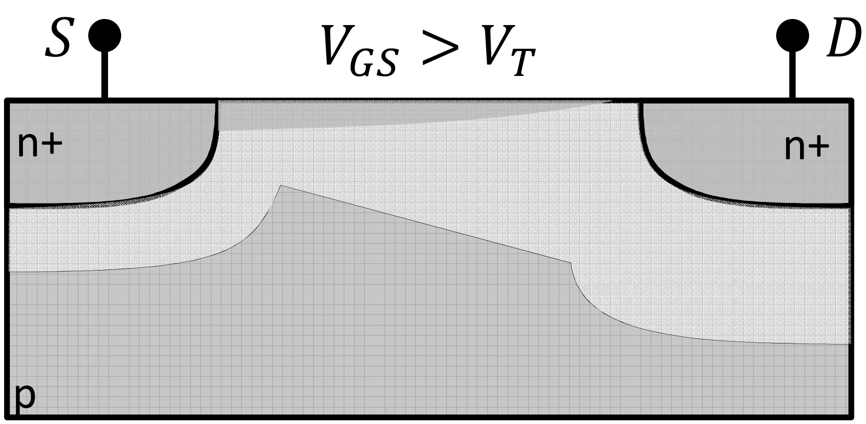


$V_{DS} \approx 0$





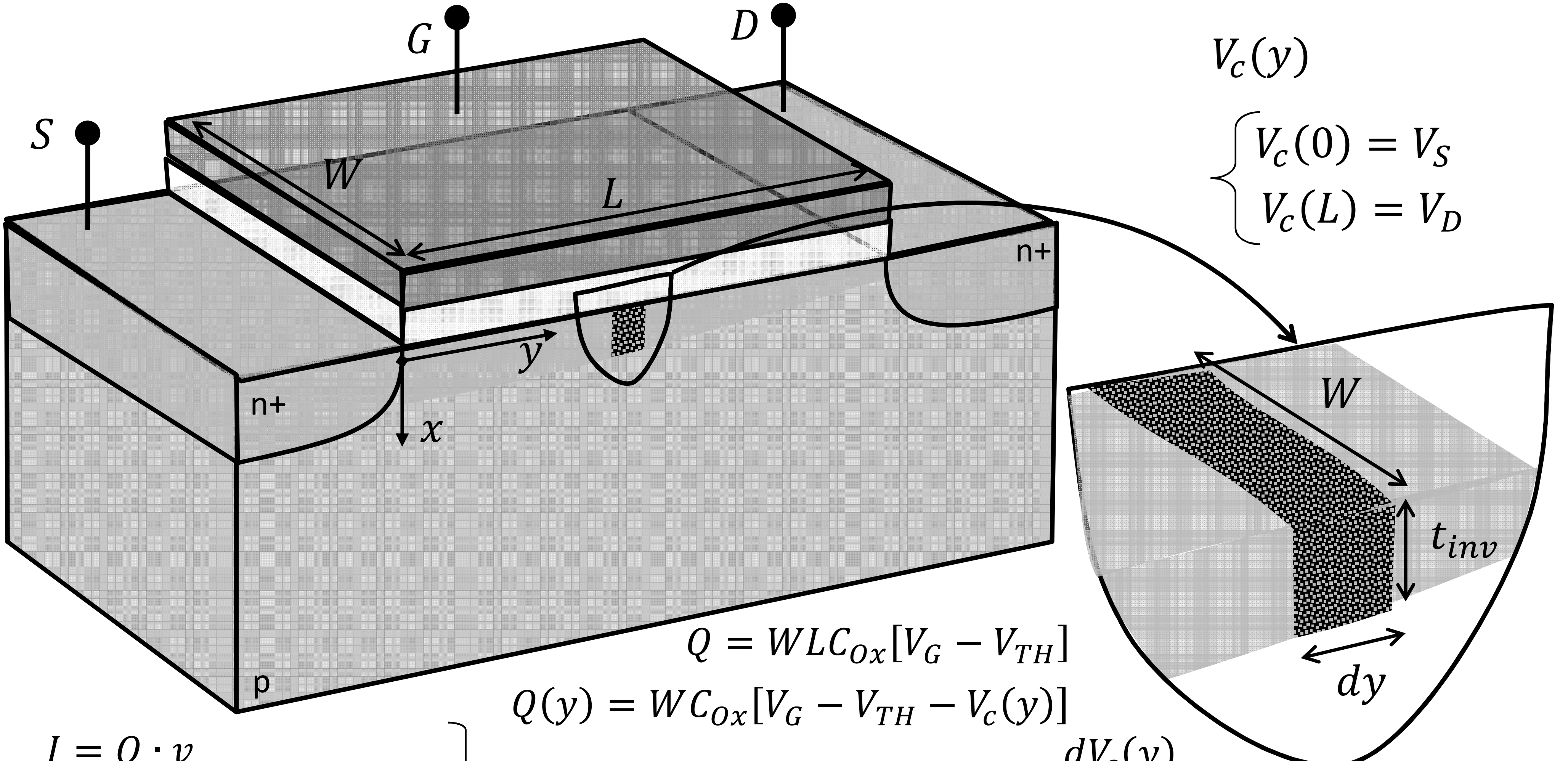
$V_{DS} > 0$



$V_{DS} > V_{GS} - V_T$

$$V_{DS_{sat}} = V_{GS} - V_T$$

MOSFET I-V Curve



$$V_c(y)$$

$$\begin{cases} V_c(0) = V_S \\ V_c(L) = V_D \end{cases}$$

$$Q = WLC_{Ox}[V_G - V_{TH}]$$

$$Q(y) = WC_{Ox}[V_G - V_{TH} - V_c(y)]$$

$$\left.\begin{aligned} I &= Q \cdot v \\ v &= -\mu_n \mathcal{E} = \mu_n \frac{dV_c(y)}{dy} \end{aligned}\right\} I_D = WC_{Ox}[V_G - V_{TH} - V_c(y)]\mu_n \frac{dV_c(y)}{dy}$$

$$\int_{y=0}^{y=L} I_D dy = \int_{V_C=0}^{V_C=V_{DS}} W\mu_n C_{Ox}[V_G - V_{TH} - V_c(y)]dV_c(y)$$

MOSFET I-V Curve

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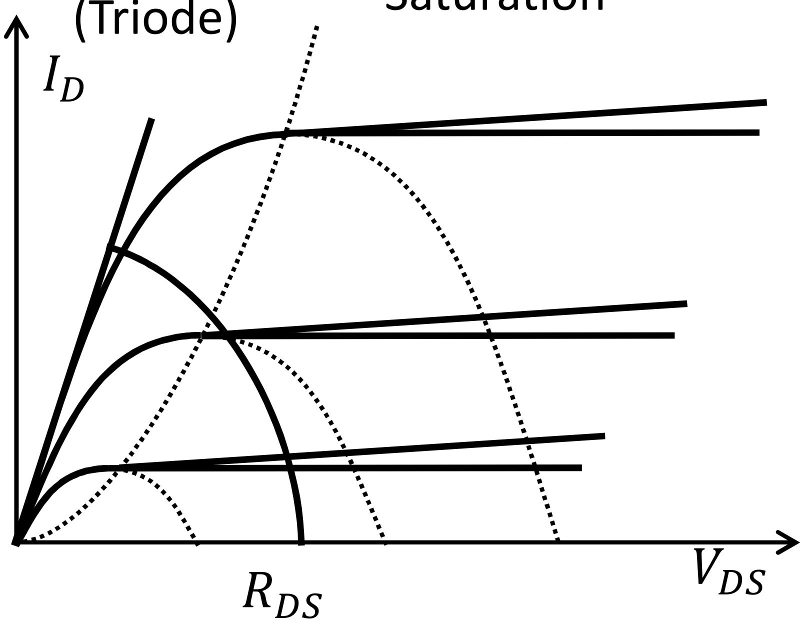
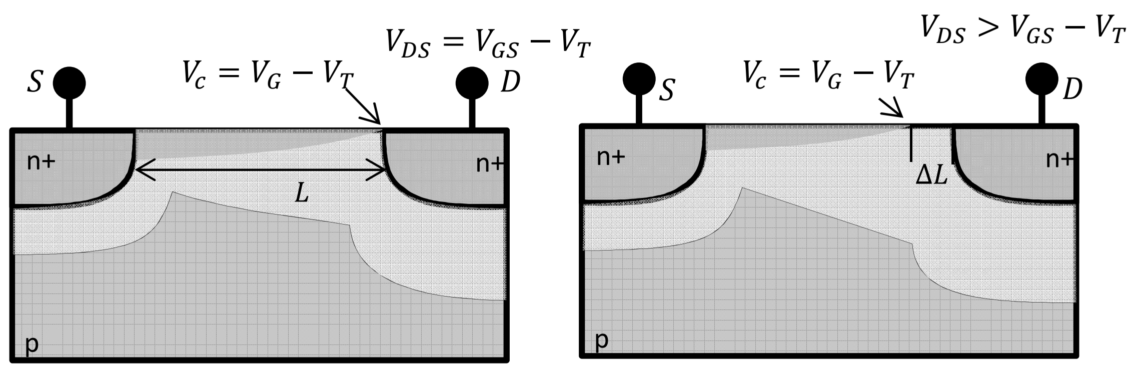
$$I_{DS} = \frac{1}{2}\frac{W}{L}\,\mu_{eff}C_{Ox}\,[2(V_{GS}-V_{TH})V_{DS}-V_{DS}^2]$$

$$I_{DS} = \begin{cases} \dfrac{1}{2}\dfrac{W}{L}\,\mu_{eff}C_{Ox}[2(V_{GS}-V_{TH})V_{DS}-V_{DS}^2] & V_{DS} < V_{DS_{sat}} \quad \text{Linear} \\ \dfrac{1}{2}\dfrac{W}{L}\,\mu_{eff}C_{Ox}(V_{GS}-V_T)^2 & V_{DS} > V_{DS_{sat}} = V_{GS}-V_T \quad \text{Saturation} \end{cases}$$



$$R_{DS} = \left(\left.\frac{\partial I_{DS}}{\partial V_{DS}}\right|_{V_{DS}=0}\right)^{-1} = \left(\frac{W}{L}\,\mu_{eff}C_{Ox}(V_{GS}-V_T)\right)^{-1}$$



"channel-length modulation"

Voltage Amplifier

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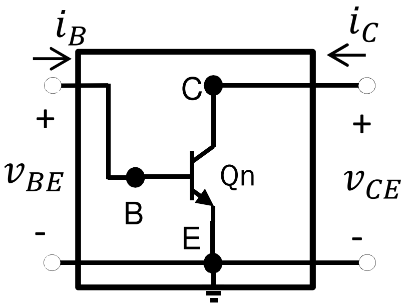
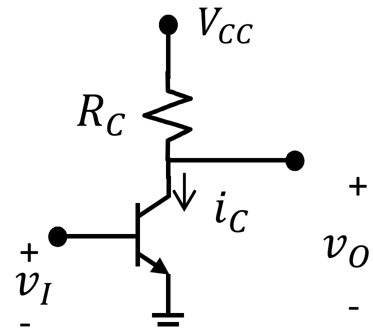
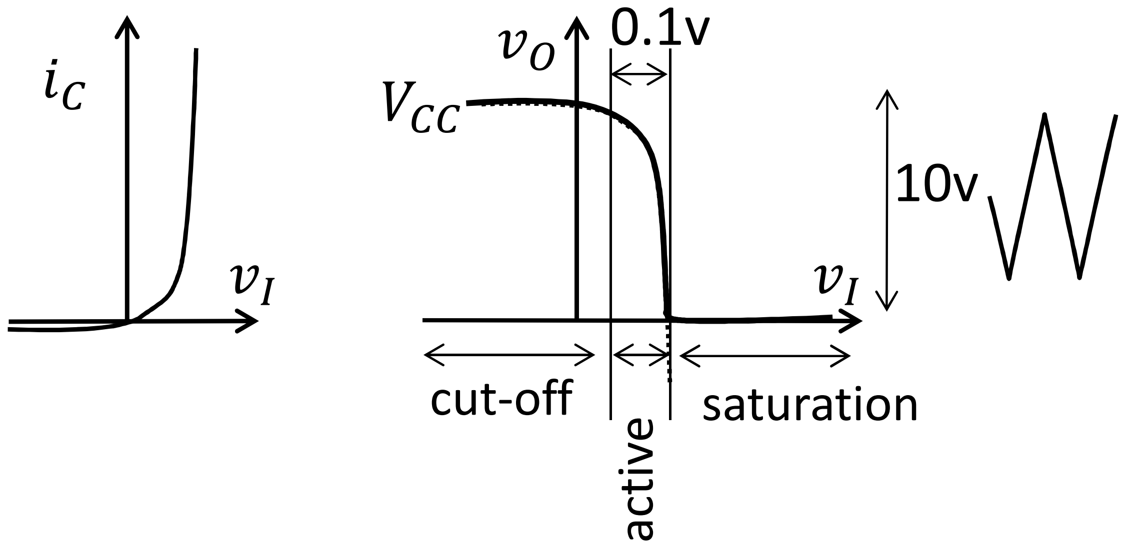
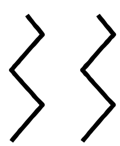




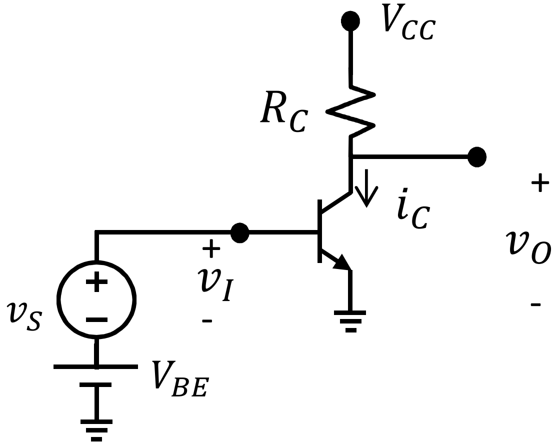
$$v_O = V_{CC} - R_C i_C$$

$$i_C = \beta i_B = I_s(e^{\frac{v_I}{V_T}} - 1)$$

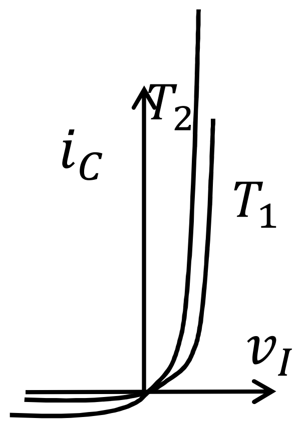


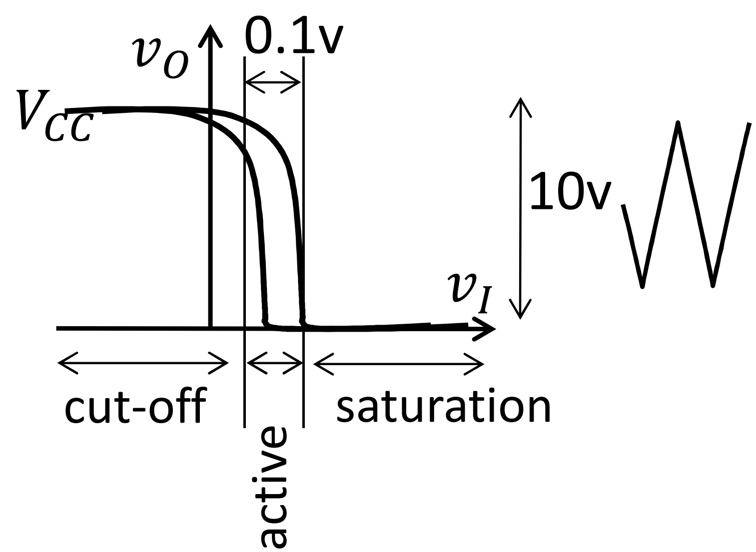


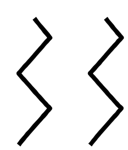
Biasing: V_{BE} = cte



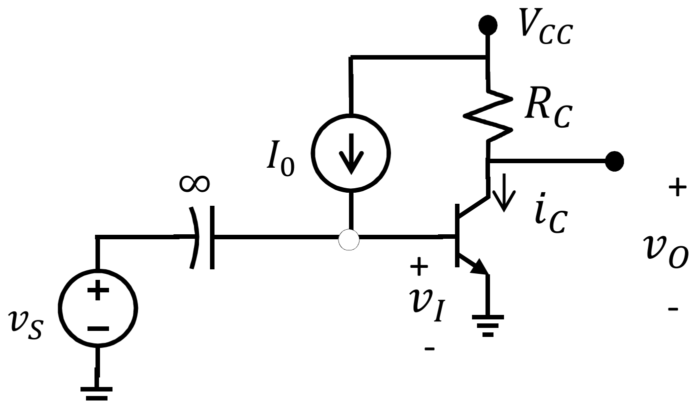
√ DC-coupled







Biasing: I_B = cte

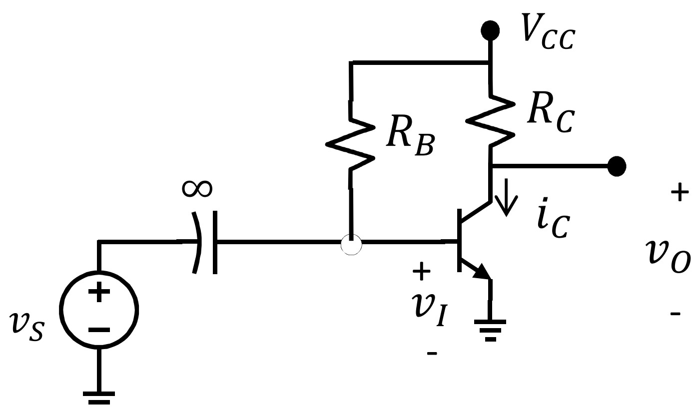


$$I_B = I_0$$
$$I_C = \beta I_B$$

$$V_{CC} = 10V$$
$$I_C = 1mA \qquad \beta = 100$$
$$I_0 = 10\mu A$$

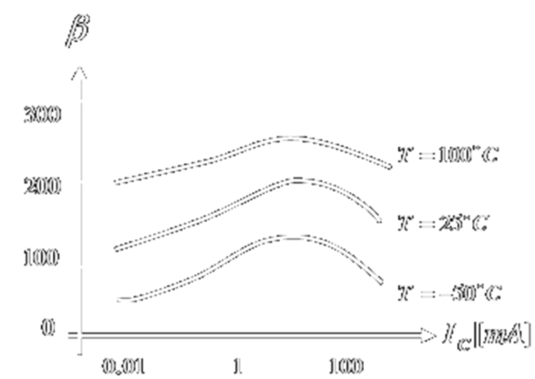
For max swing: $V_C \sim 5V \quad \rightarrow R_C \sim 5k\Omega$

For max gain: $V_C \sim 0.3V \quad \rightarrow R_C \sim 9.7k\Omega$



$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

What is the problem?

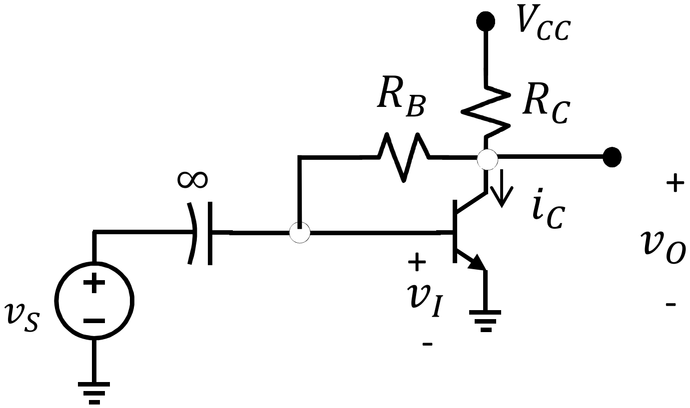


$$V_{CC} = 10V$$
$$I_C = 1mA$$
$$\rightarrow R_B = 930k\Omega$$

Replace it with transistor with $\beta = 250$

$I_C = 2.5mA \qquad V_{CE} = 10 - 5 \times 2.5 = -2.5 < V_{CEsat}$

Biasing: I_B = cte



$V_{CC} = 10V \quad I_C = 1mA \quad \beta = 100$

For max swing: $V_C \sim 5V$

$$R_B = \frac{V_C - V_{BE}}{I_B} = \beta \frac{V_C - V_{BE}}{I_C}$$

$\rightarrow R_B = 430k\Omega$

Negative feedback!

$\beta \uparrow \quad \rightarrow I_C \uparrow$

$\rightarrow V_C \downarrow$

$\rightarrow I_B \downarrow$

$\rightarrow I_C \downarrow$

Biasing: I_E = cte

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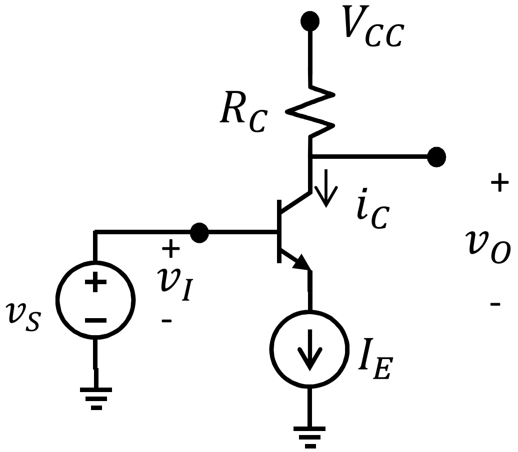
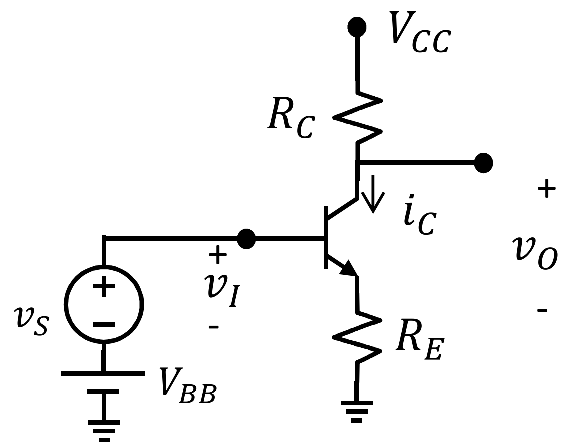
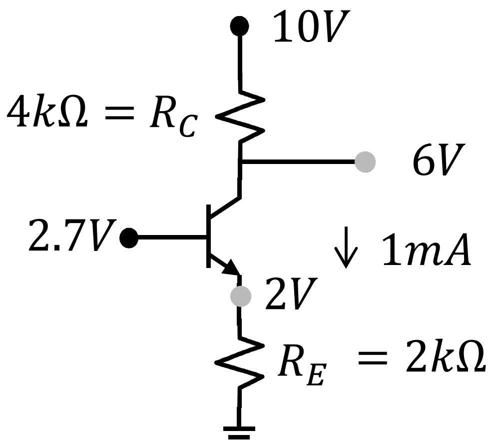


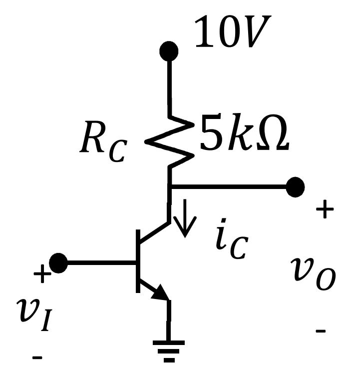


$$I_E = \frac{V_{BB} - V_{BE}}{R_E}$$

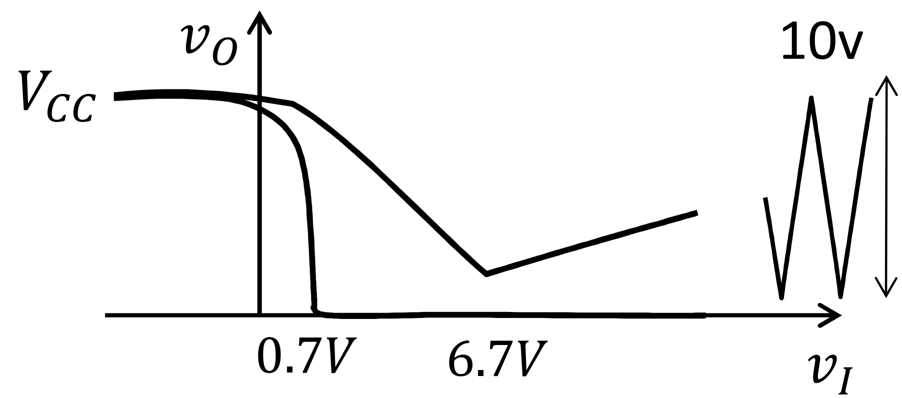
I_E independent of V_{BE}

$V_{BB} \gg V_{BE}$ $\quad$ $V_{BB} \sim 2.7V$

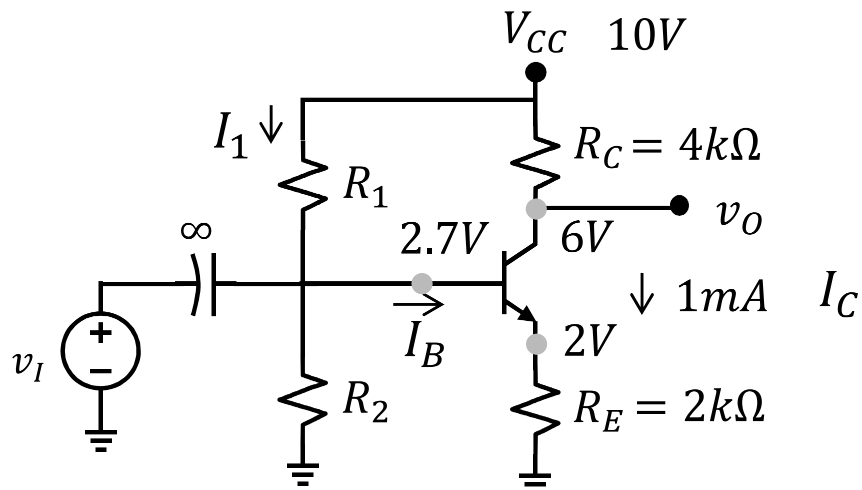




Where is the trade-off?



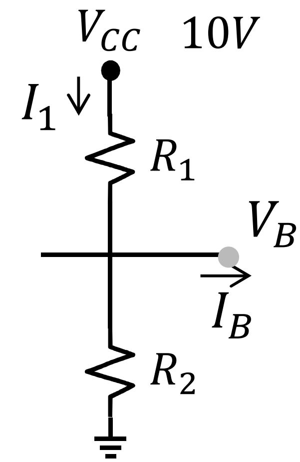
Biasing: I_E = cte

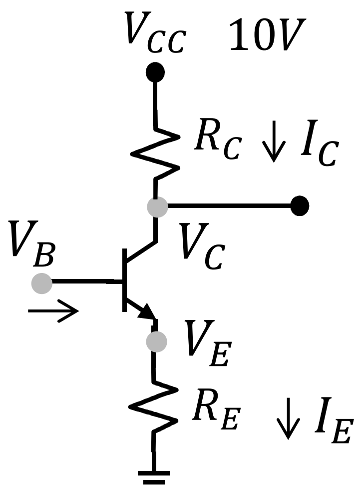


$$I_B = \frac{I_C}{\beta}$$

Assume: $I_1 \gg I_B$

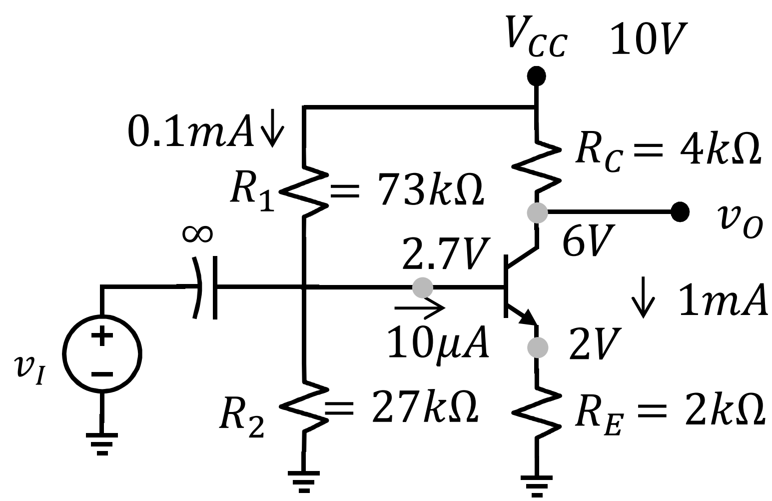
$$V_B = \frac{R_2}{R_2 + R_1} V_{CC}$$





$$V_E = V_B - V_{BEOn}$$

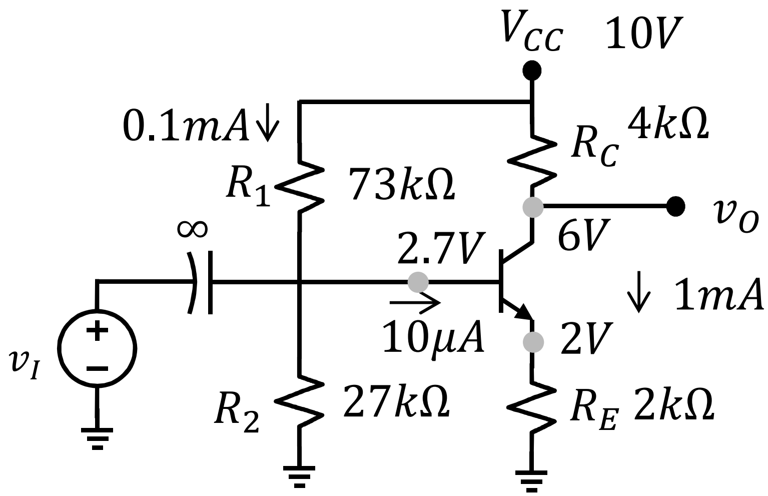
$$I_E = \frac{V_E}{R_E} \qquad I_C = \alpha I_E \approx I_E \qquad V_C = V_{CC} - I_C R_C$$



?

This is for design how about analysis

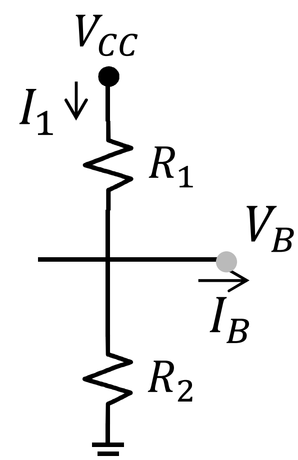
Biasing: I_E = cte



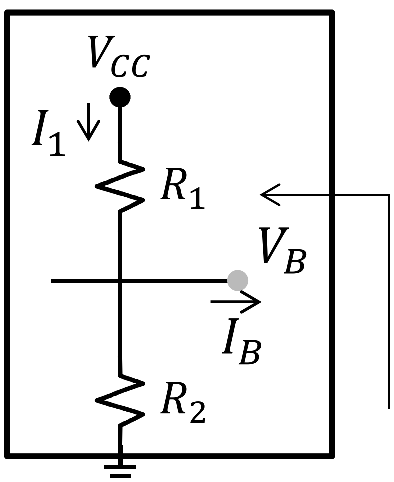
Assume: $I_1 \gg I_B$ $\quad I_1 = 0.1mA$

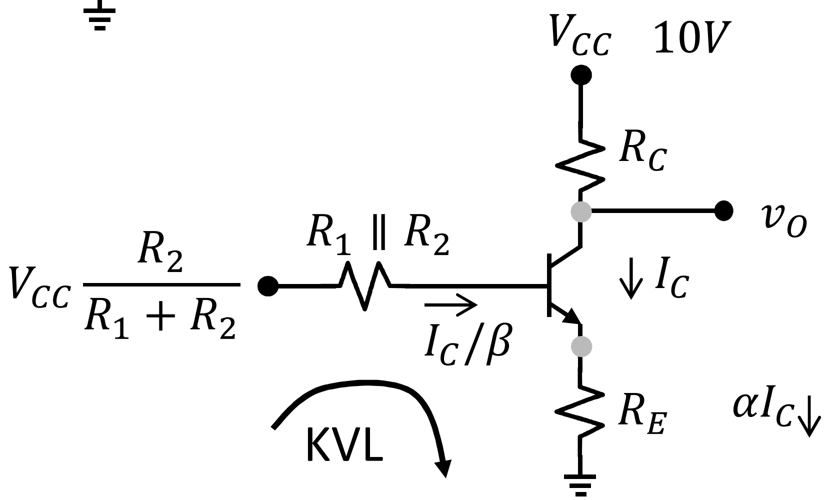
$V_B = 2.7V \quad V_E = 2V$

$I_E = 1mA \quad I_B = 0.01mA$



What if β was 10!





$$\frac{R_2}{R_1+R_2}V_{CC} = I_C\frac{R_1 \parallel R_2}{\beta} + V_{BEon} + \alpha R_E I_C \qquad \rightarrow I_C = \cdots$$

For the above numbers: $I_C = \dfrac{2}{0.99 \times 2k + 0.01 \times 19.7k} = 0.92mA$

Biasing: I_E = cte

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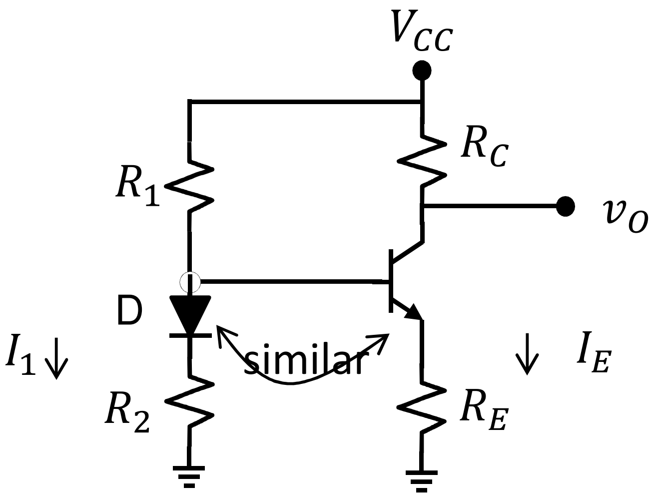
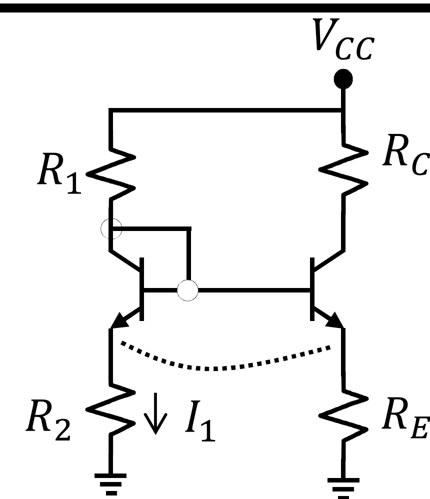


$$I_1 R_2 + V_D = V_{BEon} + I_E R_E$$

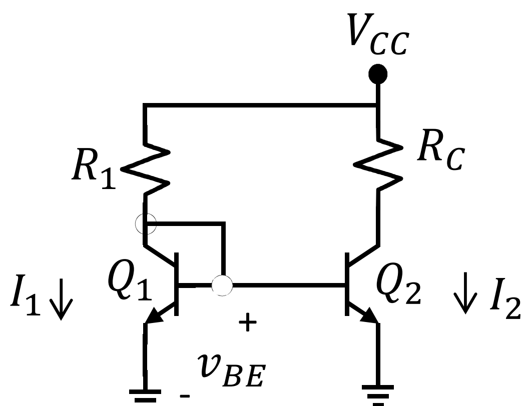
$$I_1 \approx \frac{V_{CC}}{R_1 + R_2} \qquad I_E = \frac{R_E}{R_2} I_1$$



$$I_1 R_2 = I_E R_E$$

$$I_1 = \frac{V_{CC} - V_{BEon}}{R_1 + R_2}$$

Only in Integrated Circuits!



$$I_1 = \frac{V_{CC} - V_{BEon}}{R_1}$$

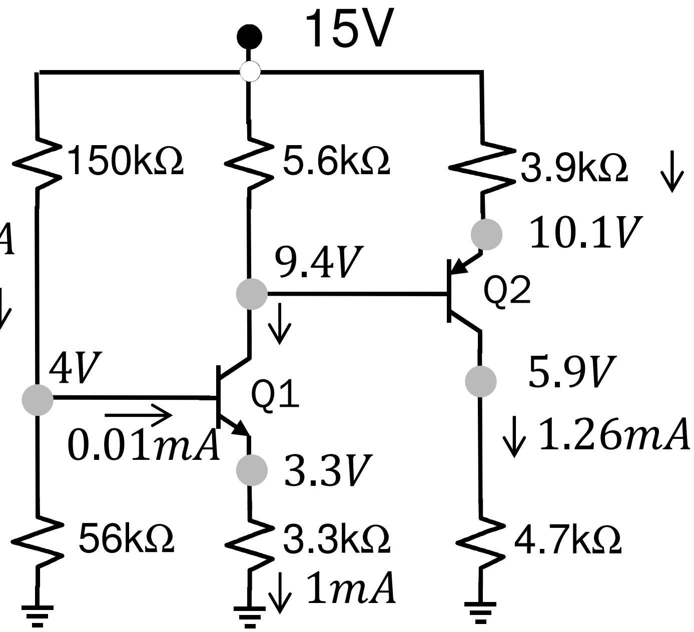
$$I_1 = I_{S1}\left(e^{v_{BE}/V_T} - 1\right)$$

$$I_2 = I_{S2}\left(e^{v_{BE}/V_T} - 1\right)$$

$$\frac{I_2}{I_1} = \frac{I_{S2}}{I_{S1}} = \frac{A_{Q2}}{A_{Q1}}$$

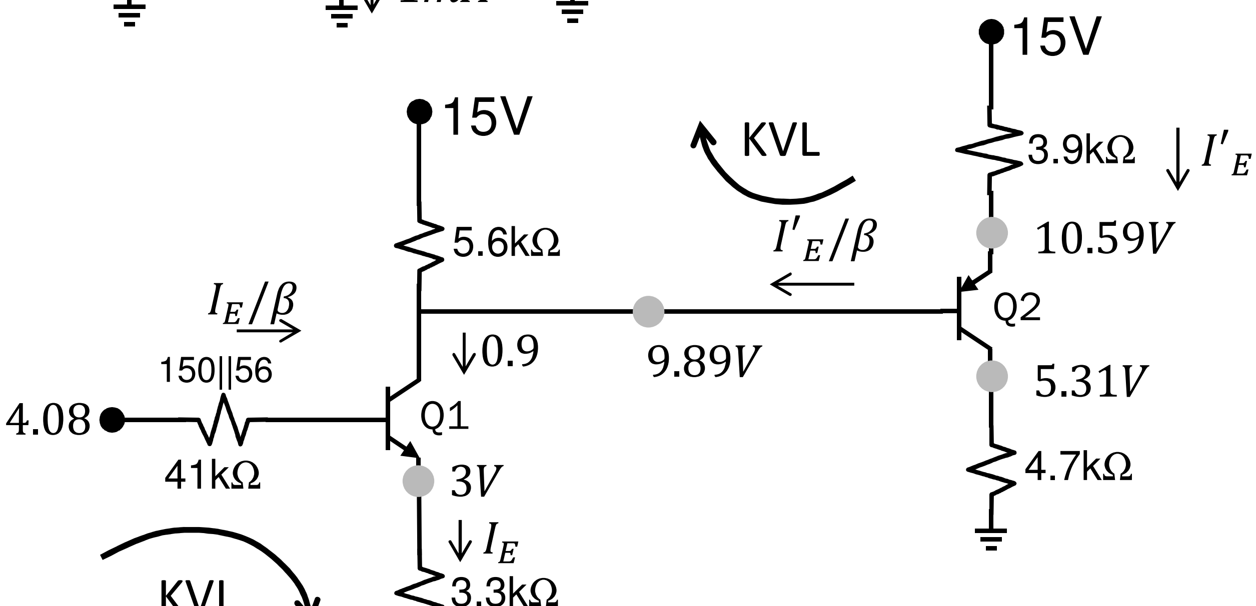
area

Biasing: Example 01



$$\beta_1 = \beta_2 = 100 \qquad V_{BE_{on}} = 0.7$$

	Q_1	Q_2
I_C[mA]	1	1.26
V_{CE}[V]	6.1	−4.2



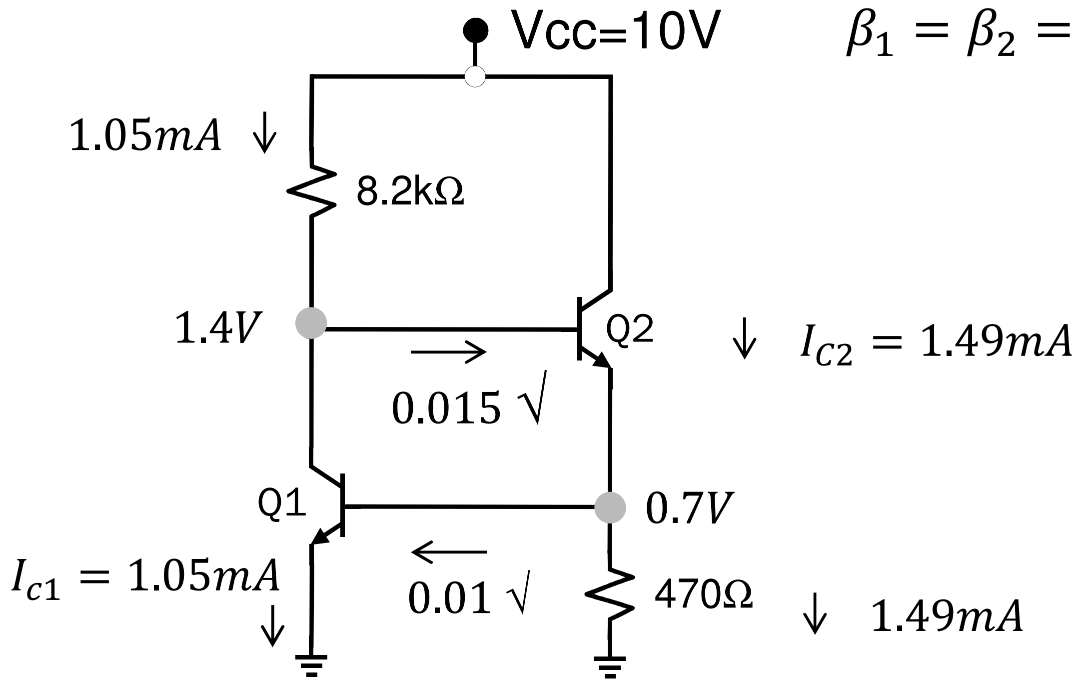
$$3.9I'_E + 0.7 = 5.6\left(0.9 - \frac{I'_E}{\beta}\right)$$

$$I'_E = 1.13mA$$

$$I_E = \frac{4.08}{3.3 + 40.8/101} = 0.91mA$$

Biasing: Example 02

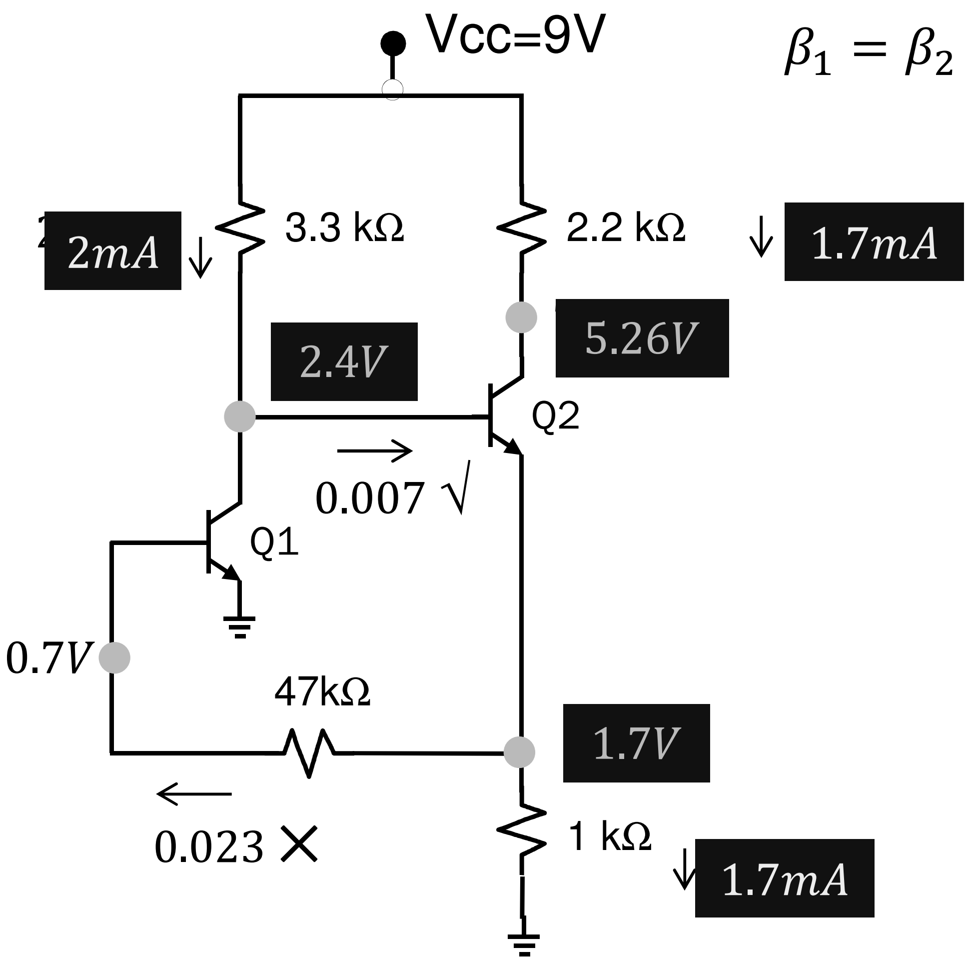
$\beta_1 = \beta_2 = 100 \quad V_{BE_{on}} = 0.7$



	Q_1	Q_2
I_C[mA]	1.05	1.49
V_{CE}[V]	1.4	9.3

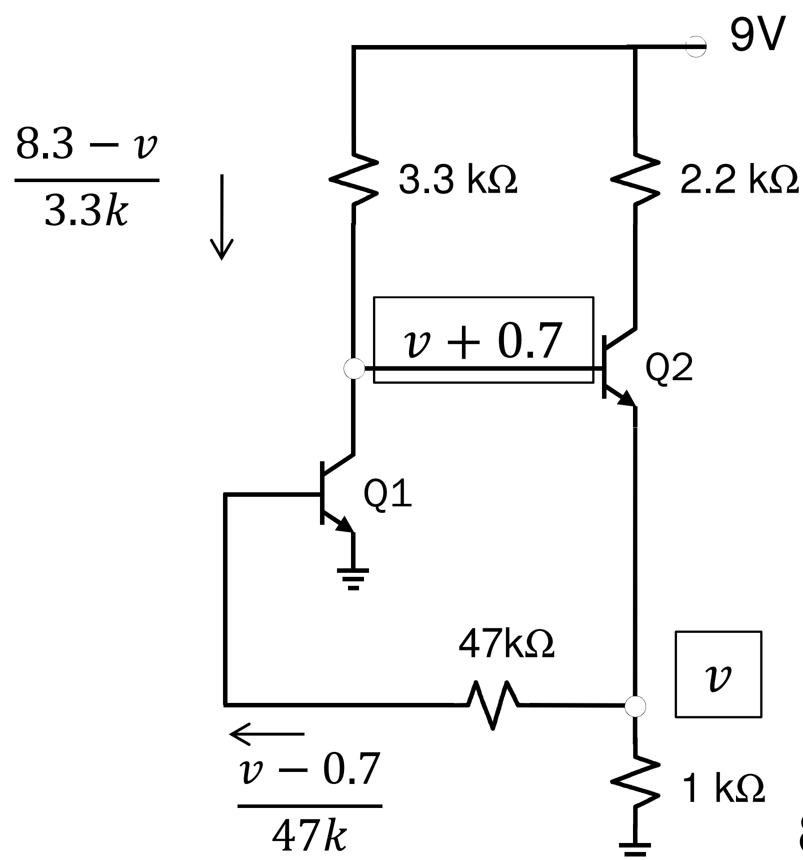
Biasing: Example 03

1.
2.
3.
4.
5.



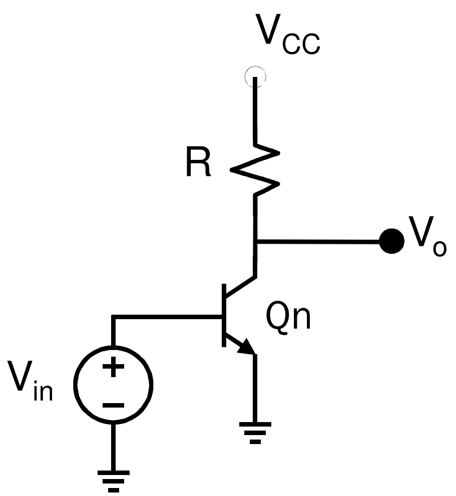
$\beta_1 = \beta_2 = 100 \quad V_{BE_{on}} = 0.7$

	Q_1	Q_2
I_C[mA]	2	1.7
V_{CE}[V]	2.4	3.56



$$\frac{8.3 - v}{330} = \frac{v - 0.7}{47} \quad \Rightarrow \quad v = 1.65$$

Linear BJT Amplifier



$$V_T = 26mV \qquad V_A = 200V$$

$$i_C = I_S e^{\frac{v_{BE}}{nV_T}}\left(1 + \frac{v_{CE}}{V_A}\right) \approx I_S e^{\frac{v_{BE}}{nV_T}}$$

$$v_{BE} = V_B + \hat{v}_i \sin \omega t$$

$$v_O = V_{CC} - Ri_C = V_{CC} - R\underbrace{I_S e^{\frac{V_B}{V_T}}}_{I_C} e^{\frac{\hat{v}_i \sin \omega t}{V_T}}$$

$$= V_{CC} - RI_C\left(1 + \frac{\hat{v}_i}{V_T}\sin \omega t + \frac{\hat{v}_i^2}{2V_T^2}\sin^2 \omega t + \cdots\right)$$

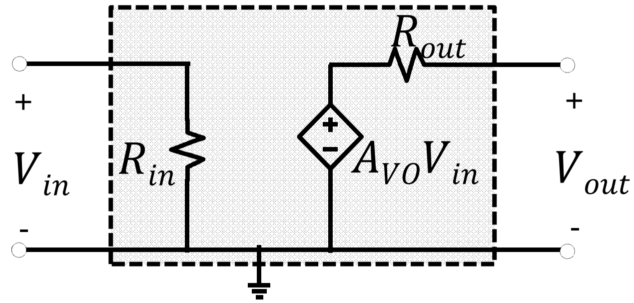
$$\approx \underbrace{V_{CC} - RI_C}_{V_O} \underbrace{- RI_C \frac{\hat{v}_i}{V_T}\sin \omega t}_{v_o}$$

$$A_V = \frac{v_o}{v_{in}} = \frac{-R_C I_C}{V_T} = -g_m R_C \qquad g_m = \frac{I_C}{V_T}$$

Amplifiers

1.
2.
3.
4.
5.

Voltage Amplifier



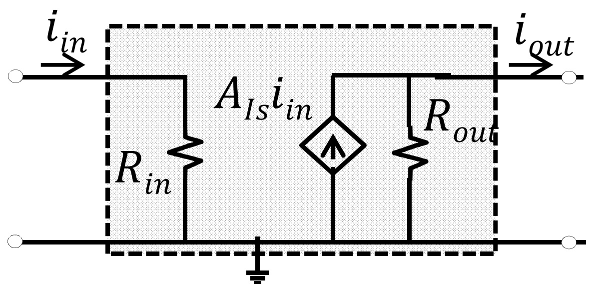
$$A_{Vo} = \left.\frac{V_{out}}{V_{in}}\right|_{i_{out}=0}$$

open circuit voltage gain

Ideal:

$$R_{in} = \infty$$
$$R_{out} = 0$$

Current Amplifier

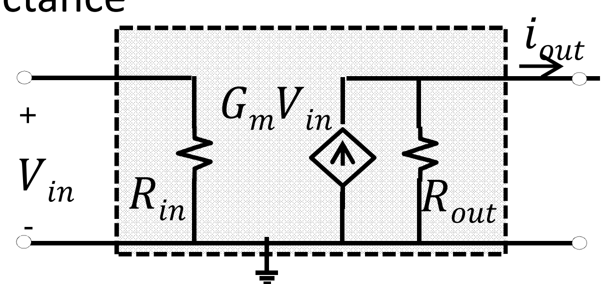


$$A_{Is} = \left.\frac{i_{out}}{i_{in}}\right|_{V_{out}=0}$$

short circuit current gain

$$R_{in} = 0$$
$$R_{out} = \infty$$

Trance-Conductance Amplifier

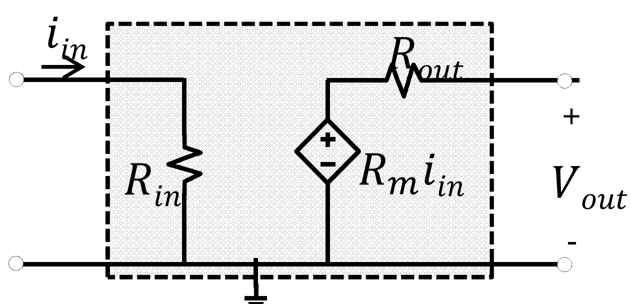


$$G_m = \left.\frac{i_{out}}{V_{in}}\right|_{V_{out}=0}$$

short circuit Trans-conductance

$$R_{in} = \infty$$
$$R_{out} = \infty$$

Trance-Resistance Amplifier



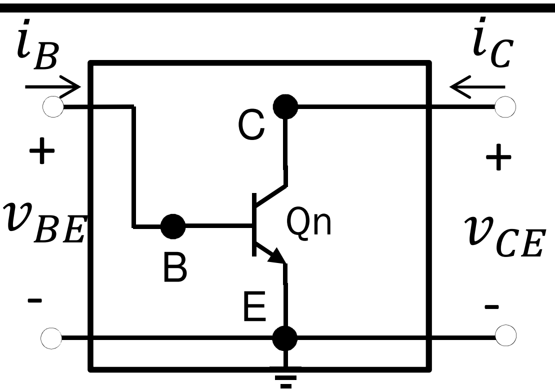
$$R_m = \left.\frac{V_{out}}{i_{in}}\right|_{i_{out}=0}$$

open circuit Trans-resistance

$$R_{in} = 0$$
$$R_{out} = 0$$

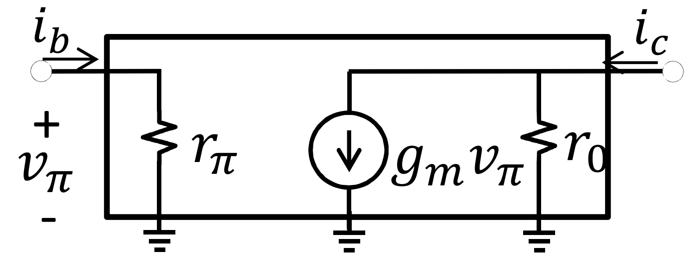
BJT Small Signal Model (h-π)

1.
2.
3.
4.
5.



$$i_C = I_S\left(e^{\frac{v_{EB}}{V_T}} - 1\right)\left(1 + \frac{v_{CE}}{V_A}\right)$$

$$\cong \underbrace{I_S e^{\frac{v_{EB}}{V_T}}}_{I_C}\left(1 + \frac{v_{CE}}{V_A}\right)$$



Input resistance:

$$r_\pi \equiv \frac{\partial v_{BE}}{\partial i_B} = \left(\frac{\partial i_B}{\partial v_{BE}}\right)^{-1} = \beta\left(\frac{\partial i_C}{\partial v_{BE}}\right)^{-1} = \beta\left(\frac{I_S}{V_T} e^{\frac{v_{EB}}{V_T}}\right)^{-1} = \beta\frac{V_T}{I_C} = \frac{\beta}{g_m} = \beta r_m$$

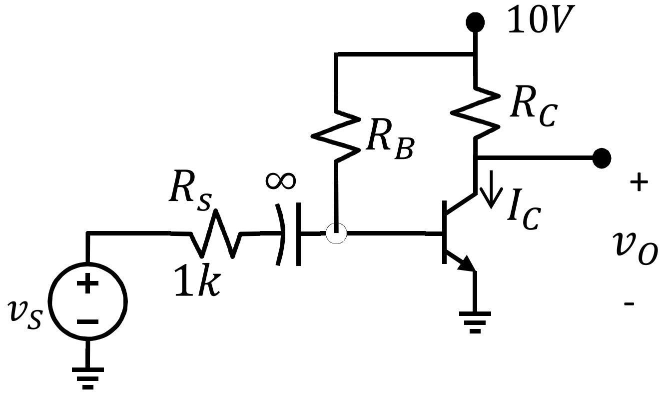
Output resistance:

$$r_o \equiv \frac{\partial v_{CE}}{\partial i_C} = \left(\frac{\partial i_C}{\partial v_{CE}}\right)^{-1} = \left(\frac{I_C}{V_A}\right)^{-1} = \frac{V_A}{I_C}$$

Trance-Conductance:

$$g_m \equiv \frac{\partial i_C}{\partial v_{BE}} = \frac{I_S}{V_T} e^{\frac{v_{EB}}{V_T}} = \frac{I_C}{V_T} = \frac{1}{r_m}$$

Example 01 - CE



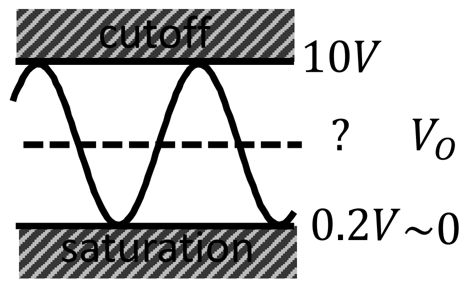
Assume $\beta = 100 \quad V_A \sim \infty$

Design for $I_C = 1mA$ and maximum swing

Find A_v, R_{in}, R_{out}

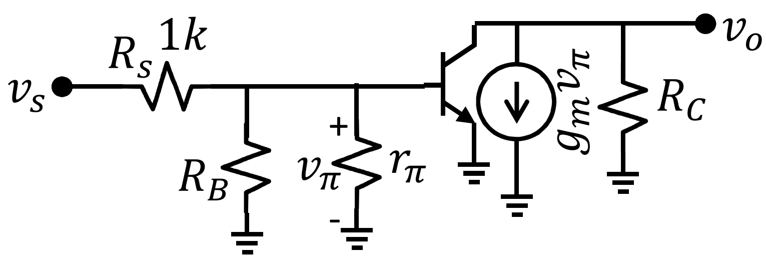
DC: $R_B = \dfrac{10 - 0.7}{0.01mA} = 930k\Omega$

window for v_O



$V_O = 5 \qquad R_C = 5k\Omega$

AC:



$$v_\pi = v_s \frac{r_\pi \parallel R_B}{r_\pi \parallel R_B + R_s} \sim v_s \frac{r_\pi}{r_\pi + R_s}$$

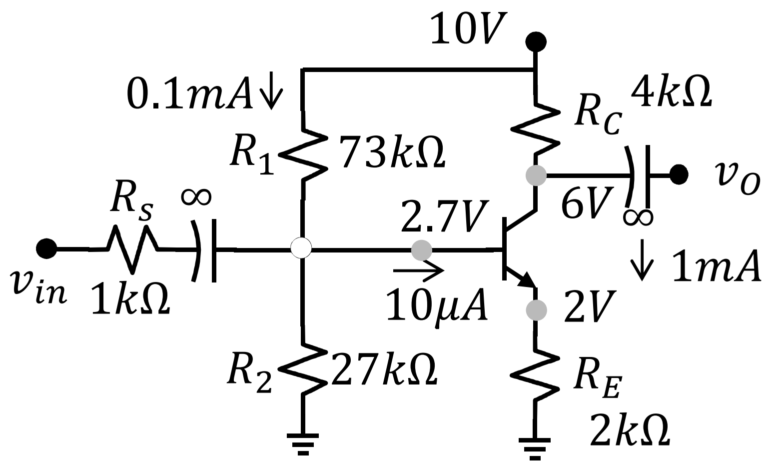
$$v_0 = -g_m v_\pi R_C$$

$$A_v = \frac{v_0}{v_s} = -g_m R_C \frac{r_\pi}{r_\pi + R_s} = \frac{-\beta R_C}{r_\pi + R_s} = \frac{-R_C}{r_m + R_s/\beta} = -\frac{\text{Collector resistance}}{\text{Emitter's circuit resistance}}$$

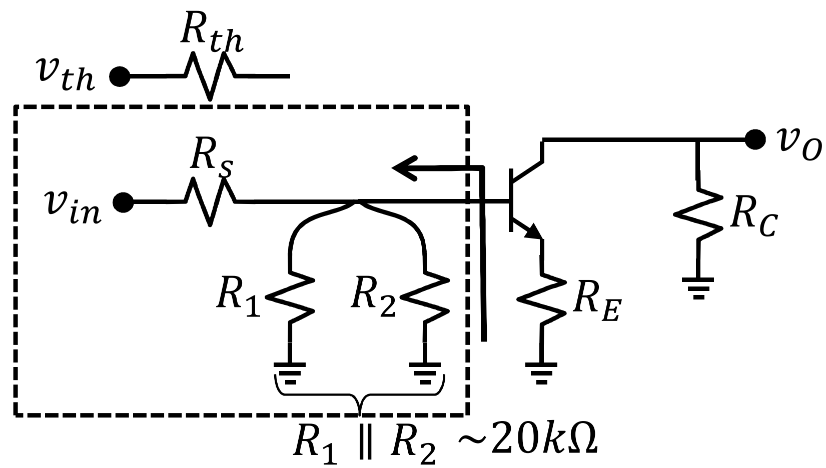
if $R_s \to 0$: $\quad A_v = -g_m R_C$

Example 02 - CE

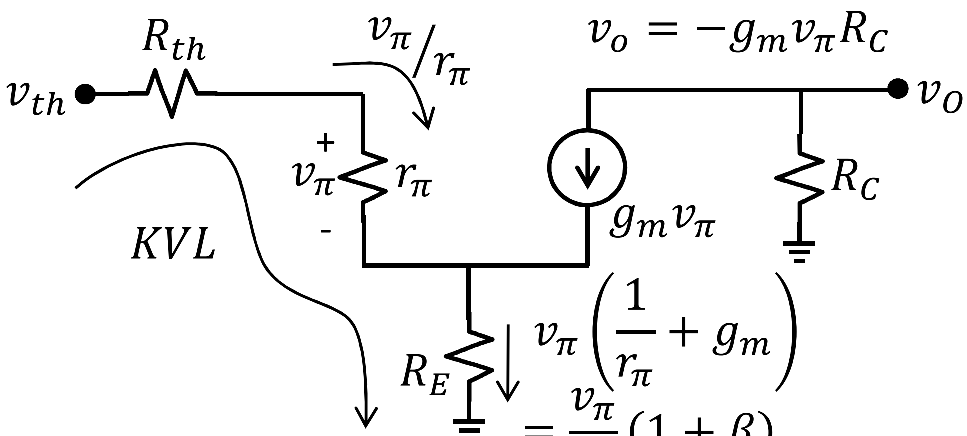
Assume $\beta = 100$ $\quad V_A \sim \infty$ $\quad$ Find A_v, R_{in}, R_{out}



AC circuit



$v_o = -g_m v_\pi R_C$



$$v_\pi\left(\frac{1}{r_\pi} + g_m\right) = \frac{v_\pi}{r_\pi}(1+\beta)$$

$$v_{th} = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_s} v_{in} \qquad R_{th} = R_1 \parallel R_2 \parallel R_s$$

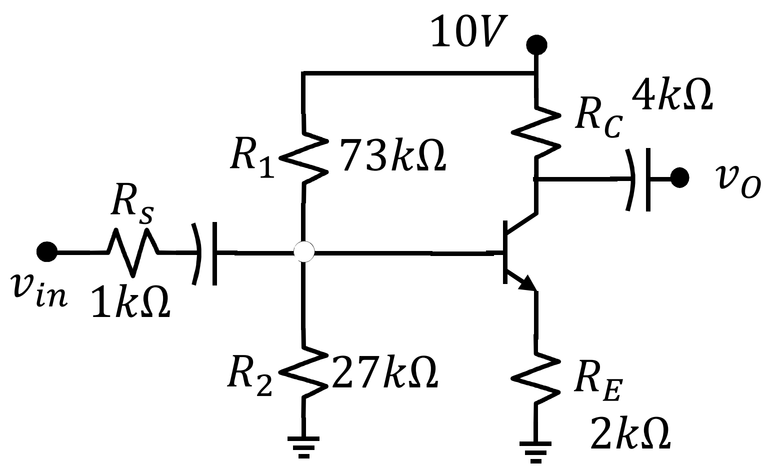
$$KVL: \quad -v_{th} + R_{th}\frac{v_\pi}{r_\pi} + v_\pi + R_E\frac{v_\pi}{r_\pi}(1+\beta) = 0$$

$$v_\pi = v_{th}\frac{r_\pi}{R_{th} + r_\pi + R_E(1+\beta)}$$

$$A'_v = \frac{v_o}{v_{th}} = \frac{-\beta R_C}{R_{th} + r_\pi + R_E(1+\beta)} = \frac{-R_C}{\frac{R_{th} + r_\pi}{\beta} + R_E\left(\frac{1+\beta}{\beta}\right)}$$

$$A_v = \frac{v_o}{v_s} = \frac{v_{th}}{v_s}\cdot\frac{v_o}{v_{th}} = -\frac{20}{21}\cdot\frac{4}{\frac{3.5}{100} + 2\times\frac{101}{100}} = -1.8$$

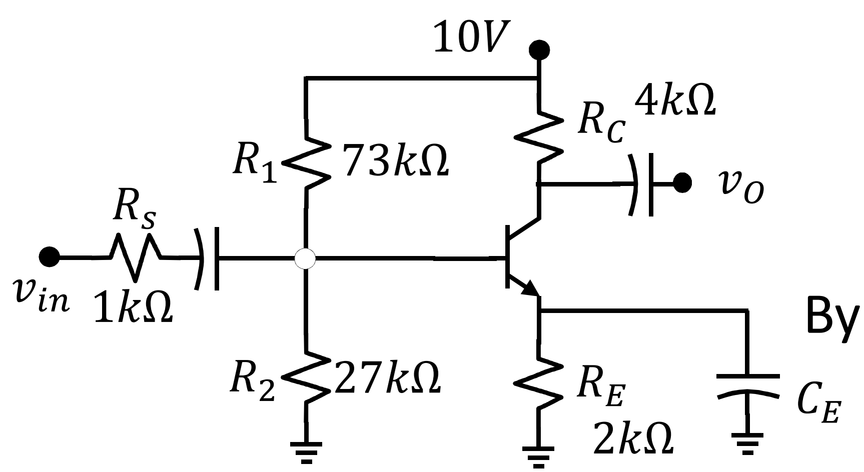
Example 02 - CE



$A_v = -1.8$

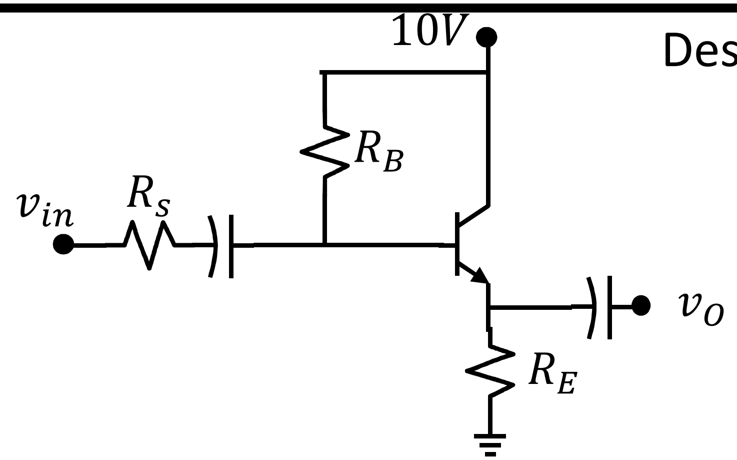
[redacted]

How we can increase gain?



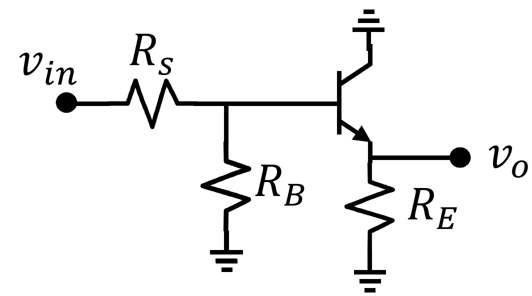
Bypass capacitance

Example 03 - CC

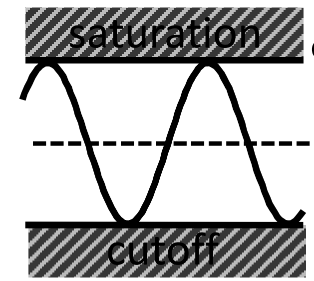


Design a buffer $I_C = 1mA$

AC circuit

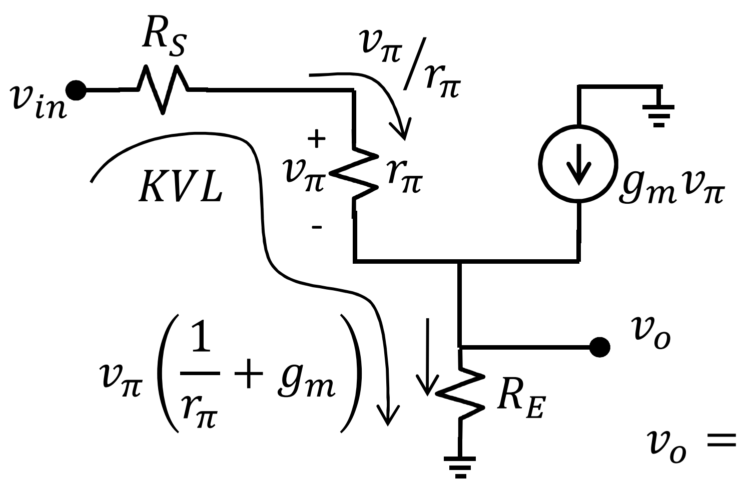


window for v_E



$R_E = 5k\Omega$

$$R_B = \frac{10 - 5.7}{0.01m} = 430k\Omega$$



KVL:

$$-v_{in} + R_S \frac{v_\pi}{r_\pi} + v_\pi + R_E \left(\frac{v_\pi}{r_\pi} + g_m v_\pi\right) = 0$$

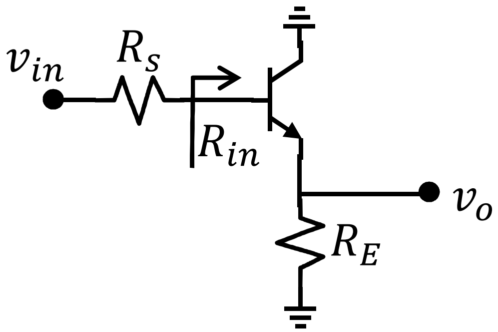
$$v_\pi = \frac{v_{in}}{\frac{R_S}{r_\pi} + 1 + R_E \left(g_m + \frac{1}{r_\pi}\right)}$$

$$v_o = v_\pi \left(\frac{1}{r_\pi} + g_m\right) R_E$$

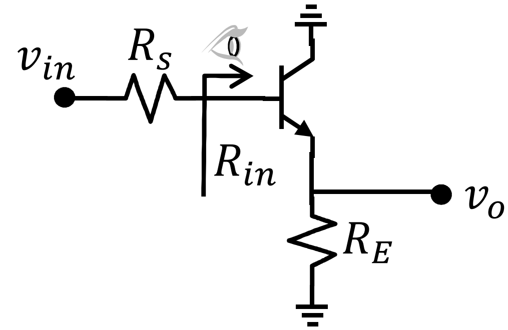
$$A_v = \frac{v_o}{v_{in}} = \frac{\left(\frac{1}{r_\pi} + g_m\right) R_E}{\frac{R_S}{r_\pi} + 1 + R_E \left(g_m + \frac{1}{r_\pi}\right)} = \frac{R_E}{\frac{R_S + r_\pi}{1 + \beta} + R_E} \sim 1$$

Example 03 - CC

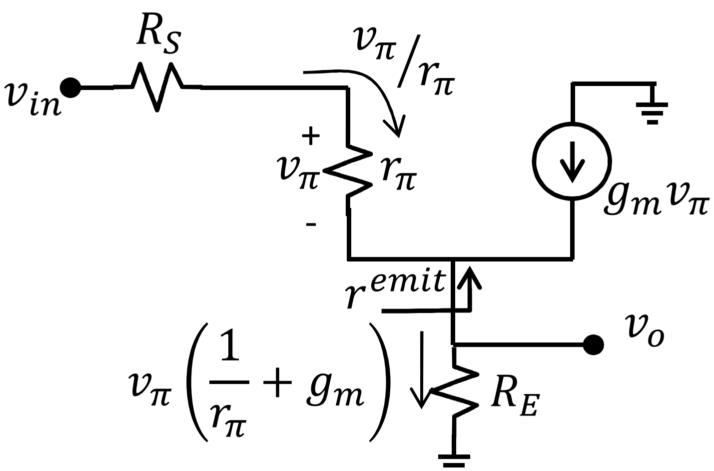
1.
2.
3.
4.
5.



$$\frac{v_o}{v_{in}} = \frac{R_E}{\frac{R_S + r_\pi}{1+\beta} + R_E}$$

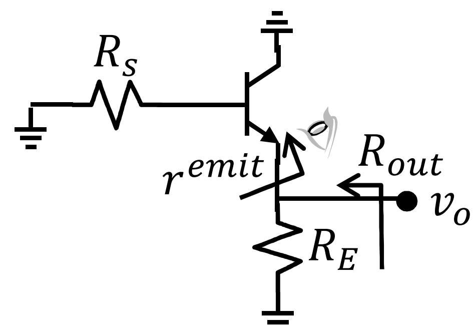


$$R_{in} = r_\pi + R_E(1+\beta)$$



$$R_{in} = \frac{v_\pi + R_E\left(\frac{v_\pi}{r_\pi} + g_m v_\pi\right)}{\frac{v_\pi}{r_\pi}}$$

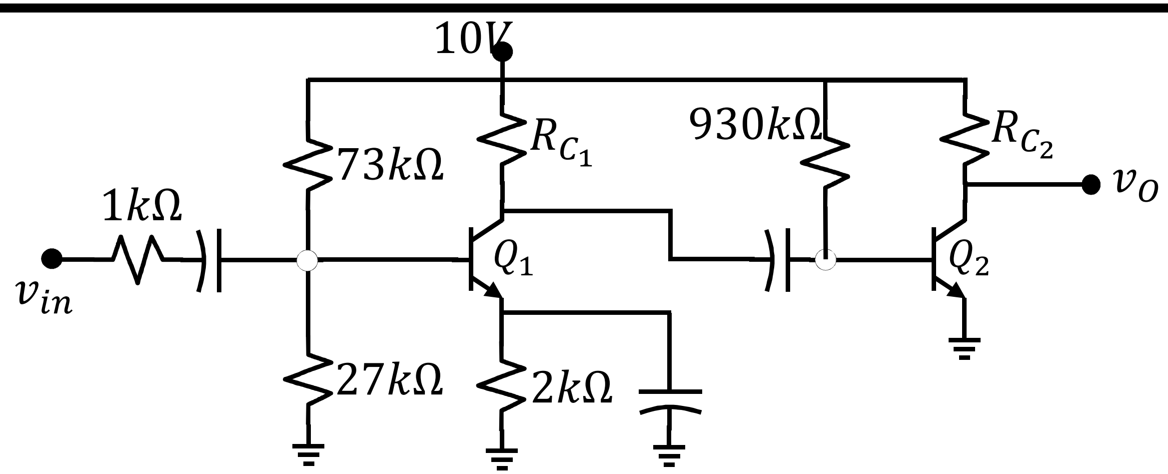
$$= r_\pi + R_E(1+\beta)$$



$$R_{out} = R_E \parallel r^{emit}$$

$$= R_E \parallel \frac{R_S + r_\pi}{1+\beta}$$

Example 04 – Multi-stage Amplifier



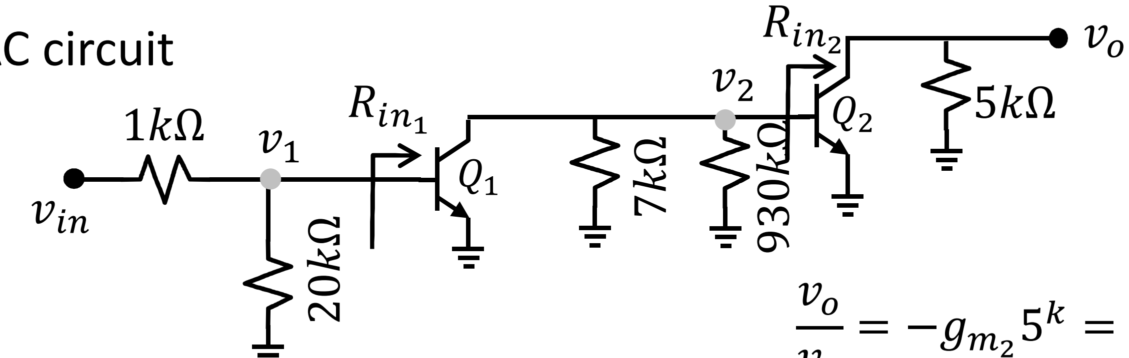
Design an amplifier:

$I_C = 1mA$
$\beta = 100$
$A_v \geq 1000$
$V_{cc} = 10V$
$R_S = 1k\Omega$

$$R_{C_1} = \frac{10-3}{1m} = 7k\Omega$$

$$R_{C_2} = \frac{10-5}{1m} = 5k\Omega$$

AC circuit



$27 \parallel 73$

$$A_v = \frac{v_o}{v_i} = \frac{v_1}{v_i} \cdot \frac{v_2}{v_1} \cdot \frac{v_o}{v_2}$$

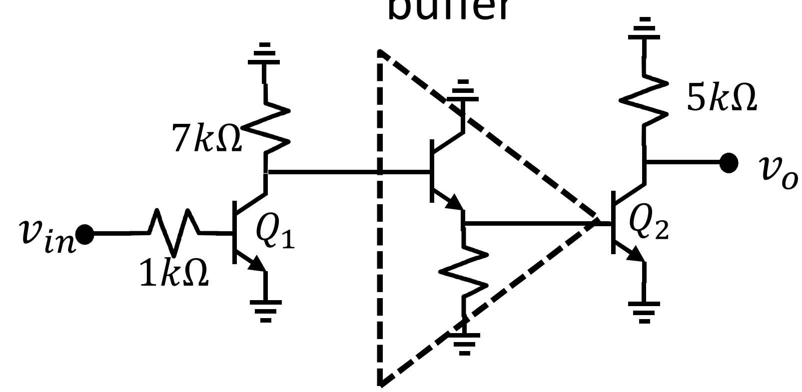
$$A_v = 10143$$

$$\frac{v_o}{v_2} = -g_{m_2} 5^k = -200$$

$$\frac{v_2}{v_1} = -g_{m_1}(7 \parallel 930 \parallel r_{\pi_2}) = -73$$

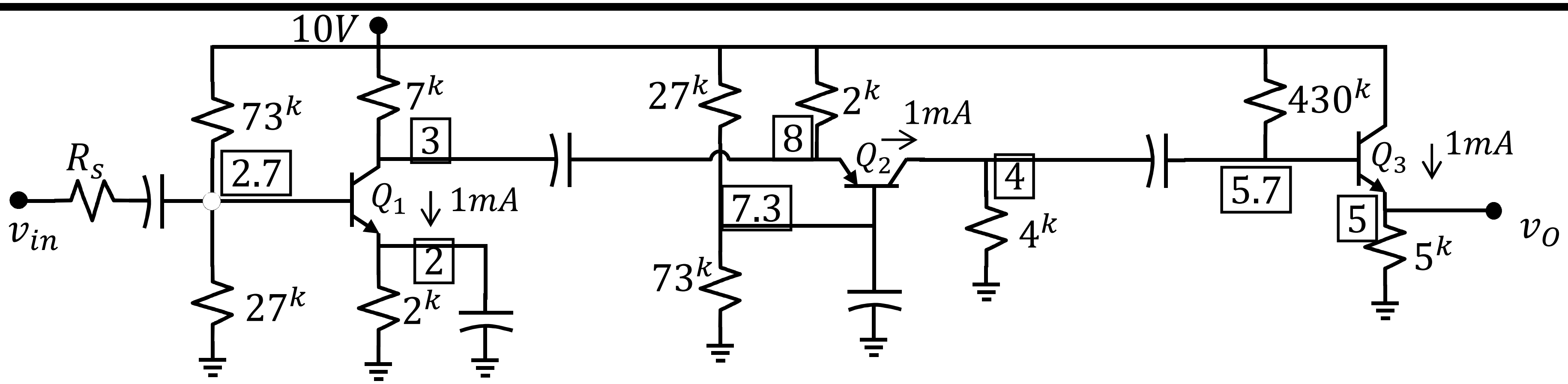
$$\frac{v_1}{v_i} = \frac{20^k \parallel r_{\pi_1}}{20^k \parallel r_{\pi_1} + 1^k} = 0.69$$

buffer

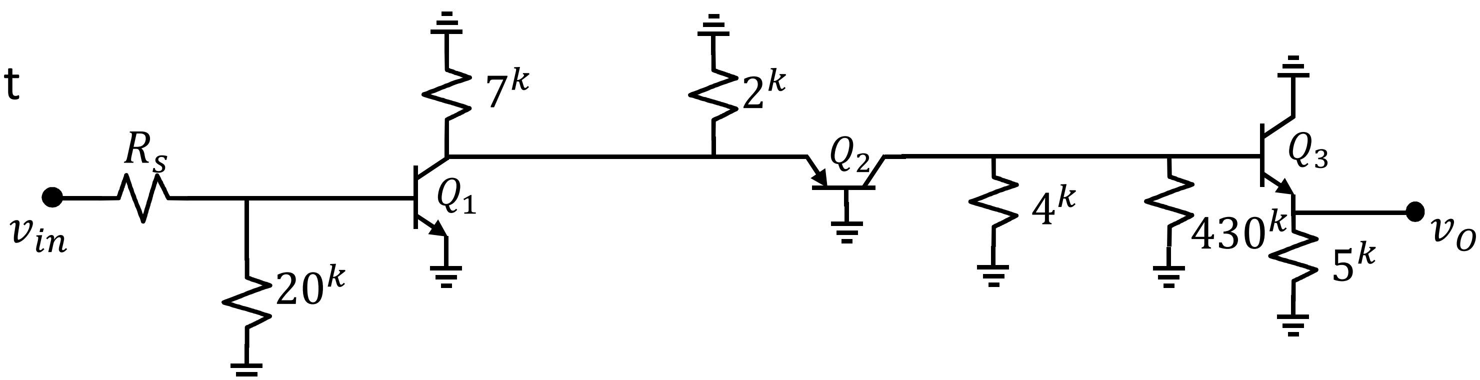


Example 05 – CE , CB, CC

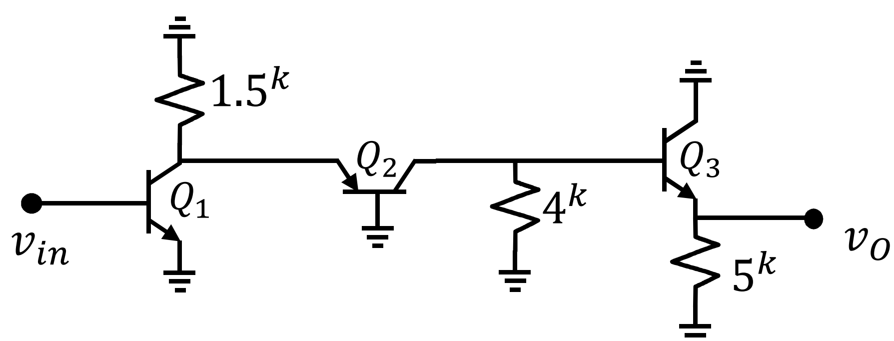
1. □□□□□□□□□□□□
2. □□□□□□□□□□
3. □□□□□□□
4. □□□□
5. □□□□□



AC circuit

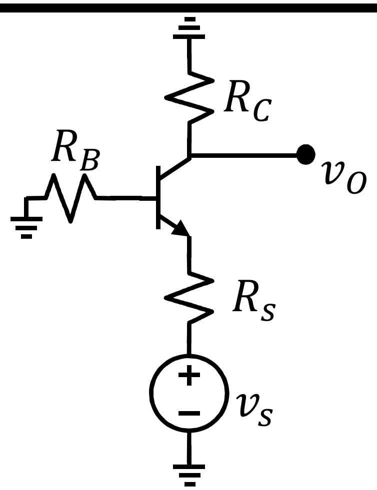


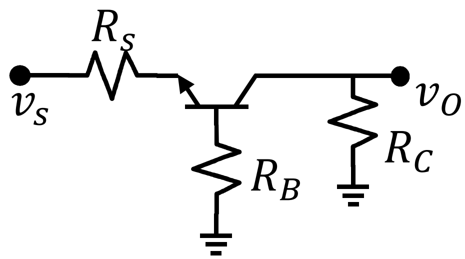
$R_s \ll ?$

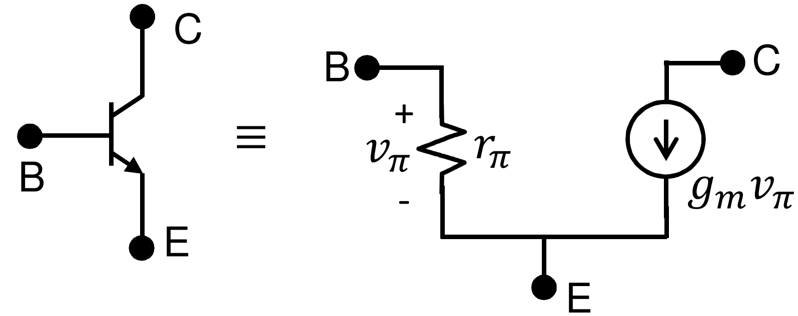


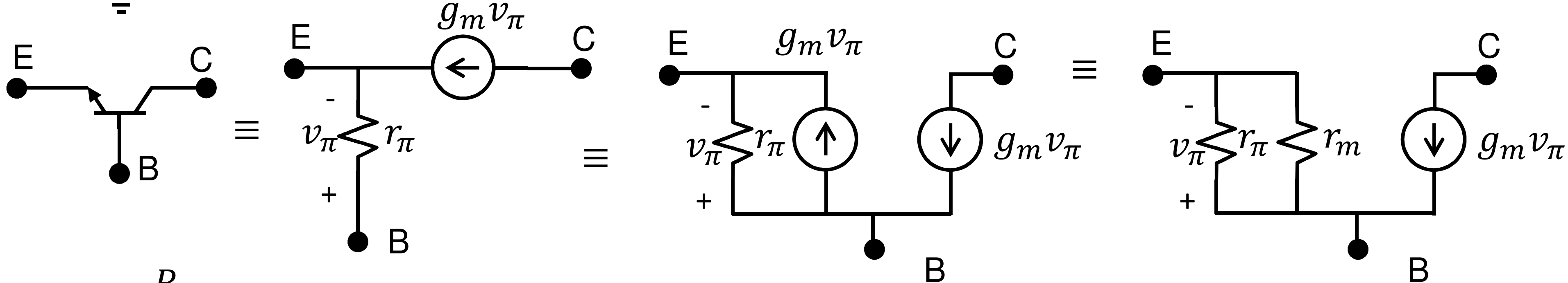
Common Base

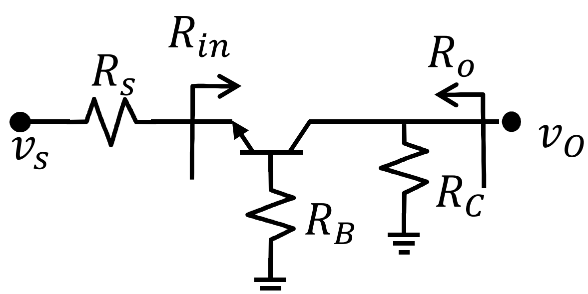
1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]







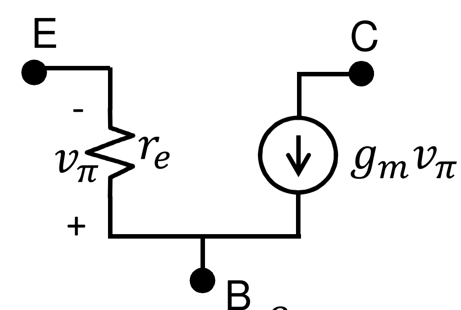




$$R_{in} = \frac{R_B + r_\pi}{\beta + 1}$$

$$R_o = R_c$$

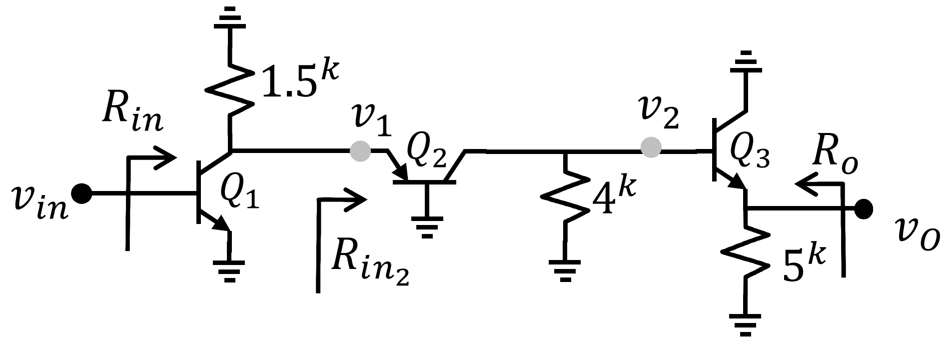
$$A_v = \frac{v_o}{v_s} = \frac{R_C}{\frac{R_B + r_\pi}{\beta + 1} + R_s}$$



$$r_e = r_m \parallel r_\pi = r_m \frac{\beta}{\beta + 1} \cong r_m$$

Example 05 – CE , CB, CC

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]



≈ r_m

$$\frac{v_1}{v_s} = -g_m\left(1.5^k \parallel R_{in_2}\right) = -1$$

$$\frac{v_2}{v_1} = +g_m\left(4^k \parallel (r_{\pi_2} + \beta 5^k)\right) = 160$$

$$\frac{v_o}{v_2} = \frac{5^k}{5^k + \frac{r_{\pi_2}}{\beta}} \cong 1$$

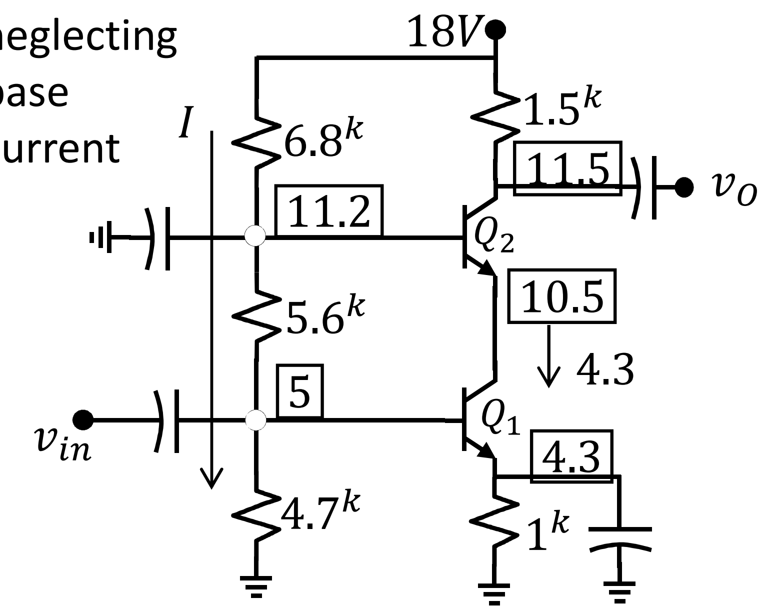
$$A_v = \frac{v_o}{v_s} = \frac{v_1}{v_s} \cdot \frac{v_2}{v_1} \cdot \frac{v_o}{v_2} = -160$$

$$R_{in} = r_{\pi_1} = 2.5^k$$

$$R_o = 4^k \parallel \frac{4^k + r_{\pi_2}}{\beta + 1} = 63\Omega$$

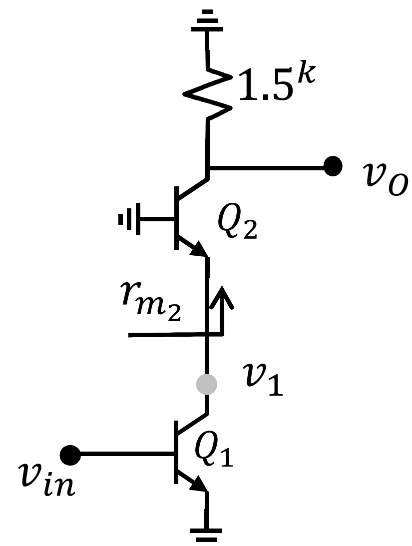
Cascode Amplifier , CE-CB

neglecting base current



$$I = \frac{18}{6.8 + 5.6 + 4.7} = 1.1^{mA}$$

AC circuit:



$$\frac{v_1}{v_{in}} = -g_{m_1}\left(r_{m_2}\right) = -1$$

$$\frac{v_o}{v_1} = \frac{1.5^k}{\frac{r_{\pi_2}}{\beta}} = 245$$

$$A_v = -245$$

Some Notes:

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2. [illegible]
3. [illegible]
4. [illegible]
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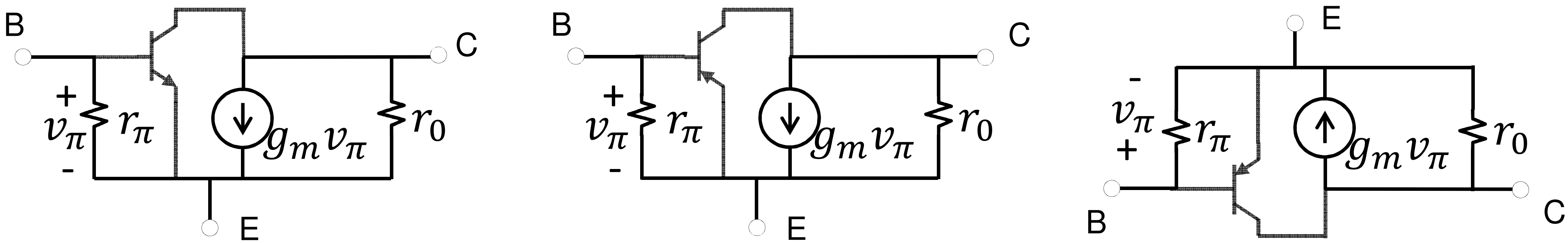
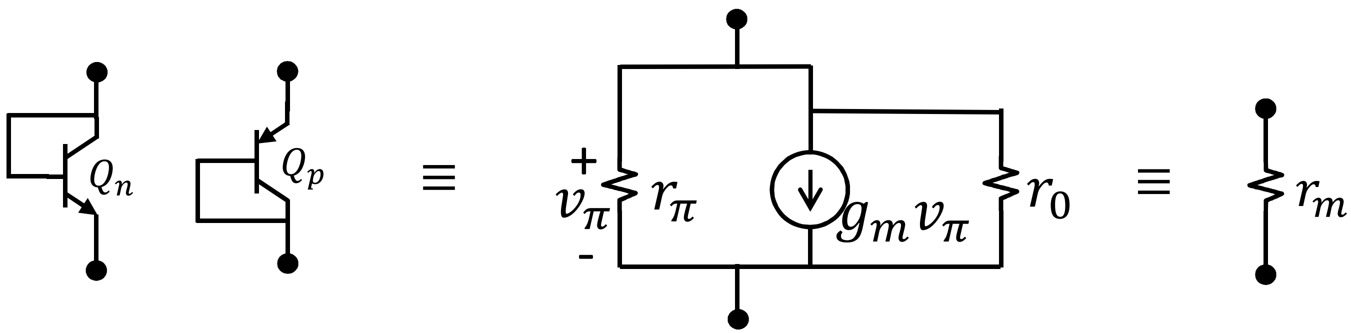
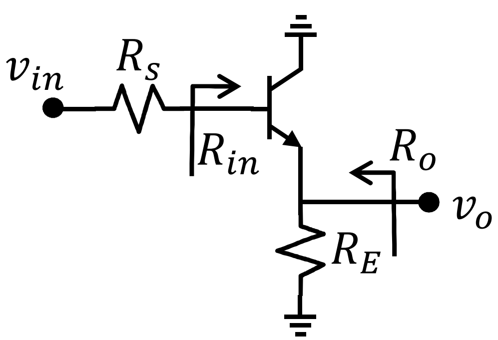




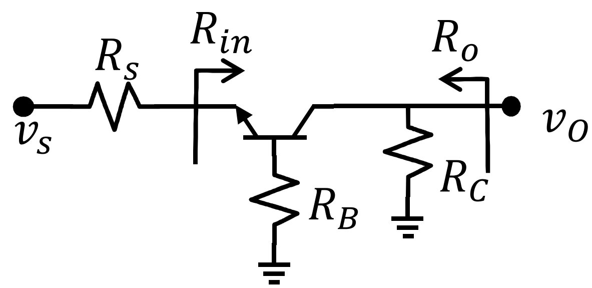
Summary



$$\frac{v_o}{v_{in}} = \frac{R_E}{\frac{R_S + r_\pi}{1+\beta} + R_E}$$

$$R_{in} = r_\pi + R_E(1+\beta)$$

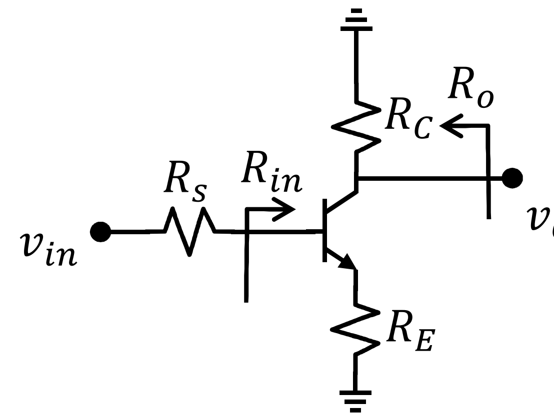
$$R_o = R_E \parallel \frac{R_S + r_\pi}{1+\beta}$$



$$\frac{v_o}{v_s} = \frac{R_C}{\frac{R_B + r_\pi}{\beta+1} + R_s}$$

$$R_{in} = \frac{R_B + r_\pi}{\beta + 1}$$

$$R_o = R_c$$



$$\frac{v_o}{v_s} = \frac{-R_C}{\frac{R_s + r_\pi}{\beta+1} + R_E}$$

$$R_{in} = r_\pi + R_E(1+\beta)$$

$$R_o = R_C$$

? V_A

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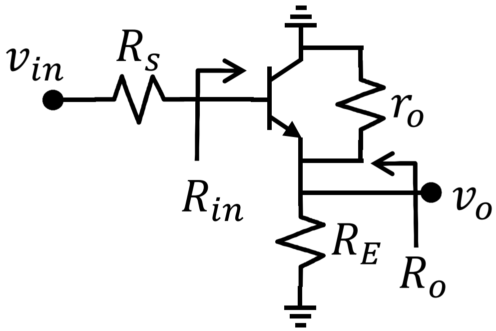
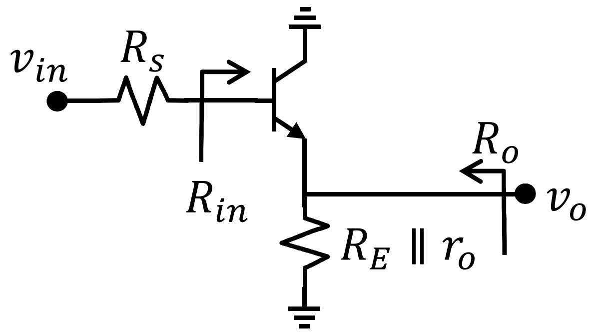


$r_o = \infty$



$$\frac{v_o}{v_{in}} = \frac{R_E}{\frac{R_S + r_\pi}{1+\beta} + R_E}$$

$$\frac{v_o}{v_{in}} = \frac{R_E \parallel r_o}{\frac{R_S + r_\pi}{1+\beta} + R_E \parallel r_o}$$

$$R_{in} = r_\pi + R_E(1+\beta)$$

$$R_{in} = r_\pi + (R_E \parallel r_o)(1+\beta)$$

$$R_o = R_E \parallel \frac{R_S + r_\pi}{1+\beta}$$

$$R_o = R_E \parallel r_o \parallel \frac{R_S + r_\pi}{1+\beta}$$

Input / Output Impedances

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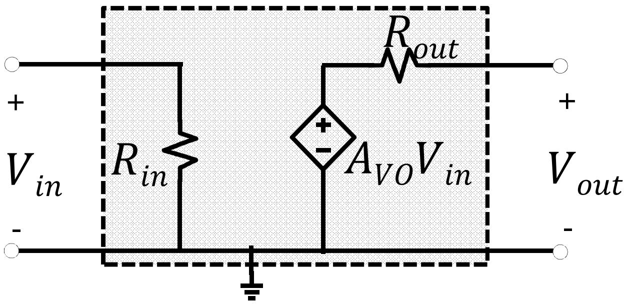

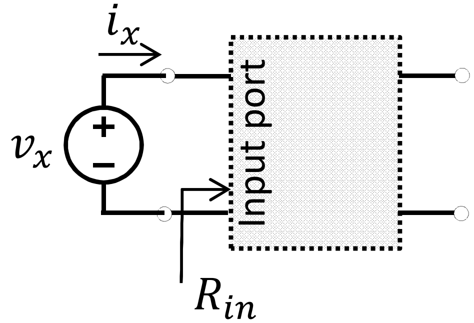

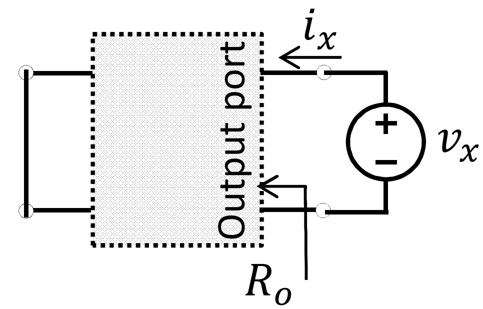

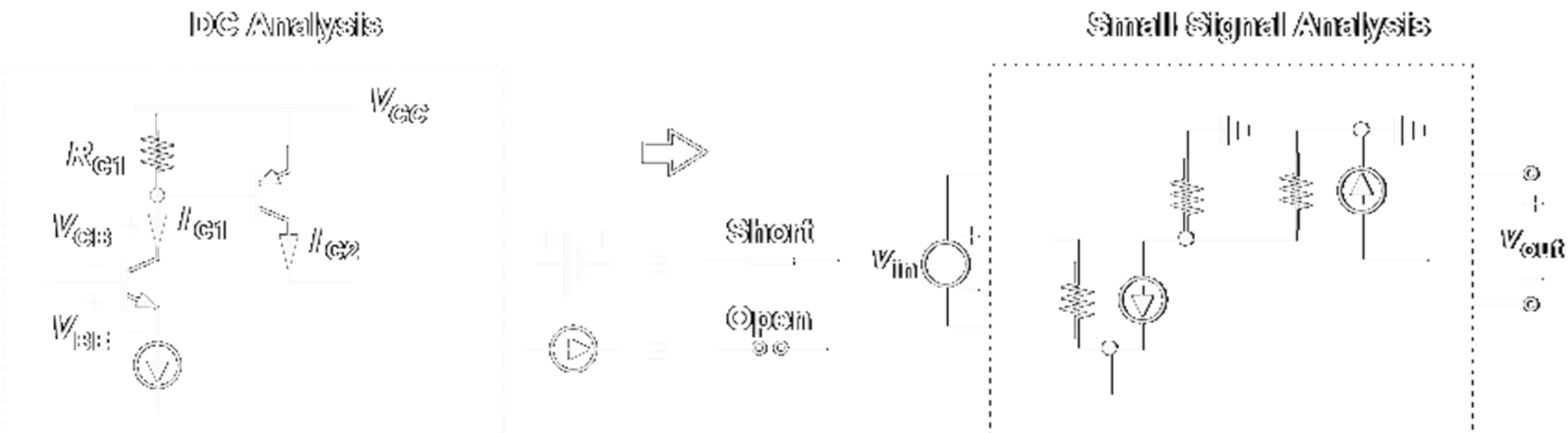

? V_A

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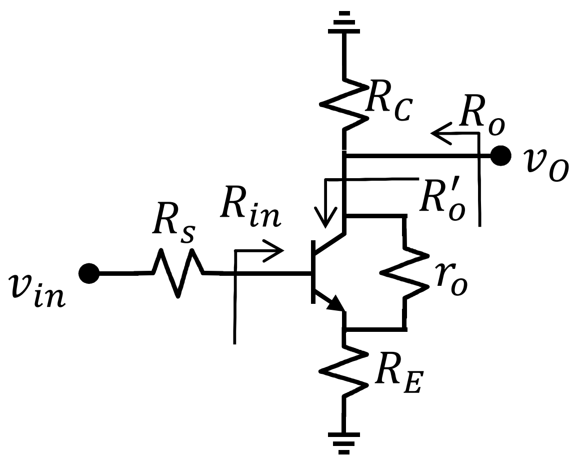
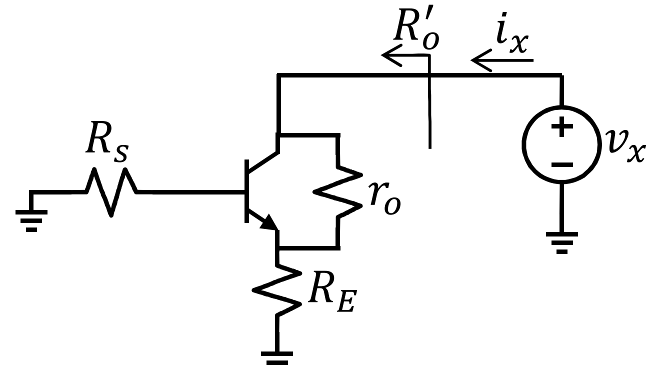
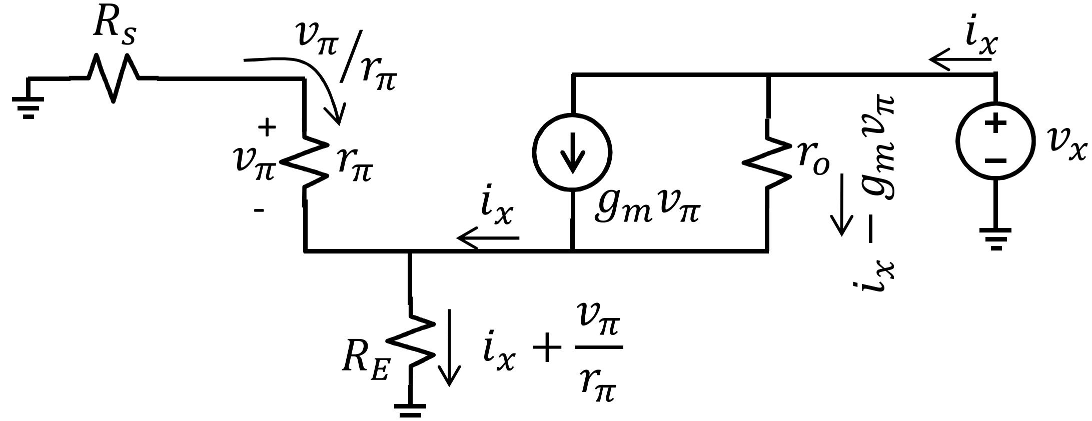


$$R_o = R_C \parallel R_o'$$



$$R_o' = \frac{v_x}{i_x}$$



$$\begin{cases} R_s \dfrac{v_\pi}{r_\pi} + v_\pi + R_E\left(i_x + \dfrac{v_\pi}{r_\pi}\right) = 0 \\ v_x = r_o(i_x - g_m v_\pi) + R_E\left(i_x + \dfrac{v_\pi}{r_\pi}\right) \end{cases}$$

$$R_o' = \frac{v_x}{i_x} = r_o\left(1 + \frac{\beta R_E}{R_s + r_\pi + R_E}\right) + R_E \parallel (R_s + r_\pi)$$

$$\cong r_o\left(1 + \frac{\beta R_E}{R_s + r_\pi + R_E}\right)$$

$$R_s = 0 \rightarrow \begin{cases} R_E \ll r_\pi : R_o' = r_o(1 + g_m R_E) \\ R_E \gg r_\pi : R_o' = \beta r_o \end{cases}$$

Nonlinear Transfer Function
Biasing

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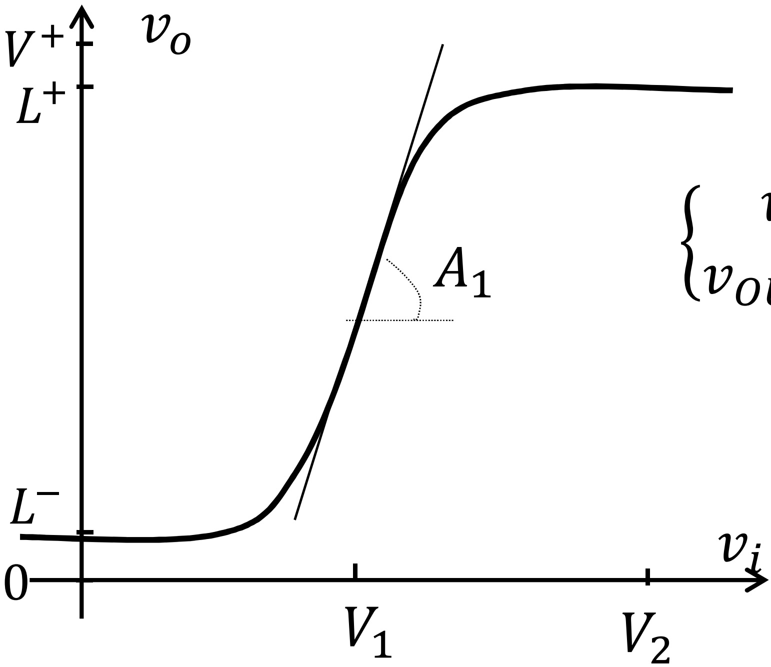
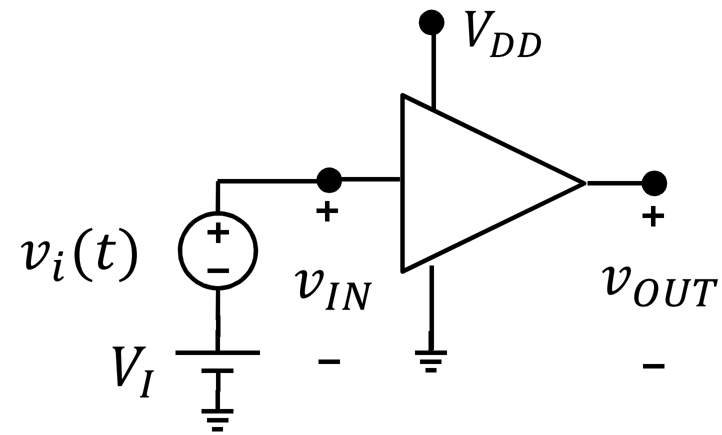


$$\begin{cases} v_{IN} = V_I + v_i(t) \\ v_{OUT} = V_O + A_v v_i(t) \end{cases}$$

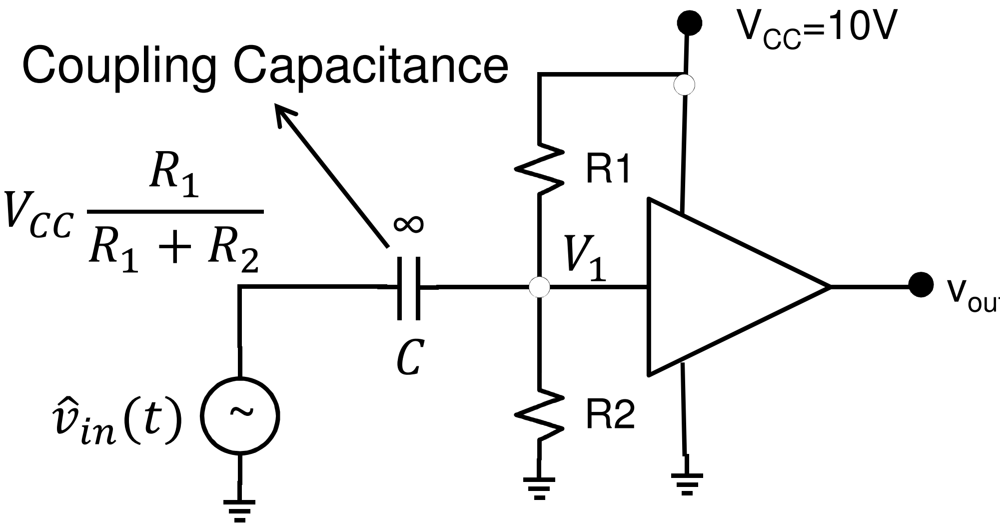


$$V_I = 0 \ \rightarrow A_v = 0$$

$$V_I = V_1 \ \rightarrow A_v = A_1$$

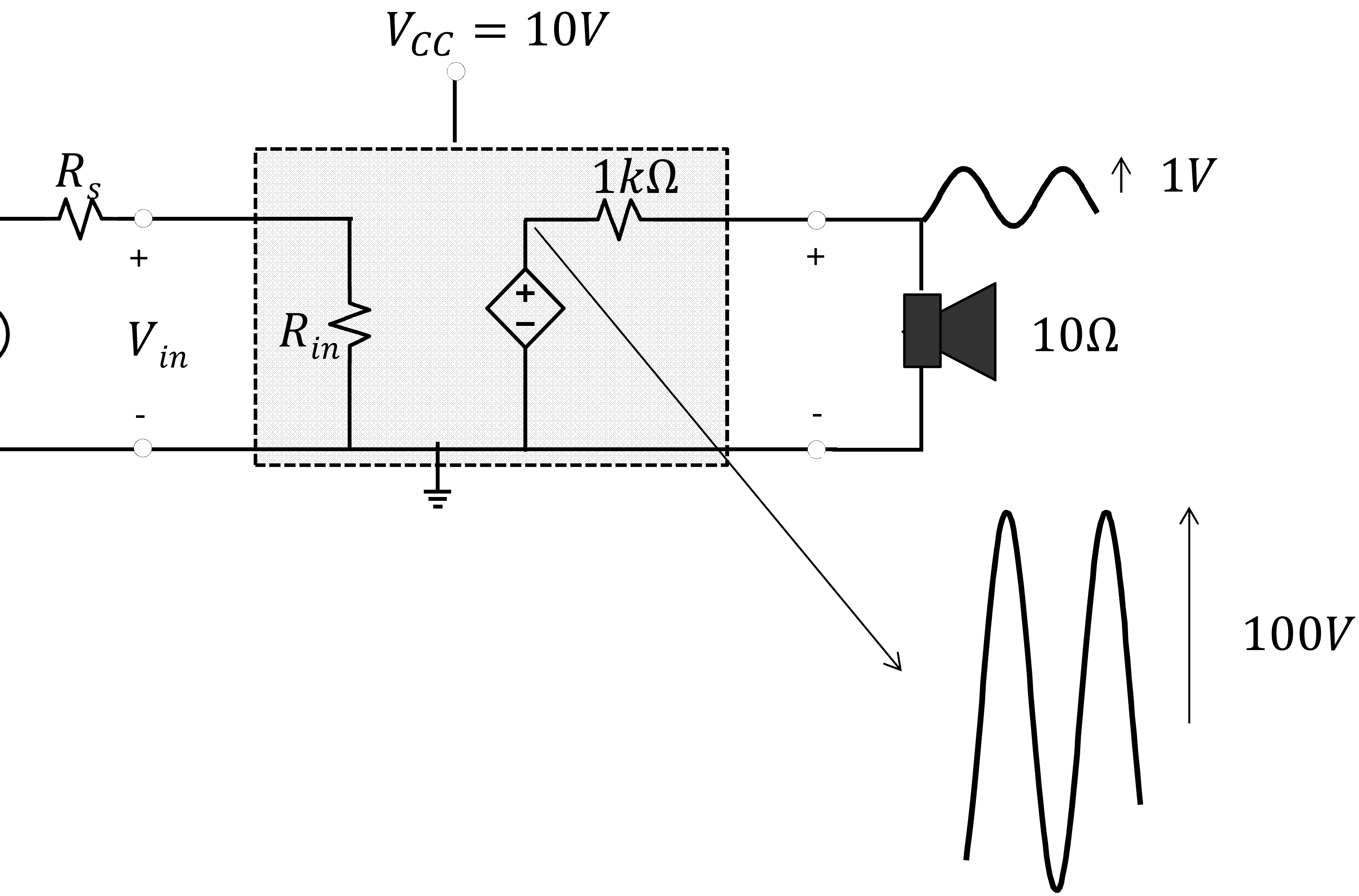
$$V_I = V_2 \ \rightarrow A_v = 0$$

$$V_1 = V_{CC}\frac{R_1}{R_1 + R_2}$$

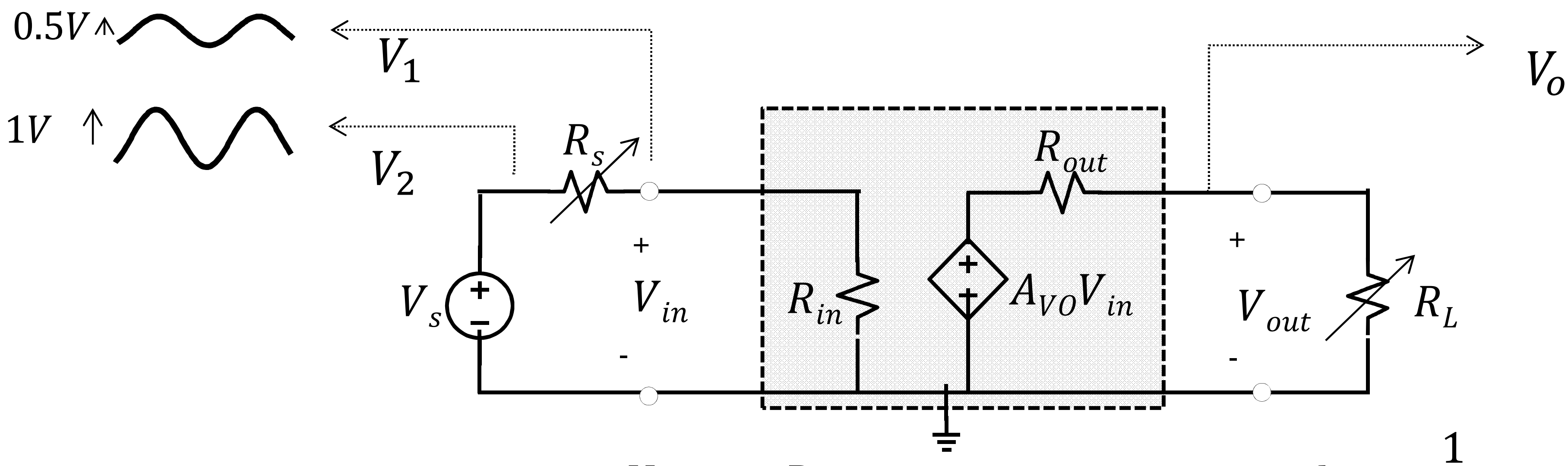


$$v_{out}(t) = V_{out} + A_v \hat{v}_{in}(t)$$

Practical Consideration



Practical Consideration: Input / Output Resistance

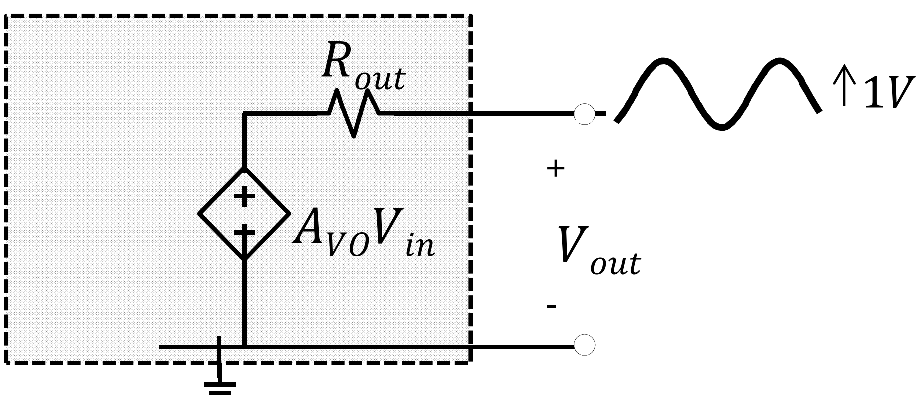


$$R_S = 1k\Omega \rightarrow \frac{V_1}{V_2} = \frac{R_{in}}{R_{in} + 1k}$$

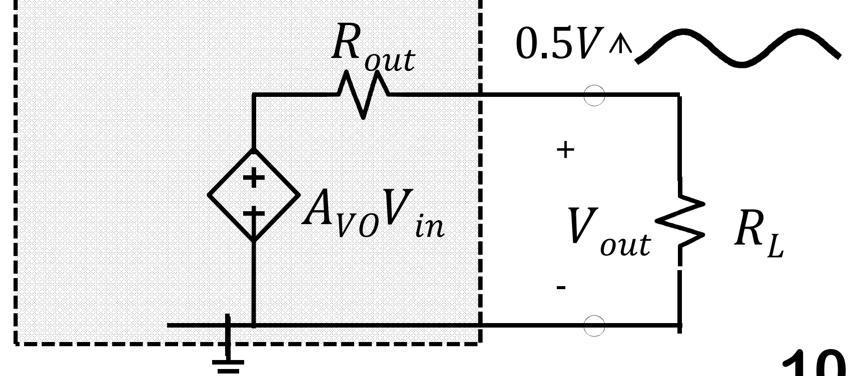
$$R_{in} = 1k \times \frac{1}{\frac{V_2}{V_1} - 1}$$

NOTE: You need to make sure circuit is in its linear operation regime

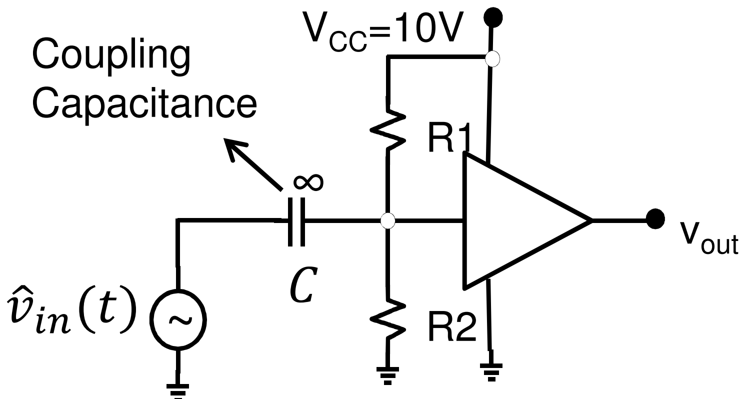
How about R_{out}?

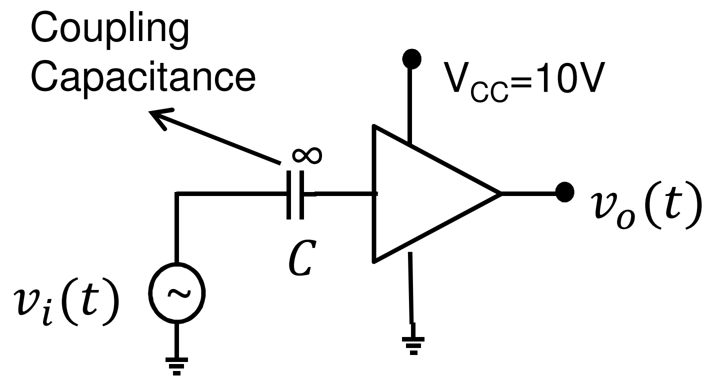


$R_L = R_{out}$



Amplifier Frequency Response





$v_i(t) = V_i \sin \omega t$ $v_o(t) = V_o \sin(\omega t + \varphi)$

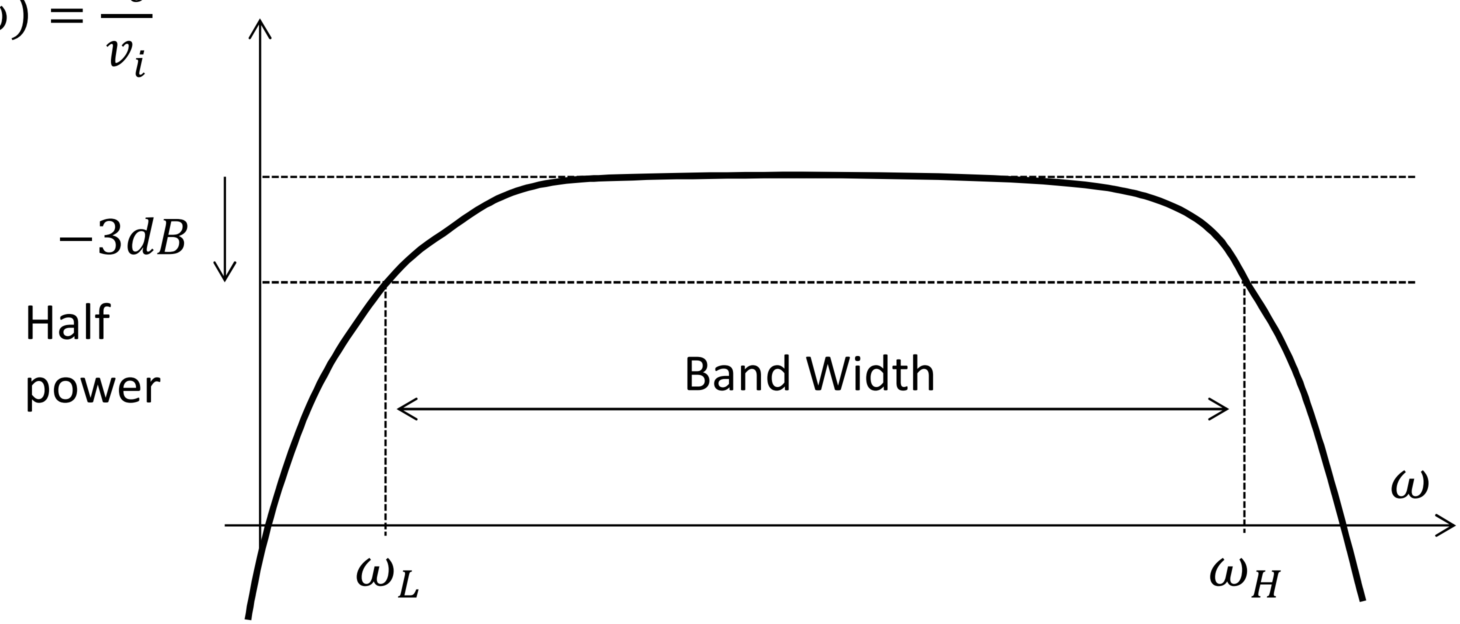
Transfer Function: $T(\omega) = \frac{v_o}{v_i}$

$|T(\omega)| = \frac{V_o}{V_i}$

Amplitude in dB

$\measuredangle T(\omega) = \varphi$

Phase



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