# ECE606: Solid State Devices
## Lecture 11
## Interface States Recombination
## Carrier Transport

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1) **SRH formula adapted to interface states**
2) Surface recombination in depletion region
3) Conclusion

# Surface Recombination Current

For single level bulk traps ….

$$R_{bulk} = \frac{np - n_i^2}{\frac{1}{c_p N_T}(n+n_1) + \frac{1}{c_n N_T}(p+p_1)} = \frac{(np - n_i^2) N_T}{\frac{1}{c_p}(n+n_1) + \frac{1}{c_n}(p+p_1)}$$

For single level interface trap at E …

$$R(E) = \frac{(n_s p_s - n_i^2) D_T(E) dE}{\frac{1}{c_{ps}}(n_s + n_{1s}) + \frac{1}{c_{ns}}(p_s + p_{1s})}$$

$$R = \int_{E_V}^{E_C} R(E) dE$$



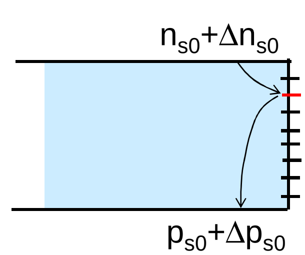
# Case 1: Minority Carrier Recombination

Donor doped

$$R(E) = \frac{\left[(n_{s0} + \Delta n_{s0})(p_{s0} + \Delta p_{s0}) - n_i^2\right] D_{IT}(E) dE}{\frac{1}{c_{ps}}(n_{s0} + \Delta n_{s0} + n_{1s}) + \frac{1}{c_{ns}}(p_{s0} + \Delta p_{s0} + p_{1s})}$$

$$= \frac{n_{s0} \Delta p_{s0} D_{IT}(E) dE}{n_{s0}\left[\frac{1}{c_{ps}} + \frac{n_{1s}}{c_{ps} n_{s0}} + \frac{p_{1s}}{c_{ns} n_{s0}}\right]}$$

$$= \frac{c_{ps} \Delta p_{s0} D_{IT}(E) dE}{\left[1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}} \frac{p_{1s}}{n_{s0}}\right]}$$



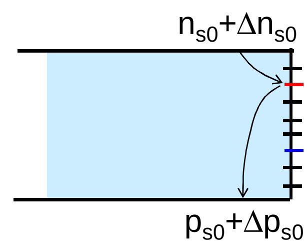
## Consider the Denominator …

$$D = 1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{n_{s0}} = 1 + \frac{n_{1s}}{N_D} + \frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{N_D}$$

$$= 1 + \frac{n_i e^{(E-E_i)\beta}}{n_i e^{(E_F-E_i)\beta}} + \frac{c_{ps}}{c_{ns}}\frac{n_i e^{-(E-E_i)\beta}}{n_i e^{(E_F-E_i)\beta}}$$

$$= 1 + e^{(E-E_F)\beta} + \frac{c_{ps}}{c_{ns}} e^{(E_F-E)\beta}$$

$$= 1 + e^{x} + a e^{-x} \qquad x \equiv \beta\left(E - E_F\right)$$

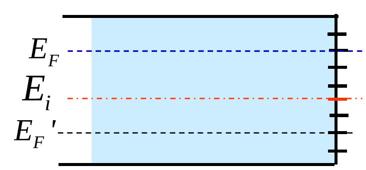





## Consider the Denominator …

$$D = 1 + \frac{n_i e^{(E-E_i)\beta}}{N_D} + \frac{c_{ps}}{c_{ns}}\frac{n_i e^{-(E-E_i)\beta}}{N_D}$$

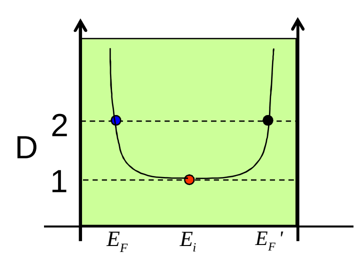
$$= 1 + e^{(E-E_F)\beta} + \frac{c_{ps}}{c_{ns}} e^{(E_F-E)\beta}$$




At $E = E_i \Rightarrow \quad D = 1 + \frac{n_i}{N_D} + \frac{c_{ps}}{c_{ns}}\frac{n_i}{N_D} \approx 1$

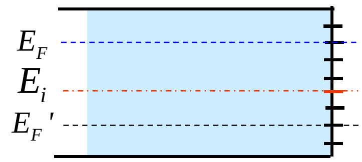
At $E = E_F > E_i, x = 0 \quad D = 1 + 1 + \frac{c_{ps}}{c_{ns}} \times small \approx 2$

At $E = E_F' < E_i, \quad D = 1 + small + 1 = 2$

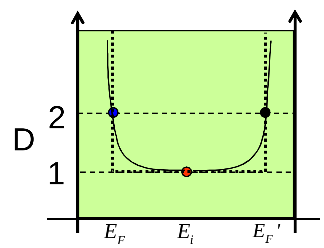


## Approximate the Denominator …

$$D = 1 + \frac{n_i e^{(E-E_i)\beta}}{N_D} + \frac{c_{ps}}{c_{ns}} \frac{n_i e^{-(E-E_i)\beta}}{N_D}$$




$$D \approx \begin{bmatrix} 1 & \text{for} \quad E_F \le E \le E_F' \\ \infty & \qquad\qquad \text{otherwise} \end{bmatrix}$$



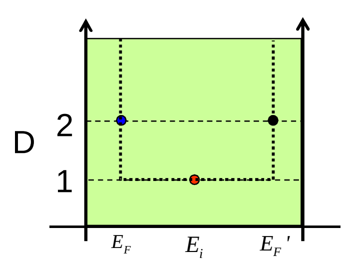





## Integrated Recombination

$$R = \int_{E_V}^{E_C} R(E) = \int_{E_V}^{E_C} \frac{c_{ps}\Delta p_{s0} D_{IT}(E)\, dE}{\left[1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}} \frac{p_{1s}}{n_{s0}}\right]}$$

$$\approx \int_{E_F'}^{E_F} c_{ps}\Delta p_{s0} D(E)\, dE$$

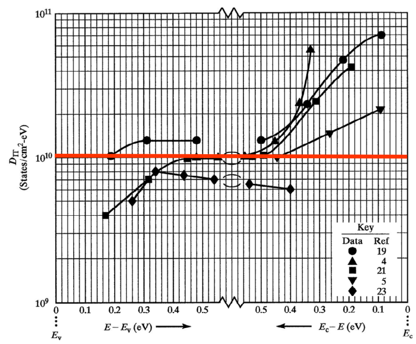


## Surface Recombination Velocity

$$R \approx \int_{E_F}^{E_F} c_{ps} \Delta p_{s0} D_{IT}(E)\, dE$$

$$= c_{ps} D_{IT} \left(E_F - E_F^{'}\right) \Delta p_{s0}$$

$$= s_g \Delta p_{s0}$$




Surface recombination velocity



## Outline

1) Nature of interface states
2) SRH formula adapted to interface states
3) **Surface recombination in depletion region**
4) Conclusion

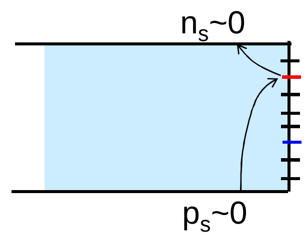
## Case 2: Recombination in Depletion

$$R(E) = \frac{\left(n_s p_s - n_i^2\right) D_{IT}(E)\, dE}{\frac{1}{c_{ps}}\left(n_s + n_{1s}\right) + \frac{1}{c_{ns}}\left(p_s + p_{1s}\right)}$$

$$= -\frac{n_i}{\frac{n_i e^{(E-E_i)\beta}}{c_{ps}} + \frac{n_i e^{-(E-E_i)\beta}}{c_{ns}}} n_i D_{IT}(E)\, dE$$

$$= -c_{ns} D_{IT} n_i \frac{e^{(E-E_i)\beta} dE}{\frac{c_{ns}}{c_{ps}} e^{2(E-E_i)\beta} + 1}$$

$$R = -c_{ns} D_{IT} n_i \int_{E_V}^{E_C} \frac{e^{(E-E_i)\beta} dE}{\frac{c_{ns}}{c_{ps}} e^{2(E-E_i)\beta} + 1}$$





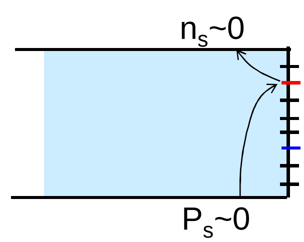

## Case 2: Recombination in Depletion

$$R = -c_{ns} D_{IT} n_i \int_{E_V}^{E_C} \frac{e^{(E-E_i)\beta} dE}{\frac{c_{ns}}{c_{ps}} e^{2(E-E_i)\beta} + 1}$$

$$= -c_{ns} D_{IT} n_i \int_{-\infty}^{+\infty} \frac{e^{(E-E_i)\beta} dE}{\frac{c_{ns}}{c_{ps}} e^{2(E-E_i)\beta} + 1}$$

$$= -c_{ns} D_{IT} n_i \phi \sqrt{\frac{c_{ps}}{c_{ns}}} \int_{0}^{+\infty} \frac{dx}{x^2 + 1}$$

$$= -\sqrt{c_{ns} c_{ps}}\, D_{IT} n_i \beta \frac{\pi}{2}$$



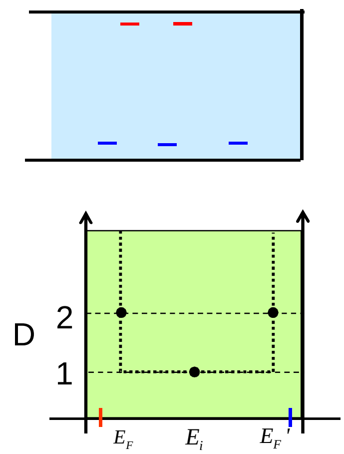
## Why do donors/acceptors *not* act as R-G Centers?

$$R(E_D) = \frac{c_{ps}\Delta p_{s0} D(E)\,dE}{\left[1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{n_{s0}}\right]}$$

$$= \frac{c_{ps}\Delta p_{s0} N_D}{D(E_D)} \rightarrow 0$$

$$R(E_A) = \frac{c_{ps}\Delta p_{s0} D(E)\,dE}{\left[1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{n_{s0}}\right]}$$

$$= \frac{c_{ps}\Delta p_{s0} N_A}{D(E_A)} \rightarrow 0$$






## Summary

$$R = -\sqrt{c_{ns}c_{ps}}\, D_{IT}\, \beta \frac{\pi}{2} \times n_i \qquad \text{Interface (depletion)}$$

$$R = c_{ps} D_{IT} \left(E_F - E_F'\right) \Delta p_s \qquad \text{Interface (minority)}$$

$$R = c_p N_T \Delta p \qquad \text{Bulk (minority)}$$

# ECE606: Solid State Devices
## Lecture 12
## Carrier Transport

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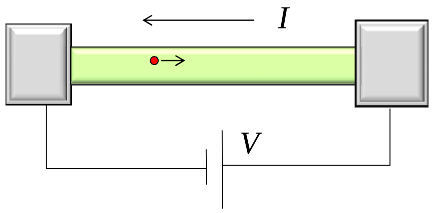
## Outline

1) **Overview**
2) Drift Current
3) Physics of Mobility
4) High field effects
5) Conclusion

REF: Advanced Device Fundmentals, Pages 175- 192

## Current Flow Through Semiconductors




$$I = G \times V$$

$$= q \times n \times v \times A$$

Carrier Density (n) — velocity (v)

Depends on chemical composition, crystal structure, temperature, doping, etc.

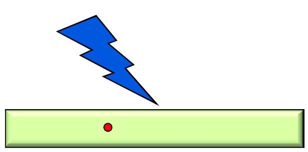
**Quantum Mechanics + Equilibrium Statistical Mechanics**
⇒ Encapsulated into concepts of effective masses and occupation factors (Ch. 1-4)

**Transport with scattering, non-equilibrium Statistical Mechanics**
⇒ Encapsulated into drift-diffusion equation with recombination-generation (Ch. 5 & 6)



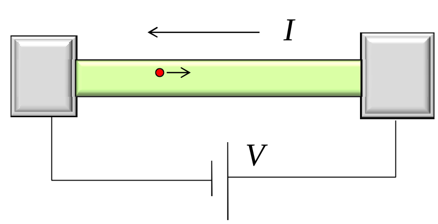
## Non-equilibrium Systems

Chapter 5



vs.

**Chapter 6**



## Summary of Transport Equations …

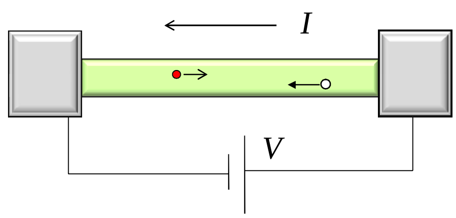
$$\nabla \bullet D = q\left(p - n + N_D^+ - N_A^-\right)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \bullet \mathbf{J}_N - r_N + g_N$$

$$\mathbf{J}_N = qn\mu_N E + qD_N \nabla n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \bullet \mathbf{J}_P - r_P + g_P$$

$$\mathbf{J}_P = qp\mu_P E - qD_P \nabla p$$






## Outline

1) Overview
2) **Drift Current**
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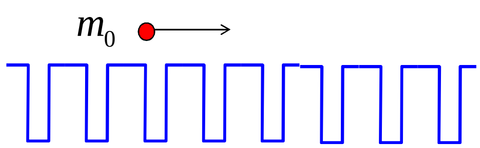
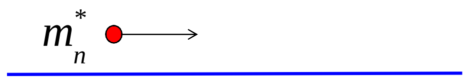
## Meaning of Effective Mass …




$$\left(-\frac{\hbar^2}{2m_0}\frac{d^2}{dx^2}+U_{crys}(x)+U_{ext}(x)\right)\psi=E\psi$$




$$\left(-\frac{\hbar^2}{2m_0^*}\frac{d^2}{dx^2}+U_{ext}(x)\right)\phi=E\phi$$



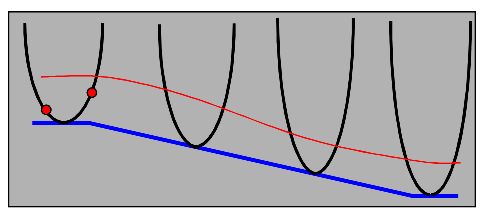


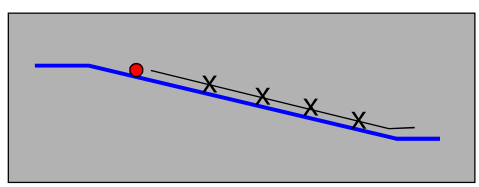
## Drift by Electric field ….

$$J_n=qn\mu_n\mathcal{E}$$

$$\frac{d(m_n^*\upsilon)}{dt}=-q\mathcal{E}-\frac{m_n^*\upsilon}{\tau_n}$$

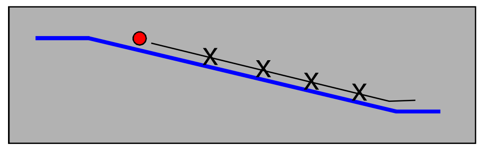
$$\upsilon(t)=-\frac{q\tau_n}{m_n^*}\mathcal{E}\left[1-e^{-\frac{t}{\tau_n}}\right]$$





## Drift by Electric field ....

$$\upsilon(t) = -\frac{q\tau_n}{m_n^*}\mathcal{E}\left[1 - e^{-\frac{t}{\tau_n}}\right]$$

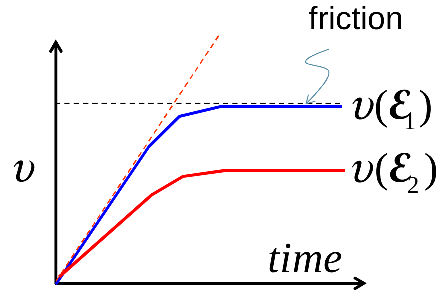


$$= -\frac{q\tau_n}{m_n^*}\mathcal{E} \quad (t \to \infty,\ \text{1-2 ps})$$

$$\equiv \mu_n \mathcal{E}$$

$$J_n = qn\mu_n \mathcal{E}$$

(Theory valid once t > 1-2 ps)

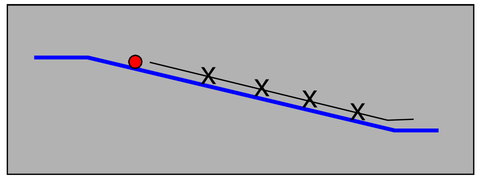




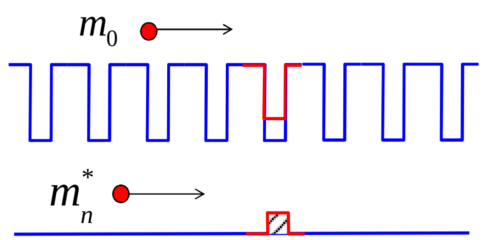

## Outline

1) Overview
2) Drift Current
3) **Physics of Mobility**
4) High field effects
5) Conclusion

## Mobility and Physics of Scattering Time



$$\mu_n = \frac{q\tau_n}{m_n^*}$$




Fermi's Golden rule …

$$\tau_n^{-1} \sim \left| \frac{2\pi}{\hbar} \int_{-\infty}^{\infty} \psi^*(x) U(x) \psi(x) dx \right|^2$$



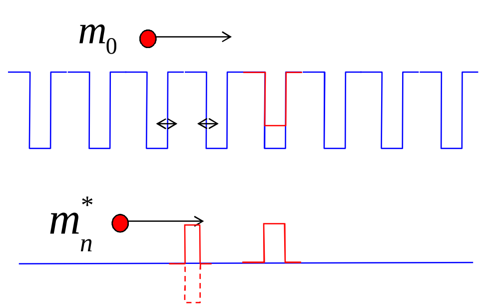
## Phonon and Ionized Impurity Scattering

Ionized impurity

$$\tau_n \sim \frac{T^{3/2}}{N_D}$$

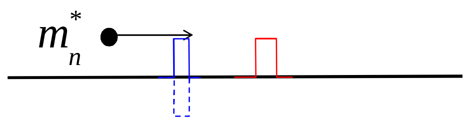
Higher temperature,
more phonon scattering

$$\tau_n \sim T^{-3/2}$$



## Multiple Scattering Events

- Ionized impurity
- Phonon scattering
- others ….




$$\frac{1}{\mu_n} = \frac{1}{\mu_{ph}} + \frac{1}{\mu_{II}}$$

$$\Rightarrow \mu_n = \frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}}$$

$$= \mu_{\min} + \left( \frac{\mu_{ph}\mu_{II}}{\mu_{ph} + \mu_{II}} - \mu_{\min} \right)$$

$$= \mu_{\min} + \left( \frac{\mu_0}{1 + (N_I/N_0)^{\alpha}} \right)$$

$$\frac{1}{\tau_n} = \frac{1}{\tau_{II}} + \frac{1}{\tau_{ph}} + \frac{1}{\tau_s} + \cdots$$

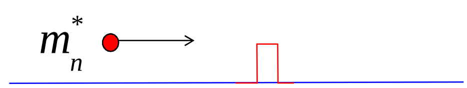
$$\frac{1}{\mu_n} = \frac{m_n^*}{q\tau_n}$$

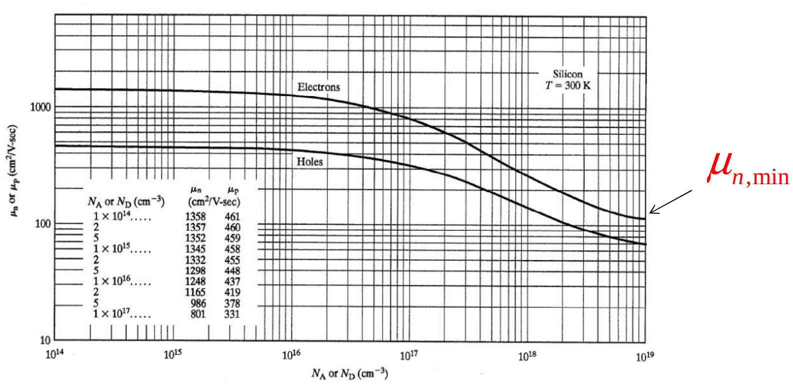
Matthession Rule ….



## Model for Ionized impurity Scattering

$$\mu_n = \mu_{n,\min} + \left( \frac{\mu_{0,n}}{1 + (N_I/N_{0,n})^{\alpha_n}} \right)$$

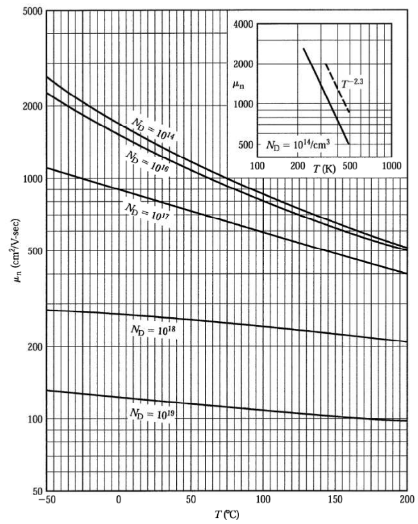



| $N_A$ or $N_D$ (cm$^{-3}$) | $\mu_n$ (cm$^2$/V-sec) | $\mu_p$ (cm$^2$/V-sec) |
|---|---|---|
| $1 \times 10^{14}$….. | 1358 | 461 |
| 2 | 1357 | 460 |
| 5 | 1352 | 459 |
| $1 \times 10^{15}$….. | 1345 | 458 |
| 2 | 1332 | 455 |
| 5 | 1298 | 448 |
| $1 \times 10^{16}$….. | 1248 | 437 |
| 2 | 1165 | 419 |
| 5 | 986 | 378 |
| $1 \times 10^{17}$….. | 801 | 331 |

## Temperature-dependent Mobility

$$\mu_n \sim \tau_n \sim T^{-3/2}$$

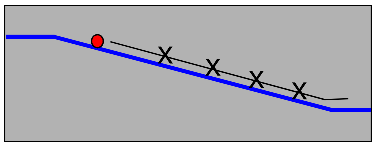




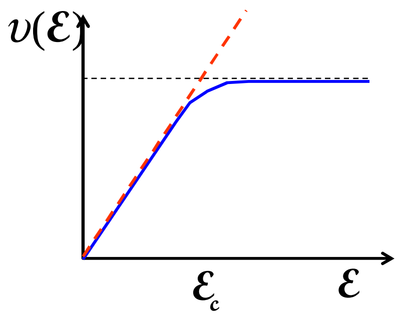

## Outline

1) Overview
2) Drift Current
3) Physics of Mobility
4) **High Field Effects**
5) Conclusion

# Mobility at High Fields?



$$\upsilon = \frac{q\tau_N}{m_N^*}\mathcal{E}$$



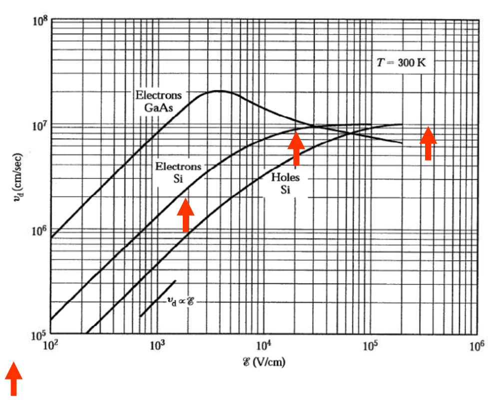

What causes velocity saturation at high fields?

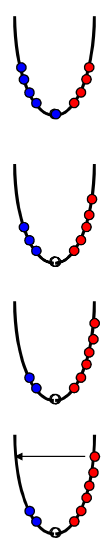
Where does all the mobility formul
in device simulator come from?





# Velocity Saturation in Si/Ge



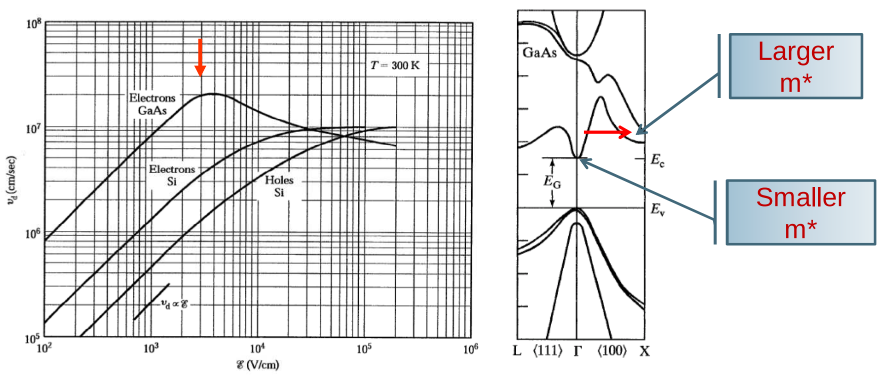
$\mathcal{E} = 0 \quad J_1 = J^+ - J^- = 0$

$\mathcal{E} \ll \mathcal{E}_c \quad J_2 = J^+ - J^- > J_1$

$\mathcal{E} \approx \mathcal{E}_c \quad J_3 = J^+ - J^- > J_2$

$\mathcal{E} \gg \mathcal{E}_c \quad J_4 = J^+ - J^- \approx J_3$

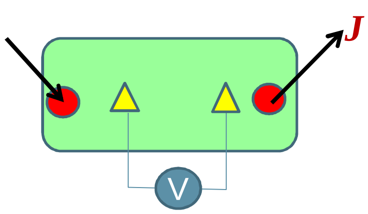
## Velocity Overshoot & Inter-valley Transfer




What type of scattering would you need for inter-valley transfer?



## Doping dependent Resistivity



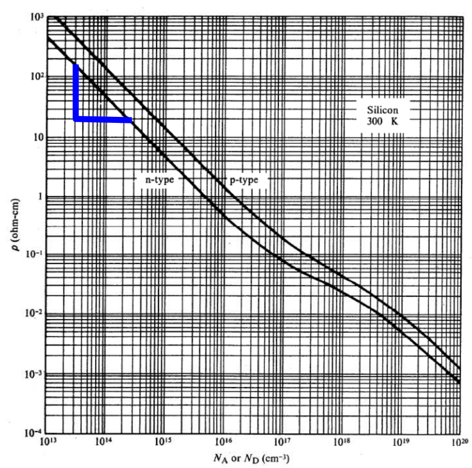

$$\mathcal{E} = \rho J$$

$$J = q(\mu_n n + \mu_p p)\mathcal{E}$$

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$$= \frac{1}{q\mu_n N_D} \ldots \text{for n-type}$$

$$= \frac{1}{q\mu_p N_A} \ldots \text{for p-type}$$



# Conclusion

1) Poisson and drift-diffusion equations form a complete semi-classical transport model that can explain wide variety of device phenomena.

2) Drift current results from response of electrons/holes to electric field. The physics of mobility is complex and material dependent.

3) Constancy of low-field mobility can be checked by experiments.