Compact Distributed RLC Interconnect Models—Part I: Single Line Transient, Time Delay, and Overshoot Expressions

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***Abstract*—Novel compact expressions that describe the transient response of a high-speed distributed resistance, inductance, and capacitance (rlc) interconnect are rigorously derived with on-chip global interconnect boundary conditions. Simplified expressions enable physical insight and accurate estimation of transient response, time delay, and overshoot for high-speed global interconnects with the inclusion of inductance.**

***Index Terms*—Bessel functions, inductance, interconnections, time domain analysis, transmission line theory.**

I. Introduction

INTERCONNECT models must incorporate distributed self and mutual inductance to accurately estimate time delay and crosstalk in a multilevel network for multi-GHz gigascale integration (GSI) [1]. Sakurai has rigorously derived compact expressions for the transient response of a distributed resistance capacitance (rc) interconnect [2]. This work significantly extends his expressions to include self and mutual inductance in models of high-speed GSI interconnects. Novel compact expressions for transient response describe the time delay and overshoot of a distributed resistance, inductance, and capacitance (rlc) transmission line model of a high-speed, on-chip interconnect. In a companion paper, these results are extended to describe the worst-case time delay and crosstalk of two and three coupled lines in a multilevel wiring network [3].

II. Transient Voltage of Distributed RLC Interconnects

Sakurai solved a partial differential equation (PDE) that describes a single distributed rc interconnect [2]. In this section, the transient response of a single distributed rlc interconnect is rigorously derived. From this solution, the transient responses of two and three coupled interconnect are determined in a companion paper [3].

A. Semi-Infinite Line

The transient response of a single semi-infinite distributed rlc interconnect with arbitrary source impedance and driven by a step input voltage as seen in Fig. 1(a) is first determined. The PDE that describes a single distributed rlc line is given by

$$\frac{\partial^2}{\partial x^2} V(x,t) = lc\frac{\partial^2}{\partial t^2} V(x,t) + rc\frac{\partial}{\partial t} V(x,t) \qquad (1)$$

where

- r distributed resistance per unit length;
- l distributed inductance per unit length;
- c distributed capacitance per unit length.

Using a single-sided Laplace transform of $V(x,t)$, the differential equation in (1) becomes an ordinary differential equation. It is assumed that the initial values of the voltage and the current on the transmission line are zero which gives

$$\frac{\partial^2}{\partial x^2} V(x,s) = V(x,s)lcs\left(s+\frac{r}{l}\right). \qquad (2)$$

The general solution to this expression in the Laplace domain is

$$V(x,s) = Ae^{-x\sqrt{lc}\sqrt{s(s+(r/l))}} + Be^{x\sqrt{lc}\sqrt{s(s+(r/l))}}. \qquad (3)$$

The coefficient B must be zero so that the solution of (3) is well-behaved and finite at infinity. Likewise, the coefficient A is determined from the boundary condition at $x = 0$ where $V_{\text{inf}}(x=0,s)$ is equal to the input voltage, $V_{\text{in}}(s)$, minus the voltage across the source impedance. After applying these boundary conditions, the voltage at a position x along the line is given by

$$V_{\text{inf}}(x,s) = V_{\text{in}}(s)\frac{Z(s)}{Z(s)+R_{\text{tr}}}e^{-x\sqrt{lc}\sqrt{s(s+(r/l))}} \qquad (4)$$

where $Z(s)$ is defined as the *lossy* characteristic impedance and is given by

$$Z(s) = \sqrt{\frac{r+sl}{sc}} = Z_o\sqrt{\frac{s+\frac{r}{l}}{s}} \qquad (5)$$

where the *lossless* characteristic impedance is $Z_o = \sqrt{(l/c)}$.

To determine the series solution of (4) in the time domain, the following inverse Laplace transformation is used [4]:

$$\frac{\sigma^{\nu}e^{-k\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}\left(\sqrt{s^2-\sigma^2}+s\right)^{\nu}} \rightarrow \left(\frac{t-k}{t+k}\right)^{(\nu/2)} I_{\nu}\left(\sigma\sqrt{t^2-k^2}\right)u_o(t-k) \qquad (6)$$

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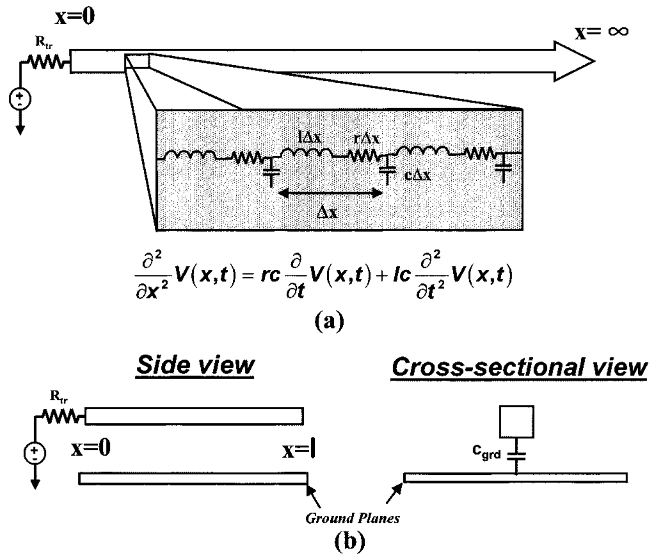

Fig. 1. (a) Semi-infinite and (b) finite distributed rlc line

where $u_0(t)$ is a unit step and $I_\nu[-]$ is a νth-order modified Bessel function.

Two transformations used by Heaviside in [5] are used to obtain the time domain series of (4). The first transformation is given by letting $s \rightarrow s - \sigma$. Performing this transformation on s in $V(x, s)$ produces a new function, $V'(x, s-\sigma)$, that is related to $V(x, s)$ in the time domain according to the following relation:

$$e^{\sigma t}V(x, t) \rightarrow V(x, s-\sigma) = V'(x, s). \quad (7)$$

Therefore, the time domain solution to $V'(x, s)$ is related to $V(x, t)$ by

$$V(x, t) = e^{-\sigma t}V'(x, t). \quad (8)$$

Making this transformation in (4) where $\sigma = r/(2l)$and simplifying gives

$$V'_{\rm inf}(x, s) = V_{dd}\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}}\left[\frac{s+\sigma}{s-\sigma}\frac{Z_o}{Z_o\sqrt{\frac{s+\sigma}{s-\sigma}}+R_{\rm tr}}\right]. \quad (9)$$

Now a second variable transformation is performed which is given by

$$s = \frac{\sigma}{2}(a + a^{-1}). \quad (10)$$

Making this temporary substitution only in the bracketed quantity in (9) and simplifying gives

$$V'_{\rm inf}(x, s) = V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \times\left\{\frac{(a+1)^2}{a^2-2\frac{R_{\rm tr}}{(Z_o+R_{\rm tr})}a+\frac{R_{\rm tr}-Z_o}{Z_o+R_{\rm tr}}}\right\}. \quad (11)$$

Factoring the term in the curly brackets in (11) gives

$$V'_{\rm inf}(x, s) = V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \times\left\{\frac{(a+1)^2}{(a-a_+)(a-a_-)}\right\} \quad (12)$$

where

$$a_\pm = \frac{R_{\rm tr} \pm Z_o}{Z_o + R_{\rm tr}}. \quad (13)$$

Using a partial fraction expansion on the term in the curly brackets in (12) gives

$$V'_{\rm inf}(x, s) = V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \times\left\{1+\frac{(a_++1)^2}{(a_+-a_-)(a-a_+)}-\frac{(a_-+1)^2}{(a_+-a_-)(a-a_-)}\right\}. \quad (14)$$

Making the substitution of (13) into (14) and recognizing the reflection coefficient $\Gamma = (R_{\rm tr} - Z_o)/(R_{\rm tr} + Z_o)$ gives

$$V'_{\rm inf}(x, s) = V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \times\left\{1+\frac{1}{(1-\Gamma)}\left(\frac{4}{(a-1)}-\frac{(\Gamma+1)^2}{(a-\Gamma)}\right)\right\}. \quad (15)$$

To determine a series representation of (15), the following series definition is used [4]:

$$\frac{1}{1-x} = \sum_{k=0}^{k=\infty}(x)^k. \quad (16)$$

Using (16) and the definition of the reflection coefficient, Γ, (15) becomes

$$V'_{\rm inf}(x, s) = V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \times\left\{1+\frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\left(\frac{1}{a}\right)^k(4-(1+\Gamma)^2\Gamma^{k-1}\right\}. \quad (17)$$

Solving for a in (10) gives

$$a = \frac{1}{\sigma}\left(s+\sqrt{s^2-\sigma^2}\right), \quad (18)$$

and substituting (18) into (17) leads to

$$V'_{\rm inf}(x, s) = V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \times\left\{1+\frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\frac{\sigma^k}{\left(s+\sqrt{s^2-\sigma^2}\right)^k} \times(4-(1+\Gamma)^2\Gamma^{k-1})\right\}. \quad (19)$$

Using the transformation presented in (6) on (19) gives the expression for $V'_{\text{inf}}(x, s)$ in the time domain seen in (20), shown on the bottom of the page. Because of the transformation $s \to s - \sigma$, (8) is utilized to determine the final expression for the voltage at a position x along a semi-infinite line, as seen in (21), shown at the bottom of the page.

To gain further insight, the expression in (21) is rewritten to highlight the two most important parameters influencing the characteristics of the infinite line. This is accomplished by replacing the time variable, t, with a new time variable, t', that is normalized to the time of flight (i.e., $t = t'x\sqrt{lc}$). Making this substitution in (21) gives (22), shown on the bottom of the page, where $R = rx$. Therefore, the only variables that affect infinite line transients are the reflection coefficient, $\Gamma = ((R_{\text{tr}}/Z_o - 1)/(R_{\text{tr}}/Z_o + 1))$, and the ratio of R/Z_o.

Letting the resistance in (21) go to zero gives (23), shown at the bottom of the page. Because the zero order modified Bessel function has a value of unity and all higher order modified Bessel functions have a value of zero, (23) becomes the travelling wave solution for the lossless line given by

$$V_{\text{inf}}(x, t) = V_{dd}\frac{Z_o}{(Z_o + R_{\text{tr}})}\, u_o\left(t - x\sqrt{lc}\right). \tag{24}$$

$$V'_{\text{inf}}(x, t) = V_{dd}\frac{Z_o}{(Z_o + R_{\text{tr}})} \times \left\{ \begin{aligned} & I_0\left(\sigma\sqrt{t^2 - \left(x\sqrt{lc}\right)^2}\right) \\ & + \frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\left(\frac{t - x\sqrt{lc}}{t + x\sqrt{lc}}\right)^{(k/2)} I_k\left(\sigma\sqrt{t^2 - \left(x\sqrt{lc}\right)^2}\right)(4 - (1+\Gamma)^2\Gamma^{k-1}) \end{aligned} \right\} u_o\left(t - x\sqrt{lc}\right). \tag{20}$$

$$V_{\text{inf}}(x, t) = V_{dd}\frac{Z_o}{(Z_o + R_{\text{tr}})}e^{-(r/2l)t} \times \left\{ \begin{aligned} & I_0\left(\sigma\sqrt{t^2 - \left(x\sqrt{lc}\right)^2}\right) \\ & + \frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\left(\frac{t - x\sqrt{lc}}{t + x\sqrt{lc}}\right)^{(k/2)} I_k\left(\sigma\sqrt{t^2 - \left(x\sqrt{lc}\right)^2}\right)(4 - (1+\Gamma)^2\Gamma^{k-1}) \end{aligned} \right\} u_o\left(t - x\sqrt{lc}\right). \tag{21}$$

$$V_{\text{inf}}(x, t') = V_{dd}e^{-(R/2Z_o)t'} \left\{ \begin{aligned} & \frac{1-\Gamma}{2} I_0\left(\frac{R}{2Z_o}\sqrt{(t')^2 - 1}\right) \\ & + \frac{1}{2}\sum_{k=1}^{k=\infty}\left(\frac{t'-1}{t'+1}\right)^{(k/2)} I_k\left(\frac{R}{2Z_o}\sqrt{(t')^2 - 1}\right)(4 - (1+\Gamma)^2\Gamma^{k-1}) \end{aligned} \right\} u_o(t'-1) \tag{22}$$

$$V_{\text{inf}}(x, t) = V_{dd}\frac{Z_o}{(Z_o + R_{\text{tr}})}e^{0} \left\{ \begin{aligned} & I_0(0) \\ & + \frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\left(\frac{t - x\sqrt{lc}}{t + x\sqrt{lc}}\right)^{(k/2)} I_k(0)(4 - (1+\Gamma)^2\Gamma^{k-1}) \end{aligned} \right\} u_o\left(t - x\sqrt{lc}\right) \tag{23}$$

Recognizing that $t = x\sqrt{lc}$ in (21), gives the expression for the voltage wave front travelling down a lossy infinite line

$$V_{\text{inf}}(x,\, t) = V_{dd}\frac{Z_o}{(Z_o + R_{\text{tr}})}\, e^{-(rx/2Z_o)}. \tag{25}$$

The expression in (25) is derived with traditional transmission line theory, but the new compact expressions are used to obtain a more accurate representation of the transient response close to the wave front on a distributed rlc line.

To derive this new near wave front expression, the expansion of a zeroth order Bessel function is used and given by

$$I_o(x) = 1 + \frac{x^2}{2^2(1!)} + \frac{x^4}{2^4(2!)} + \cdots \tag{26}$$

In addition, if the argument of the Bessel function is *much less* than the order of the Bessel function then the modified Bessel function can be approximated by

$$I_k(x) \approx \frac{1}{k!}\left(\frac{x}{2}\right)^n; \qquad k \gg x. \tag{27}$$

Substituting (26) and (27) into (22) and simplifying gives

$$\begin{aligned} &V_{\text{inf}}(x,\, t') \\ &= V_{dd}\frac{Z_o}{(Z_o + R_{\text{tr}})}\, e^{-(R/2Z_o)t'} \\ &\times \left\{ \begin{aligned} &1 + \frac{\left(\frac{R}{2Z_o}\sqrt{t'^2-1}\right)^2}{2^2(1!)} + \frac{\left(\frac{R}{2Z_o}\sqrt{t'^2-1}\right)^4}{2^4(2!)} + \cdots \\ &\frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\frac{1}{k!}\left(\frac{R}{4Z_o}\right)^k (t'-1)^k(4-(1+\Gamma)^2\Gamma^{k-1}) \end{aligned} \right\} \\ &\times u_o(t'-1). \end{aligned} \tag{28}$$

The summation over k in (28) is the difference of two exponential functions. To derive its exact form, a new function, $f(t')$, is defined that is the value of the summation from $k = 0$ to infinity, which is

$$\begin{aligned} f(t') &= \sum_{k=0}^{k=\infty}\frac{1}{k!}\left(\frac{rx}{2Z_o}\right)^k (t'-1)^k(4-(1+\Gamma)^2\Gamma^{k-1}) \\ &= 4e^{(rx/4Z_o)(t'-1)} - \frac{(1+\Gamma)^2}{\Gamma}\, e^{(rx/4Z_o)\Gamma(t'-1)}. \end{aligned} \tag{29}$$

Therefore, the value of the summation from $k = 1$ to infinity is determined by the subtraction from (29) of the $k = 0$ term, which gives

$$\begin{aligned} &\sum_{k=1}^{k=\infty}\frac{1}{k!}\left(\frac{rx}{2Z_o}\right)^k (t'-1)^k(4-(1+\Gamma)^2\Gamma^{k-1}) \\ &\quad = f(t') - f(1) \end{aligned} \tag{30}$$

where $f(t)$ is defined in (29) and $f(1) = 4 - ((1+\Gamma)^2/\Gamma)$.

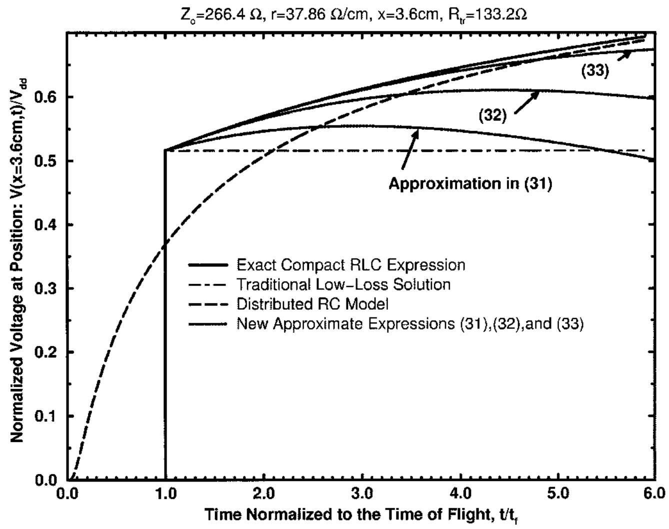

Fig. 2. Approximate transient expressions for a semi-infinite line compared to the exact compact rlc model, a distributed rc model, and a traditional low-loss solution ($Z_o = 266.4\ \Omega$, $r = 37.86\ \Omega$/cm, $x = 3.6$ cm, $R_{\text{tr}} = 133.2\ \Omega$)

Assuming that the zero order Bessel function is approximated by one, then a new simplified expression for the transient voltage near the wave front is given by

$$\begin{aligned} \frac{V_{\text{inf}}(x,\, t')}{V_{dd}} &= \frac{Z_o}{R_{\text{tr}} + Z_o} e^{-(rx/2Z_o)t'} u_o(t'-1) \\ &\quad + \frac{1}{2}\, e^{-(rx/2Z_o)t'}(f(t') - f(1))u_o(t'-1). \end{aligned} \tag{31}$$

The first term of the expression in (31) is a fast rising attenuated travelling wave solution. The second term is a slow rising waveform that is more indicative of traditional distributed rc solutions. This near wave front approximation is compared to the exact compact solution in Fig. 2.

To capture transient behavior further from the edge of the wave front, zero order and first order modified Bessel function approximations are used. Using (22) and (29) these zero and first order modified Bessel function approximations, respectively, are derived as

$$\begin{aligned} \frac{V_{\text{inf}}(x,\, t')}{V_{dd}} = &\left[\frac{Z_o}{R_{\text{tr}} + Z_o}\, e^{-(rx/2Z_o)t'} I_o\left(\frac{rx}{2Z_o}\sqrt{t'^2-1}\right)\right. \\ &\left. + \frac{1}{2}\, e^{-(rx/2Z_o)t'}(f(t') - f(1))\right] u_o(t'-1) \end{aligned} \tag{32}$$

and (33), shown at the bottom of the next page.

These additional approximations are compared to the exact compact model, a distributed rc model, and a traditional low loss model that is derived in (25). Physically, the addition of each modified Bessel function in summation in (22) provides a greater accuracy further from the wave front. In addition, Fig. 2 illustrates that the distributed rc model significantly overestimates the 50% time delay for this example.

B. Finite Line

The most appropriate boundary conditions for a global interconnect for GSI are given by a finite line with an arbitrary source

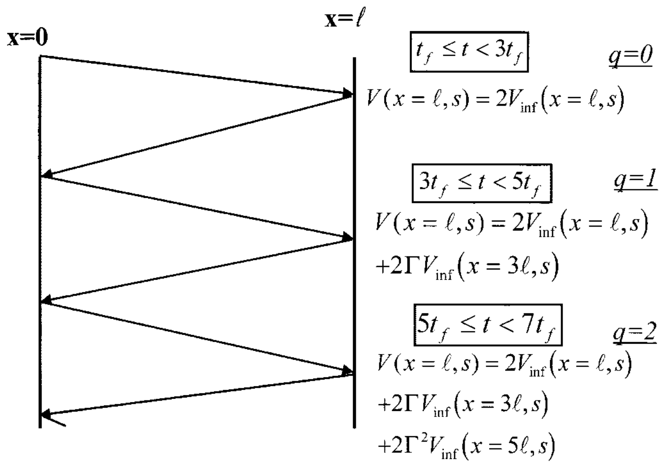

Fig. 3. Reflection diagram for finite line.

impedance and an open circuit termination at the receiving end of the line as seen in Fig. 1(b) [6]. Using the reflection diagram in Fig. 3, the expression for the voltage *at the end of a line of finite length*, ℓ, in the Laplace domain is determined to be

$$V_{fin}(x=\ell,\, s) = 2V_{\rm inf}(\ell,\, s) + 2\sum_{n=1}^{q} V_{\rm inf}((2n+1)\ell,\, s)\left(\frac{R_{\rm tr}-Z(s)}{R_{\rm tr}+Z(s)}\right)^n \tag{34}$$

where

$Z(s)$ defined in (5);
n reflection number;
q maximum reflection number as seen in Fig. 3.

Following (9), making the transformation $s \to s-\sigma$ in (34) where $\sigma = r/(2l)$ leads to

$$V'_{fin}(x=\ell,\, s) = 2V'_{\rm inf}(\ell,\, s) + 2\sum_{n=1}^{q} V'_{\rm inf}((2n+1)\ell,\, s) \times \left(\frac{R_{\rm tr}-Z_o\sqrt{\frac{s+\sigma}{s-\sigma}}}{R_{\rm tr}+Z_o\sqrt{\frac{s+\sigma}{s-\sigma}}}\right)^n. \tag{35}$$

Using the second substitution of $s = (\sigma/2)(a+a^{-1})$ and simplifying gives

$$V'_{fin}(x=\ell,\, s) = 2V'_{\rm inf}(\ell,\, s) + 2\sum_{n=1}^{q} V'_{\rm inf}\left((2n+1)\ell,\, \frac{\sigma}{2}(a+a^{-1})\right) \times \left(\frac{R_{\rm tr}-Z_o\frac{a+1}{a-1}}{R_{\rm tr}+Z_o\frac{a+1}{a-1}}\right)^n. \tag{36}$$

Simplifying (36) in terms of source reflection coefficient, Γ, gives

$$V'_{fin}(x=\ell,\, s) = 2V'_{\rm inf}(\ell,\, s) + 2\sum_{n=1}^{q} V'_{\rm inf}((2n+1)\ell,\, s)\Gamma^n\left(\frac{1-\frac{1}{a\Gamma}}{1-\frac{\Gamma}{a}}\right)^n. \tag{37}$$

To evaluate the series solution, the following series definition is used [4]:

$$\left(\frac{1-x}{1-y}\right)^n = \sum_{i=0}^{n}\sum_{j=0}^{\infty}\frac{n(n-1+j)!}{i!j!(n-i)!}(-1)^i x^i y^j \quad \text{where } (n>0) \tag{38}$$

where

n assumed to be a positive integer;
i index associated with the expansion of the numerator;
j index associated with the expansion of the denominator of (38).

Substituting (38) into (37) gives

$$V'_{fin}(x=\ell,\, s) = 2V'_{\rm inf}(\ell,\, s) + 2\sum_{n=1}^{q}\sum_{i=0}^{n}\sum_{j=0}^{\infty}\frac{n(n-1+j)!}{i!j!(n-i)!} \times (-1)^i V'_{\rm inf}((2n+1)\ell,\, s)\Gamma^{n-i+j}\left(\frac{1}{a}\right)^{i+j}. \tag{39}$$

Substituting the expression for $V'_{\rm inf}(x,\, s)$ from (17) into (39)

$$\frac{V_{\rm inf}(x,\, t')}{V_{dd}} = \left[\frac{Z_o}{R_{\rm tr}+Z_o}e^{-(rx/2Z_o)t'}I_o\left(\frac{rx}{2Z_o}\sqrt{t'^2-1}\right) + \frac{1}{2}e^{-(rx/2Z_o)t'}\left(\frac{t'-1}{t'+1}\right)^{0.5}(4-(1+\Gamma^2))I_1\left(\frac{rx}{2Z_o}\sqrt{t'^2-1}\right) + \frac{1}{2}e^{-(rx/2Z_o)t'}(-(t-1)f'(1)+f(t')-f(1))\right] \times u_o(t'-1) \tag{33}$$

and simplifying leads to

$$V'_{fin}(x=\ell,\, s) = 2V'_{\rm inf}(\ell,\, s)+2V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\sum_{n=1}^{q}\sum_{i=0}^{n}\sum_{j=0}^{\infty}\frac{n(n-1+j)!}{i!j!(n-i)!} \times(-1)^i\Gamma^{n-i+j}\frac{e^{-(2n+1)\ell\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \times\left\{\left(\frac{1}{a}\right)^{i+j}+\frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\left(\frac{1}{a}\right)^{k+i+j}(4-(1+\Gamma)^2\Gamma^{k-1})\right\}. \tag{40}$$

Making the substitution from (18) and using the transformation in (6), leads to the following expression in the time domain, as seen in (41), shown at the bottom of the page. Using (8) gives the final expression for the voltage at the end of a finite distributed rlc interconnect, as seen in (42), shown at the bottom of the page, where q, which is defined as the maximum reflection number for a given time, is written as a function of time according to

$$q=\left\langle 0.5\left(\frac{t}{x\sqrt{lc}}+1.0\right)\right\rangle-1.0 \tag{43}$$

where the notation $\langle x\rangle$ is defined as the decimal truncation of x (i.e., $\langle 2.7\rangle = 2$).

This new expression in (42) is compared to HSPICE simulation of an interconnect using 1, 10, 50, and 500 lumped RLC elements in Fig. 4(a)–(d). This interconnect has a length of 3.6 cm, a 2.1 μm by 2.1 μm cross-sectional metal dimension (giving a resistance per unit length to be approximately 37.9 Ω/cm), a driver resistance of 133.2 Ω, and a lossless characteristic impedance of 266.5 Ω. The interconnect metal is composed of copper with a surrounding low k dielectric. The inductance per unit length was derived using a quasi-TEM mode approximation. Fig. 4(a)–(d) illustrates that as the number of lumped elements is increased in the HSPICE simulation, HSPICE results converge to the compact distributed rlc solutions. Unlike HSPICE, the compact expressions can be used with various searching algorithms to calculate directly without intermediate steps interconnect characteristics such as time delay and overshoot.

In addition, special cases can be explored of (42) to provide further insight into the distributed rlc interconnect operation. For example, consider (42) when the source resistance is equal to the *lossless* characteristic impedance of the line, Z_o. The reflection coefficient, Γ, in this case becomes zero. The only summation terms that survive in (42) are when $\Gamma^0=1$ is satisfied.

$$V'_{fin}(x=\ell,\, t) = 2V'_{\rm inf}(x=\ell,\, t)+2V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}\sum_{n=1}^{q}\sum_{i=0}^{n}\sum_{j=0}^{\infty}\frac{n(n-1+j)!}{i!j!(n-i)!}(-1)^i\Gamma^{n-i+j} \times\left\{\begin{aligned}&\left(\frac{t-(2n+1)\ell\sqrt{lc}}{t+(2n+1)\ell\sqrt{lc}}\right)^{(i+j)/2)} I_{i+j}\left(\sigma\sqrt{t^2-\left((2n+1)\ell\sqrt{lc}\right)^2}\right)\\ &+\frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\left(\frac{t-(2n+1)\ell\sqrt{lc}}{t+(2n+1)\ell\sqrt{lc}}\right)^{(i+j+k)/2)} I_{i+j+k}\left(\sigma\sqrt{t^2-\left((2n+1)\ell\sqrt{lc}\right)^2}\right)(4-(1+\Gamma)^2\Gamma^{k-1})\end{aligned}\right\} \times u_o\left(t-(2n+1)\ell\sqrt{lc}\right) \tag{41}$$

$$V_{fin}(x=\ell,\, t) = 2V_{\rm inf}(x=\ell,\, t)+2V_{dd}\frac{Z_o}{(Z_o+R_{\rm tr})}e^{-(r/2l)t}\sum_{n=1}^{q}\sum_{i=0}^{n}\sum_{j=0}^{\infty}\frac{n(n-1+j)!}{i!j!(n-i)!}(-1)^i\Gamma^{n-i+j} \times\left\{\begin{aligned}&\left(\frac{t-(2n+1)\ell\sqrt{lc}}{t+(2n+1)\ell\sqrt{lc}}\right)^{(i+j)/2)} I_{i+j}\left(\sigma\sqrt{t^2-\left((2n+1)\ell\sqrt{lc}\right)^2}\right)\\ &+\frac{1}{1-\Gamma}\sum_{k=1}^{k=\infty}\left(\frac{t-(2n+1)\ell\sqrt{lc}}{t+(2n+1)\ell\sqrt{lc}}\right)^{(i+j+k)/2)} I_{i+j+k}\left(\sigma\sqrt{t^2-\left((2n+1)\ell\sqrt{lc}\right)^2}\right)(4-(1+\Gamma)^2\Gamma^{k-1})\end{aligned}\right\} \times u_o(t-(2n+1)\ell\sqrt{lc}) \tag{42}$$

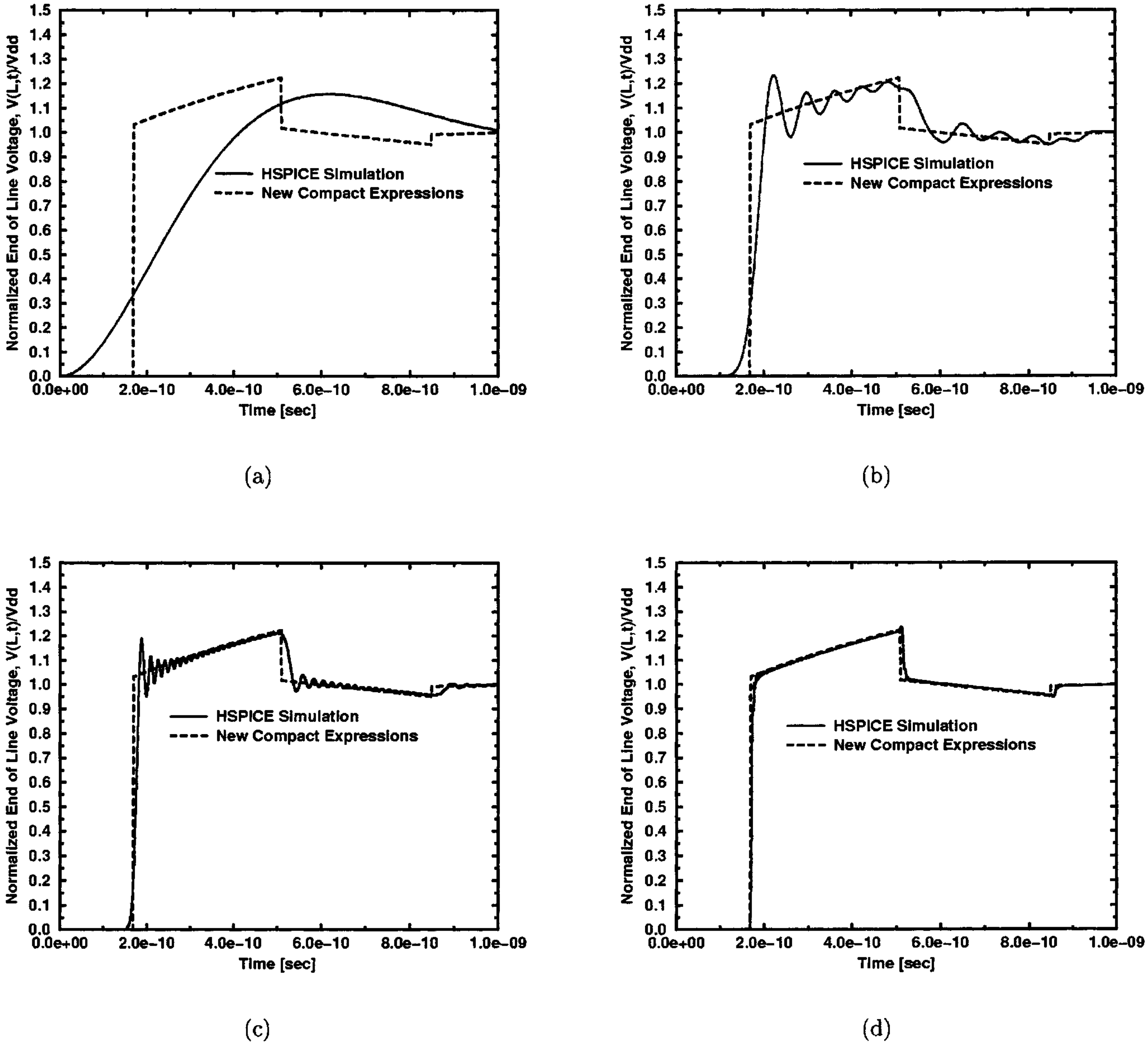

Fig. 4. Compact expression compared to HSPICE Simulation of (a) 1, (b) 10, (c) 50, and (d) 500 lumped rlc elements ($Z_o = 266.5\ \Omega$, $r = 37.87\ \Omega$/cm, $\ell = 3.6$ cm, $R_{\rm tr} = 133.2\ \Omega$)

This condition occurs when

$$n - i + j = 0. \tag{44}$$

Because $n \geq i$ and $j \geq 0$ by definition, then (44) is satisfied only when $i = n$ and $j = 0$. Making this substitution into (42) gives the transient expression when the source resistance is equal to the lossless characteristic impedance.

$$\begin{aligned} V_{fin}&(x=\ell,\, t) \\ &= 2V_{\rm inf}(x=\ell,\, t) + V_{dd}e^{-(r/2l)t}\sum_{n=1}^{q}(-1)^n \\ &\times \left(\left(\frac{t-(2n+1)\ell\sqrt{lc}}{t+(2n+1)\ell\sqrt{lc}}\right)^{(n/2)} \right. \\ &\times I_n\left(\sigma\sqrt{t^2-\left((2n+1)\ell\sqrt{lc}\right)^2}\right) \\ &+ \sum_{k=1}^{k=\infty}\left(\frac{t-(2n+1)\ell\sqrt{lc}}{t+(2n+1)\ell\sqrt{lc}}\right)^{(n+k)/2} \\ &\left. \times I_{n+k}\left(\sigma\sqrt{t^2-\left((2n+1)\ell\sqrt{lc}\right)^2}\right)(4-0^{k-1}) \right) \\ &\times u_o(t-(2n+1)\ell\sqrt{lc}) \end{aligned} \tag{45}$$

where $0^0 \equiv 1$. As mentioned previously, n has the interpretation of being the reflection number where $2V_{\rm inf}(x=\ell,\, t)$ is the first reflection ($n = 0$) and the summation terms associated with $n = 1, 2, 3, \cdots$ are the later reflections that begin at $3t_f, 5t_f, 7t_f, \cdots$ (where $t_f = \ell\sqrt{lc}$), respectively. Equation

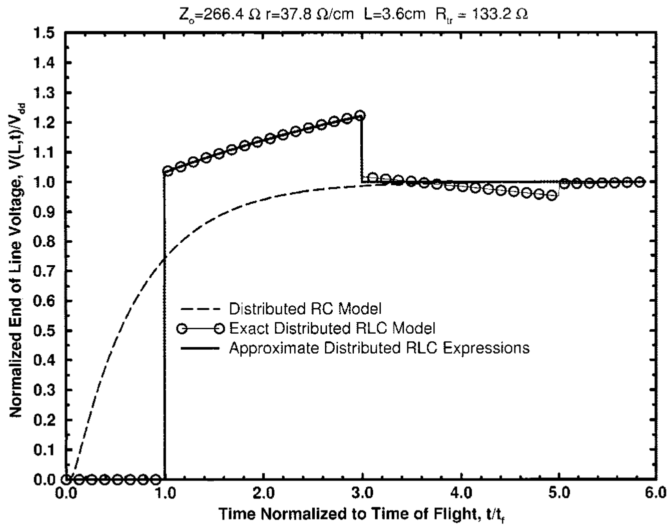

Fig. 5. Finite line first reflection approximation compared to complete compact model ($Z_o = 266.4\ \Omega$, $r = 37.8\ \Omega$/cm, $\ell = 3.6$ cm, $R_{\rm tr} = 133.2\ \Omega$)

(45) differs from traditional lossless transmission line theory in which a matched source absorbs all power from the transmission line leaving only the first reflection. A distributed rlc interconnect, however, prevents this type of perfect matching because the voltage and current ratio are out of phase AND their ratio changes with time. In lossless transmission line theory, the ratio of the voltage and the current is always a constant equal to the lossless characteristic impedance, which allows perfect impedance matching.

Even though perfect matching is not possible, it can be assumed that the first reflection provides significant information about the transient characteristics. Using the near-wave-front approximation from (31) gives the following approximate expression for the transient voltage at the end of a line with a matched source impedance as

$$\frac{V_{fin}(x=\ell,\,t)}{V_{dd}} = e^{-(r\ell/2Z_o)t'}u_o(t'-1) + \left(8e^{-(r\ell/2Z_o)}\sinh\left[\frac{r\ell}{2Z_o}(t'-1)\right] - e^{-(r\ell/2Z_o)t'}\frac{r\ell}{4Z_o}(t'-1)\right)u_o(t'-1) \quad (46)$$

where $t' = t/(\ell\sqrt{lc})$. With arbitrary source impedance, this simple first reflection approximation is given by

$$\frac{V_{fin}(x,\,t')}{V_{dd}} = \frac{2Z_o}{R_{\rm tr}+Z_o}e^{-(rx/2Z_o)t'}u_o(t'-1) + e^{-(rx/2Z_o)t'}(f(t')-f(1))u_o(t'-1) \quad (47)$$

where $f(t')$ is defined in (29). Both (46) and (47) are valid for t' slightly greater than 1, and the first term has the interpretation of being the fast rising "lc" portion and the second term is the slow rising "rc" portion.

To approximate the transient response further away from the edge of the wave front, the zero and first order modified Bessel functions are needed. Using expression (42) and (33) and normalizing the time variable, t, to the time of flight (i.e., $t = t'\sqrt{lc}\ell$), the single reflection approximation is given by (48), shown at the bottom of the page. This expression gives the detailed transient response of an interconnect in a range up to three times of flight of the signal. This is verified in Fig. 5 in which (48) is compared to the exact compact expression for a finite line. The first reflection approximation provides useful information on time delay and peak overshoot as clearly demonstrated in Fig. 5. The new approximations are also compared to a distributed rc model that significantly underestimates the 50% time delay and significantly overestimates the 90% response time of the interconnect. In addition, the distributed rc model does not predict any overshoot on the line.

III. Simplified Expression for Time Delay and Overshoot

The complete compact models provide great flexibility to calculate a variety of parameters on a distributed rlc line including 50% time delay and maximum overshoot; however, approximate expressions for time delay and maximum overshoot are also desirable for the multilevel network designer.

A. Overshoot Expression

First, a closed-form expression for the peak overshoot is proposed. Fig. 5 illustrates that the peak overshoot on a single line occurs at $t = 3t_f$, where t_f is the time of flight of an electromagnetic signal traveling down the line. Generalizing this assumption, the first reflection approximation is used to determine a closed-form expression for peak overshoot. Substituting $t = 3\ell\sqrt{lc}$ into (42) and taking the first three modified Bessel

$$V_{fin}(x=\ell,\,t) = 2V_{dd}\left\{\begin{array}{l}\dfrac{Z_o}{R_{\rm tr}+Z_o}e^{-(rx/2Z_o)t'}I_o\left(\dfrac{rx}{2Z_o}\sqrt{t'^2-1}\right)+\dfrac{1}{2}e^{(-rx/2Z_o)t'}\left(\dfrac{t'-1}{t'+1}\right)^{0.5}(4-(1+\Gamma^2))I_1\left(\dfrac{rx}{2Z_o}\sqrt{t'^2-1}\right)\\ +\dfrac{1}{2}e^{-(rx/2Z_o)t'}(-(t-1)f'(1)+f(t')-f(1))\end{array}\right\} \times (u_o(t'-1)-u_o(t'-3)) + V_{dd}u_o(t'-3) \quad (48)$$

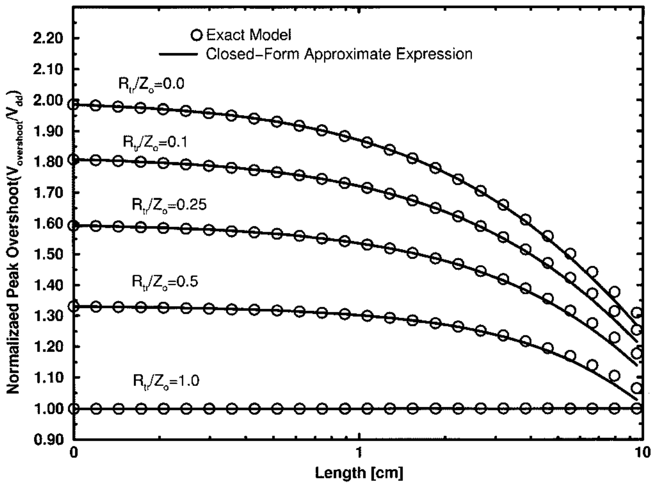

Fig. 6. Verification of simplified overshoot expression ($Z_o = 266.5\ \Omega$, $r = 37.87\ \Omega$/cm)

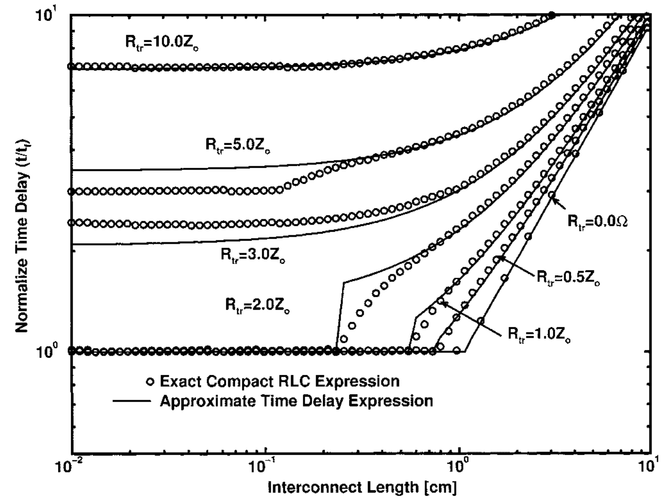

Fig. 7. Comparison of closed-form time delay expression to exact results using compact rlc model for vary interconnect lengths and driver impedance ($Z_o = 266.5\ \Omega$, $r = 668\ \Omega$/cm).

function in the expansion, which extends the accuracy for longer lines, gives

$$\frac{V(\ell, t=3t_f)}{V_{dd}} = 2\frac{Z_o}{(Z_o+R_{\rm tr})}e^{-(3r\ell/2Z_o)} \times \left\{ I_0\left(\frac{r\ell}{2Z_o}\sqrt{8}\right) + (\Gamma+3)\left(\frac{1}{2}\right)^{(1/2)} I_1\left(\frac{r\ell}{2Z_o}\sqrt{8}\right) + \frac{(\Gamma(\Gamma+3)+4)}{2} I_2\left(\frac{r\ell}{2Z_o}\sqrt{8}\right)\right\}. \quad (49)$$

The expression in (49) is valid as long as its result is greater than one. If the result is less than one, then it can be assumed that there is no overshoot; therefore, an expression for the peak overshoot is approximately given by

$$\frac{V_{overshoot}}{V_{dd}} = \max\left(1, \frac{V(\ell, t=3t_f)}{V_{dd}}\right) \quad (50)$$

where $V(\ell, t=3t_f)/V_{dd}$ is defined in (49) and max is a function that returns the maximum value of its arguments. The results of this simplified expression are compared to the compact rlc expression for various values of Z_o, $R_{\rm tr}$, and R in Fig. 6. Fig. 6 shows that this approximate expression very accurately describes the peak overshoot on a distributed rlc interconnect.

B. Time Delay Expression

Time delay is defined as the time at which the voltage at the end of an interconnect reaches 50% of its steady-state value. Sakurai has rigorously derived an expression for the time delay of a distributed rc line and it has the following form [2]:

$$\tau = 0.693R_{\rm tr}c\ell + 0.377rc\ell^2 \quad (51)$$

Using a low loss model, Sakurai's model is enhanced to include inductance. Setting (25) equal to $0.5V_{dd}$, and solving for the resistance to lossless characteristic impedance ratio gives the following condition for time of flight interconnect operation as

$$\frac{R}{Z_o} \le 2\ln\left[\frac{4Z_o}{R_{\rm tr}+Z_o}\right]. \quad (52)$$

If the source resistance is greater than $3Zo$ in (52), then the voltage launched at $x=0$ on the transmission line is less than $0.25V_{dd}$. Therefore, the voltage at the end of the line with the open circuit termination is less than $0.5V_{dd}$ at the time of flight regardless of the interconnect resistance. This means that (52) is valid only for $R_{\rm tr} < 3Z_o$. In addition, if the source impedance is matched to the characteristic impedance, the interconnect resistance must be

$$R \le 2\ln 2Z_o = 1.39Z_o \quad (53)$$

to have a time delay equal to the time of flight.

An improvement to Sakurai's model is given by

REGION I: $(R/Z_o) \le \ln[4Z_o/(R_{\rm tr}+Z_o)]$ AND $R_{\rm tr} < 3Z_o$

$$\frac{\tau}{t_f} = 1.0 \quad \text{or } \tau = \frac{\ell}{\sqrt{lc}} \quad (54)$$

REGION II: $(R/Z_o) \ge 2\ln[4Z_o/(R_{\rm tr}+Z_o)]$ OR $R_{\rm tr} > 3Z_o$

$$\frac{\tau}{t_f} = 0.693\frac{R_{\rm tr}}{Z_o} + 0.377\frac{r\ell}{Z_o} \quad \text{or } \tau = 0.693R_{\rm tr}c\ell + 0.377rc\ell^2. \quad (55)$$

This closed-form expression for time delay is compared to the exact compact rlc expression in Fig. 7. The simplified expression provides less than 5% error when $R_{\rm tr}/Z_o$ is less than 0.2 or R/Z_o is greater than 2.3. To get more accurate results outside this region, the compact distributed rlc expressions are needed.

IV. Conclusion

Compact expressions that describe the transient response of a single distributed rlc interconnect are rigorously derived. Simple closed-form approximations are derived that estimate the transient response of semi-infinite and finite distributed rlc interconnects. Finally, simple closed-form expressions for overshoot and time delay are derived that include the effects of inductance on a single line. The results in this paper are used in the companion paper to determine the worst-case time delay and crosstalk between three coupled lines in a multilevel network.

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