

**VLSI Interconnect – HW02 – due 28 Ordibehesht**

1. Consider a circular conductor made of Copper ( $\rho = 2.2 \mu\Omega cm$ ).

a) Calculate DC resistance per unit length of the wire for  $D = [1mm, 10 \mu m, 100nm]$ .

b) Using classical formula for skin depth, calculate skin depth for Copper at  $f = [1Ghz, 10Ghz, 100Ghz]$ . Use resistance per unit length of the wires in part a) at these frequencies.

c) Use COMSOL (or any other package), and find the current density vs.  $r$  (distance from the center of the wire). Assume that the total current for the wire is constant (You may do this, by setting a constant magnetic field at the surface of the wire). Now plot current density vs.  $r$  for different frequencies. Find  $r_{skin}$ , the position at which the current density is reduced to  $1/e$  of its value at the surface. Plot  $r_{skin}$  vs. frequency and compare  $r_{skin}$  with the classical skin depth calculated in part b). If the total current is constant, is there a position in the wire that the current density is independent of the frequency? Explain why!

d) Use appropriate grid approximation, and estimate the value for the resistance per unit length of the wire at  $f = [1Ghz, 10Ghz, 100Ghz]$ . Compare the results with your answers in part b).

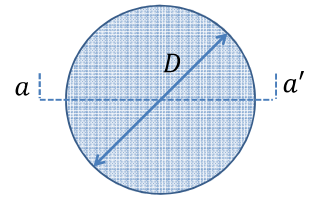


Figure 1

2. Now consider a square Copper conductor as shown in Figure 2. Repeat Problem 1, for this conductor. This time you may need to choose different grids and plot current density at different cross-sections.

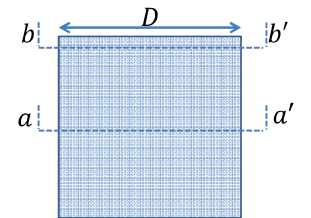


Figure 2

3. Use (electrostatic 2D model of) FEMLAB (or any other package) for calculating the interconnect! Define a  $10\mu m \times 10\mu m$  window as shown in Figure 3.a of a relative dielectric constant of  $\epsilon_r = 2$ . Then add a  $1\mu m \times 2.2\mu m$  wire, above a ground plane as shown. Assume that the ground plane is at 0 volt and the wire is at 1 volt.

a) What is the charge per unit length at each edge of the wire? What is the charge per unit length at the ground plane?

b) Use “integration over boundaries” function of the FEMLAB to calculate the total charge/capacitance per unit length of the wire. Compare your results with the empirical expression by Sakurai.

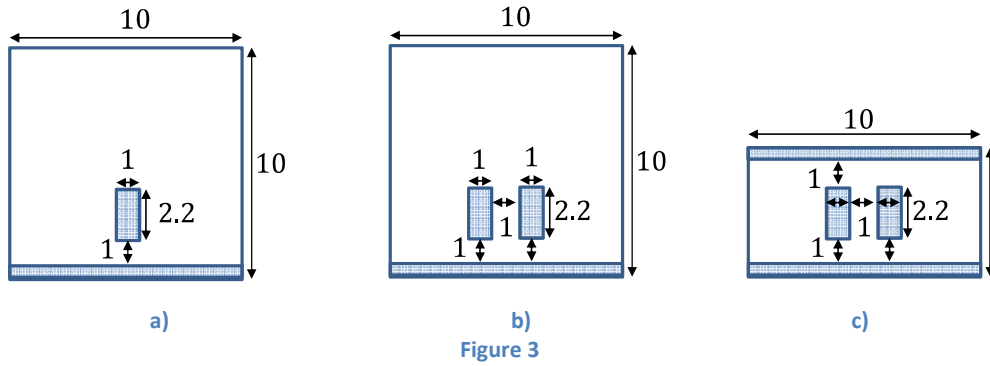
c) As shown in Figure 3.b, add another  $1\mu m \times 2.2\mu m$  wire spaced at  $1\mu m$  from the previous wire. Find  $2 \times 2$  capacitance per unit length matrix of this configuration. Compare the results with what you calculated in part b). Compare the results with the empirical expression by Sakurai.

d) As shown in Figure 3.c, add a ground plane above the two wires that is  $1\mu m$  above them. Find  $2 \times 2$  capacitance per unit length matrix of this configuration. Compare the results with what you calculated in part b). Compare the results with the empirical expression by Sakurai. (For this part, approximate that the ground capacitance is twice Sakurai’s ground capacitance expression.)

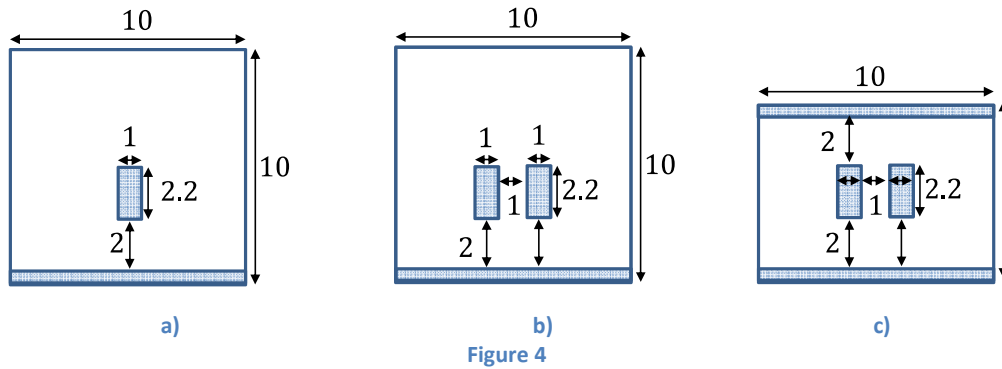
[Note: In COMSOL (or any other package for that matter), you need to set a value for potentials at the boundaries; Here the  $10\mu m \times 10\mu m$  window. Two usual choices are: i. zero boundary, and ii. periodic boundary. Explain which of these choices will over/under-estimate the capacitance. What is your suggestion?]

**(Extra credit)** e) Assume “aspect ratio” is defined as line height over width. Find elements of  $2 \times 2$  capacitance matrix of this configuration vs.  $AR$  for  $AR = 3 \dots 0$ .

**(Extra credit)** f) Repeat part d) by adding 3 more lines (and needed grids) to find  $5 \times 5$  capacitance per unit length matrix of this configuration.



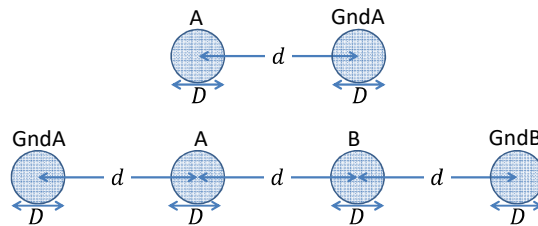
g) Repeat a) to d) while wire distance from ground changed is in Figure 4 and compare results with previous ones.



4. Drive a formula for the capacitance/inductance per unit length for the following wires. Assume that the distance between wires,  $d$ , is much larger than the inner radius of the wire,  $D$ . Also assume that wire labeled "GND" is at zero potential for capacitance calculation and is the current return path for the inductance calculation. When calculating the inductance, assume that the fields do not penetrate the conductor. (What does this assumption mean?)

a) For structure shown in Figure 4.a.

b) For structure shown in Figure 4.b. This time a  $2 \times 2$  matrix for capacitance,  $C_{2 \times 2}$  and for inductance,  $L_{2 \times 2}$ . Evaluate  $C_{2 \times 2} \cdot L_{2 \times 2}$ .



**Figure 5**

5. Using the Ruehli's formulas for the self and partial mutual inductances, calculate the total loop inductance of the following configurations. The grid dimensions are  $1 \mu\text{m} \times 1 \mu\text{m}$ , and the darkened grid points represent very thin conductors.

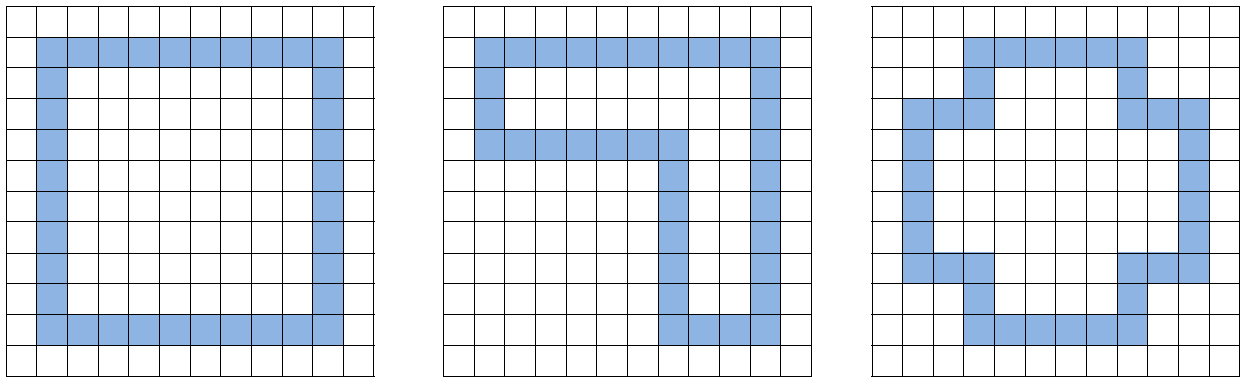


Figure 6

**(Extra credit) 6.** Repeat Problem 5 for the following structure. Assume that the loop completed by connecting squares “a” and “b” in another level (you may still consider it as a 2D problem). This structure has been used for manufacturing of on-chip inductances. Search literatures and find the formula for this structure and compare your results with that formula.

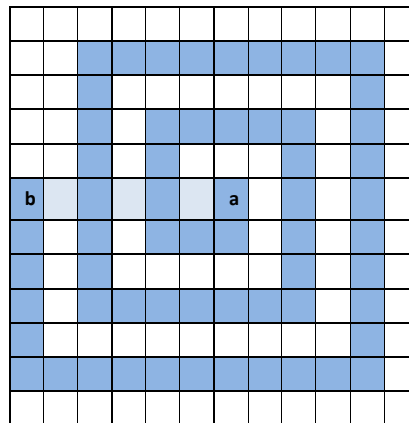


Figure 7