Transmission Line Models for Lossy Waveguide Interconnections in VLSI

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Abstract—**At high frequencies the waveguide nature of interconnections in VLSI circuits becomes important. Moreover, losses in interconnection are a major feature, not a perturbation. Here it is shown that even for such lossy waveguide structures an exactly equivalent *RLGC* transmission line can be found. Equations are given determining these transmission line parameters in terms of the waveguide propagation constant and complex average power, and also in terms of integrals over the electric and magnetic field varibles. The resulting *L*, *C*, and *G* parameters differ from the usual static values when losses are important, and *R* is not restricted to the usual formula based upon a perturbation treatment of the skin effect. Consequently, semiconductor substrates can be treated.**

"Current" and "voltage" are found to have an abstract meaning in the equivalent transmission line. For a waveguide in a medium where conductivity and permittivity vary with position (such as a many-layered medium) an explicit formula relating "current" and "voltage" to weighted averages of transverse waveguide fields is given. A brief discussion of the reformulation of Thevenin equivalent circuit parameters in terms of reflection coefficients avoids terms such as "open circuit voltage" that are difficult to interpret for the equivalent transmission line.

The framework presented allows construction of equivalent circuits for lossy waveguide interconnections, drivers, and terminations that provide correct spatial dependence in the direction of propagation and correct power relations despite the abstract nature of "current" and "voltage" in these lines.

I. INTRODUCTION

THE CRUDEST MODEL of an interconnection line is a simple capacitor. In this model, line behavior is dictated by the line capacitance and the impedances of the driver and the load. However, as frequency is increased, signal propagation is better described as a diffusion down the line, and a distributed *RC* transmission line model is satisfactory [1], [2]. At still higher frequencies the line inductance becomes important, and the interconnect is modeled as an *RLC* line [3], [4]. In general, as frequency is increased the equivalent circuit that models the interconnect becomes more and more complex, and a method for finding correct transmission line parameters becomes desirable.

The most general picture of the interconnection line is as a waveguide, for example, as a microstrip or triplate line. However, waveguide analysis is based upon detailed solution of Maxwell's equations for the electric and magnetic fields. For use with device models in circuit simulators and for continuity with ordinary *RC* and *RLC* interconnect models one prefers a transmission line model, which always can be represented as a cascade of *T*- or Π-sections of lumped elements.

The use of transmission lines to model waveguides has a long history. Among the excellent early treatments are those of Montgomery [5] and Collin [6, ch. 4]. However, these discussions are directed toward low-loss waveguides, where losses can be treated as a perturbation. For modeling of interconnections losses are important, as reflected in the *R* of the usual *RC* transmission line models. The importance of losses is even greater when lines over semiconductor substrates (rather than insulators) are considered. Thus, to model interconnections at high frequencies what is needed is a prescription for finding the correct transmission line parameters to model a *lossy* waveguide.

The most general transmission line is characterized by a series impedance $R + j\omega L$ and a shunt admittance $G + j\omega C$ where R is the resistance, L the inductance, G the conductance, and C the capacitance (all per unit length), and ω is the frequency in radians per second. In this paper, R, L, G, and C are found for a transmission line that models a lossy waveguide characterized by a complex propagation constant γ and a complex average power flow P.

Following the derivation of R, L, G, and C is a discussion of equivalent circuits for waveguide terminations and drivers. This discussion is intended to show how these equivalent circuits could be set up to mimic the actual physical waveguide termination and driver.

Appendix I of the paper considers a lossy waveguide with position-dependent complex permittivity and permeability. In Appendix I, the R, L, G, and C parameters of the equivalent transmission line are derived directly from Maxwell's equations and the requirement of identical complex power flows in the two structures. Explicit formulas for R, L, G, and C in terms of integrals of the field variables are presented that generalize known results for lossless or near lossless homogeneous media.

II. THE ANALOGY BETWEEN WAVEGUIDES AND TRANSMISSION LINES

The basis for analogy between waveguides and transmission lines is that both structures propagate waves:

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[1]We do not consider complicated cases where several modes may propagate, requiring the use of several different transmission lines to describe a single waveguide.

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waves of electric and magnetic field for the waveguide; waves of voltage and current for the transmission line [5], [6, ch. 4]. In fact, if the direction of propagation is chosen as the z-direction, then the z-dependence of the waves of transverse electric field in the waveguide and the z-dependence of the voltage in the equivalent transmission line are the same. Similarly, the z-dependence of waves of transverse magnetic field and the z-dependence of current are the same. Such results are well known for lossless waveguides [5], [6, ch. 4] and similar results are shown in (A2) and (A7) of Appendix I for an example of a lossy waveguide.

For simple cases, such as a lossless coaxial line, the electric field in the waveguide can be related directly to the voltage and the magnetic field to the current by line integrals of the fields over appropriate contours. Thus, the equivalent transmission line can be physically identified with voltages and currents that actually exist within certain regions of the waveguide structure. In fact, such an analysis leads to explicit formulas for L and C in terms of the dimensions and material parameters of the waveguide. These values agree with the static L and C for the waveguide geometry, providing a clear connection to the low-frequency equivalent circuit for the interconnection. By a perturbation method, small conductor or dielectric losses can be included to find R and G as well [5], [6].

The connection between waveguides and transmission lines is not so simple in the general case, particularly where losses are important. The presence of a longitudinal electric field (needed because of the IR drop along the guide, for example) means that the transverse field components do not satisfy the equations of a static analysis [7]. Hence, L and C cannot be inferred from simple line integrals of the fields, and may differ from the static values. The "voltage" and "current" of the equivalent transmission line are not related to easily identifiable regions in the waveguide. Rather, the transmission line becomes something of a convenient mathematical fiction that mimics the waveguide behavior.

To construct the equivalent transmission line in the lossy case, the approach based upon intuitively chosen contour integrals is abandoned. Rather, the general approach adopted here is to assume that two waveguide parameters are known for all frequencies of interest, either from measurements or by calculation of the solution to Maxwell's equations. These parameters are 1) the propagation constant γ

$$\gamma = \alpha + j\beta \tag{1}$$

where α is the attenuation constant and β is the phase constant, and 2) the complex average power traveling down the waveguide P. In terms of Poynting's vector

$$P = \tfrac{1}{2} \int dx \int dy\, (E \times H^*)_z \tag{2}$$

where the superscript "*" denotes complex conjugate quantities. Propagation occurs in the z-direction, and the fields are expressed in complex notation. For example, for a single traveling wave the electric field is the real part of

$$E(x, y, z, t) = E(x, y)\, e^{-\gamma z + j\omega t}. \tag{3}$$

Given (1) and (2), the corresponding transmission line parameters are determined by requiring: 1) the same propagation constant, that is

$$\gamma^2 = (R + j\omega L)(G + j\omega C) \tag{4}$$

and 2) the same complex average power $P(z)$, which in terms of voltage $V(z)$ and current $I(z)$ is

$$P(z) = \tfrac{1}{2}\, V(z)\, I(z)^* = \tfrac{1}{2}\, |I(z)|^2 Z_0 = \tfrac{1}{2}\, |V(z)|^2/Z_0^*. \tag{5}$$

The last two forms of $P(z)$ in (5) assume a single propagating wave (no reflections) and a single mode of propagation, allowing the use of the relation

$$Z_0 = V/I \tag{6}$$

where Z_0 is the characteristic impedance. From (5), for this case of a single propagating wave, it follows that P and Z_0 must have the same phase, that is

$$Q \equiv \frac{P_i}{P_r} = \frac{Z_{0i}}{Z_{0r}}. \tag{7}$$

Here the real and imaginary parts of P and Z_0 are introduced by

$$P = P_r + jP_i \tag{8}$$

$$Z_0 = Z_{0r} + jZ_{0i} \tag{9}$$

and (for lack of any established terminology) Q will be called the power quotient. The power quotient Q can be defined using the power at any position in the line. Because the real and imaginary parts of P attenuate at the same rate [exp $(-2\alpha z)$], the resulting Q is independent of position z.

From the transmission line equations and (4) for γ^2, Z_0 is given by

$$Z_0^2 = (R + j\omega L)/(G + j\omega C). \tag{10}$$

Using (4) and (10)

$$Z_0 = \left(\frac{R + j\omega L}{\gamma}\right) = \frac{R + j\omega L}{\alpha + j\beta}. \tag{11}$$

Using (11) in (7)

$$Q = \frac{\alpha\omega L - \beta R}{\alpha R + \beta\omega L}. \tag{12}$$

Using (4) and (10) again

$$Z_0 = \frac{\alpha + j\beta}{G + j\omega C}. \tag{13}$$

Using (12) in (7)

$$Q = \frac{\beta G - \alpha\omega C}{\beta\omega C + \alpha G}. \tag{14}$$

From (12) and (14)

$$\frac{\omega L}{R} = \frac{\alpha Q + \beta}{\alpha - \beta Q} \tag{15}$$

$$\frac{\omega C}{G} = -\frac{\alpha Q - \beta}{\alpha + \beta Q} \tag{16}$$

$$GR = \frac{\alpha^2 - \beta^2 Q^2}{1 + Q^2} \tag{17}$$

$$\omega^2 LC = -\frac{\alpha^2 Q^2 - \beta^2}{1 + Q^2} \tag{18}$$

where the right sides of (15)–(18) are known from the waveguide γ and Q, and the left sides are the required equivalent transmission line parameters. Note that R, L, G, and C are independent of position in the waveguide, even when Q is defined in terms of the power at the general position "z" because α, β, and Q are independent of position. Equations (15) and (16) are rearrangements of (12) and (14), while (17) follows from (4) using (15) and (16). Equation (18) is not independent, following from multiplication of (15), (16), and (17).

In general, γ and Q are different for each mode of propagation in the waveguide, and are dependent upon the choice of an eigenvalue derived from the boundary conditions upon the fields. Hence, a different set of R, L, G, C, parameters results for each mode of propagation.

The solution of (15)–(18) determines only three of the four parameters, because only three of the four equations are independent. For the usual case of a waveguide that propagates only one mode at low frequencies, a convenient condition upon the parameters for this mode is that the quasi-static impedance be obtained as the frequency is reduced. On the other hand, for a propagation mode that cuts off as frequency is reduced, the extra parameter is arbitrary and can be chosen to simplify the equivalent transmission line or the equivalent circuit of any termination or driver [8].

III. Discussion

Equations (15)–(18) show that an equivalent transmission line can be found for any lossy waveguide. In general, this line is not unique. However, uniqueness does result if agreement with a particular low-frequency equivalent circuit is required, as in the case of microstrip interconnections.

Of course, (15)–(18) do not guarantee the resulting R, L, G, and C will have a simple frequency dependence nor a ready interpretation in terms of an equivalent circuit. An alternative to using (15)–(18) to derive R, L, G, and C is to use these equations only to check R, L, G, and C once the equivalent circuit has been derived by a different procedure.

Finally, let it be noted that the requirements (15)–(18) based upon a single traveling wave are sufficient for the general case where reflections occur, as is shown in Appendix I.

IV. The Three-Parameter Transmission Line

To check whether a given equivalent circuit satisfies the requirements (15)–(18) that guarantee correct power flow, in general it is necessary to measure or to calculate the power quotient (7) and then check (15)–(18). However, it may be useful to note that for a three-parameter line this quotient takes a very simple form.

Using (12) for the special cases $R = 0$ and $L = 0$

$$Q = \frac{\alpha}{\beta}, \qquad R = 0 \tag{19}$$

$$= -\frac{\beta}{\alpha}, \qquad L = 0. \tag{20}$$

Also, from (14)

$$Q = -\frac{\alpha}{\beta}, \qquad G = 0 \tag{21}$$

$$= \frac{\beta}{\alpha}, \qquad C = 0. \tag{22}$$

That is, for the case of a three-parameter line, the power quotient is dictated by the propagation constant γ. Conversely, if the power quotient of a waveguide is well approximated as the ratio $\pm(\alpha/\beta)$ or its reciprocal, then this waveguide is well aproximated by a three-parameter line.

V. Equivalent Circuits for Drivers and Loads

Equations (15)–(18) determine R, L, G, and C in terms of the waveguide parameters α, β, and Q. There is no requirement made upon the meaning of the resulting current and voltage variables I and V, and no examination has been made of their relation to the fields in the corresponding waveguide. In (A25) and (A26) of Appendix I it is shown that the voltage and current of the equivalent transmission line are related to weighted averages of the transverse field components of E and H across the waveguide cross section. For lossless, homogeneous media these results reduce to those of Collin [6, ch. 4]. Thus, a simple interpretation of current and voltage is not available except in special cases. All that is known is that I and V satisfy the transmission line equations with R, L, G, and C as determined from (15)–(18).

The usual low-frequency equivalent circuit for a driver is either the Thevenin voltage–source equivalent circuit, or the Norton current–source equivalent circuit. How are these circuits to be interpreted in a waveguide context where the terms "voltage" and "current" are difficult to interpret? Revised statements of Thevenin's and Norton's theorems that avoid direct use of terms like "open circuit voltage" and "short circuit current" would be easier to use. Such restatements of these theorems can be based upon the reflection coefficients of the waveguide. As pointed out earlier [5], [6, ch. 4] for lossless waveguides, and in (A2) and (A8) of Appendix I for an example of a lossy waveguide, the z-dependence of transverse electric field in the waveguide and the z-dependence of voltage in the equivalent transmission line both are governed by the function

$$V(z) = A\, e^{-\gamma z} + B\, e^{\gamma z}. \tag{23}$$

Similarly, the z-dependence of transverse magnetic field and current both are governed by

$$I(z) = \frac{1}{Z_0} (A\, e^{-\gamma z} - B\, e^{\gamma z}). \tag{24}$$

In (23) and (24), A and B are the forward and reflected wave amplitudes that are determined by the driver and the load.

The reflection coefficient looking toward the load Γ_L is defined as

$$\Gamma_L \equiv \frac{B}{A}. \tag{25}$$

This quantity is characteristic of the waveguide termination. Thus, if the equivalent circuit for the waveguide termination consists of an impedance Z_L, a requirement upon Z_L is that it lead to the same Γ_L in the transmission line as exists in the waveguide. If the transmission line has length "l", then (23) and (24) provide at $z = l$

$$Z_L \equiv \frac{V(l)}{I(l)} = Z_0 \left(\frac{1 + \Gamma_L \exp{(2\gamma l)}}{1 - \Gamma_L \exp{(2\gamma l)}} \right). \tag{26}$$

Because both Γ_L and γ are fixed by the wave behavior in the waveguide, (26) determines the ratio Z_L/Z_0 for the equivalent circuit corresponding to the waveguide termination. But Z_0 is determined by the conditions (15)–(18) and the requirement of agreement with a low-frequency equivalent circuit. Hence, Z_L itself is determined by Γ_L through (26).

For the driver-equivalent circuit, suppose a Thevenin voltage-equivalent circuit is wanted. Then the input voltage V_{in} and an impedance Z_{in} must be chosen. Suppose we require that the equivalent circuit provide the correct complex input power P_{in} and the correct reflection coefficient from the driver, Γ_D. The input power is given by

$$\begin{aligned} P_{in} &= \frac{1}{2} I^*(0) V_{in} \\ &= \frac{1}{2} \left(\frac{V_{in}}{Z_{in} + Z_i} \right)^* V_{in} \end{aligned} \tag{27}$$

where the line input impedance is (from (23) and (24))

$$Z_i \equiv \frac{V(0)}{I(0)} = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}. \tag{28}$$

Also, let

$$Z_{in} = Z_0 \frac{1 + \Gamma_D}{1 - \Gamma_D} \tag{29}$$

which determines Z_{in} when Γ_D is given. Then (27) becomes, using (28)–(29)

$$P_{in} = \frac{1}{4} |V_{in}|^2 \frac{1}{Z_0^*} \left[\frac{(1 - \Gamma_D)(1 - \Gamma_L)}{1 - \Gamma_D \Gamma_L} \right]^* \tag{30}$$

which determines $|V_{in}|$ from the power input to the waveguide and the two reflection coefficients Γ_D, Γ_L.

In (27) or (30) only $|V_{in}|$ enters, and the phase of V_{in} does not matter. From (29), Γ_D and Z_{in} are interchangeable in the sense that one determines the other. Hence, only three equivalent circuit parameters matter: $|V_{in}|$ and the magnitude and phase of Z_{in}. On the other hand, we have proposed to use four input parameters: the real and imaginary part of Γ_D and P_{in}. It would appear that (30) cannot be satisfied because the phase of P_{in} is fixed by Γ_D, Γ_L, and cannot be taken as an independent input parameter. However, in general, one does not expect to be able to specify the phase of the input power. This phase represents the ratio of stored to dissipated power in the source and is determined by its load and its internal construction, not by external considerations. Hence, it is inevitable that only the magnitude of P_{in} can be externally specified (or, perhaps, only the real part of P_{in}). Therefore, (30) can be interpreted as allowing $|V_{in}|$ to be adjusted to fit the specified magnitude (or real part) of P_{in}, while the phase (or imaginary part) as determined from (30) is dictated by the internal structure of the source, as embodied in Γ_D, and of the load, as given by Γ_L.

It remains to decide how Γ_D, Γ_L are to be established. For example, these parameters can be found in terms of standing wave measurements in the waveguide, or by calculation in some instances [5], [6], [8], [9].

VI. Remarks

As discussed in [5], [6, ch. 4], in order to model a waveguide source of excitation the physical region chosen to enclose this driver must include a long enough section of waveguide to insure that all the higher order evanescent waveguide modes excited by the driver have attenuated to a negligible level. The same is true for the region enclosing a waveguide termination. In this way the transmission line model need describe only the energy that propagates down the waveguide in the lowest order mode. That is, because the transmission line I, V can characterize only one mode's E, H fields, we must exclude from the line any regions that support higher order modes.[1]

The evanescent modes contained in the terminal regions are the waveguide equivalent of fringing fields in the static case, and give rise to reactive contributions to the equivalent circuits [8, p. 138 ff.]. Needless to say, these evanescent modes will differ for different waveguides, leading to a dependence of the equivalent circuits for drivers and terminations upon the waveguide used. This dependence is in addition to the dependence *via* Z_0 already visible in (26), (29), and (30).

VII. Summary

A framework has been given for developing a transmission line to model interconnect at high frequencies where its waveguide nature becomes evident. This framework differs from the conventional analogy between transmission lines and waveguides because it extends to cases where losses are too large to be treated as a perturbation. Formulas for the required R, L, G, and C line parameters are provided in (15)–(18). Equivalent formulas in terms of the field variables E and H are given in Appendix I in (A35)–(A38). These R, L, G, and C parameters differ from the static values when the longitudinal field components are important, or when losses are severe.

A complication in the use of the equivalent transmission line is that the low-frequency concepts of voltage and current lose their intuitive appeal at higher frequencies where "voltage" and "current" are related in an abstract manner to the transverse electric and magnetic fields of the waveguide. The precise relation is given in (A25) and (A26) of Appendix I. Consequently the Thevenin voltage-equivalent circuit has been recast in a manner relating this circuit to reflection coefficients. A brief discussion has been presented to show that there is no difficulty in such a reformulation.

For concreteness, a detailed example of the analogy between a lossy waveguide structure in an inhomogeneous medium and its equivalent transmission line has been provided in Appendix I. Besides the results already mentioned, this example illustrates directly from the field solutions how the restrictions upon the allowed Z_0 of the equivalent transmission line arise, and provides a deeper understanding of the results of the text.

By using the analysis of this paper one can determine whether any proposed R, L, G, and C line is adequate to model a given interconnection. In addition, one can be certain that the power flow computed from an equivalent transmission line with $RLGC$ parameters that satisfy (15)–(18) is meaningful despite the abstract nature of "current" and "voltage" variables in the line at high frequencies.

Appendix I
Transmission Line Parameters From Maxwell's Equations

In this appendix the relations for R, L, G, and C of a transmission line equivalent to a lossy waveguide are derived from Maxwell's equations. For a homogeneous, lossless waveguide similar results are known [5], [6, p. 80], [7], [8], but for the inhomogeneous, lossy case results are not available. To analyze this case, the results of Kurokawa [10] are useful.

The material parameters are considered to be complex and dependent on transverse position. That is, choosing the z-direction as the direction of propagation, the dielectric permittivity, ϵ and magnetic permeability μ are given by

$$\epsilon = \kappa(x, y)\epsilon_0 + \sigma(x, y)/(j\omega) \tag{A1a}$$

$$\mu = \mu_1(x, y) + j\mu_2(x, y). \tag{A1b}$$

Here κ is the dielectric constant and σ is the conductivity. In addition, the imaginary part of the dielectric constant could be used to describe dielectric loss.

A. Basic Equations

Let the complex electric and magnetic fields at angular frequency ω be given by

$$E(x, y, z) = E_t(x, y)\, V(z) + \eta E_l(x, y)\, I(z) \tag{A2a}$$

$$H(x, y, z) = H_t(x, y)\, I(z) + \eta^{-1} H_l(x, y)\, V(z). \tag{A2b}$$

Here the subscript "t" denotes a transverse vector (with x- and y-components) and "l" denotes a longitudinal component in the z-direction.

Arbitrary constants can be placed in front of E_l and H_l. In (A2) these constants have been chosen as η and $(1/\eta)$ where η is the intrinsic impedance of empty space

$$\eta = (\mu_0/\epsilon_0)^{1/2} \approx 376.7\ \Omega.$$

As a result of this choice, the dimensions of E_t, E_l, H_t, and H_l all are inverse length.

The equations of (A2) are not the most general form of a solution to Maxwell's equations. One expects (A2) to hold whenever only one mode propagates. Otherwise, the right sides of (A2a) and (A2b) should be replaced by summations over all the modes, and E_t, H_t, V, and I would require a subscripted mode index because these functions usually would be different for each mode.

Assuming only one mode propagates, the equations determining E_t and H_t now can be found. Substituting (A2) in Maxwell's equations and separating the longitudinal and transverse components, one finds the following equations by separation of variables as explained in Appendix II (cf. [10] for the case of a single traveling wave)

$$\frac{\gamma}{Z_0} \hat{k} \times H_t + j\omega\epsilon E_t - \eta^{-1} \nabla \times H_l = 0 \tag{A3a}$$

$$\nabla \times H_t - j\omega\epsilon\eta E_l = 0 \tag{A3b}$$

$$\gamma Z_0 \hat{k} \times E_t - j\omega\mu H_t - \eta \nabla \times E_l = 0 \tag{A4a}$$

$$\nabla \times E_t + j\omega\mu\eta^{-1} H_l = 0 \tag{A4b}$$

$$\nabla \cdot (\epsilon E_t) - \frac{\gamma}{Z_0} \epsilon\eta\hat{k} \cdot E_l = 0 \tag{A5}$$

$$\nabla \cdot (\mu H_t) - \gamma Z_0 \mu\eta^{-1} \hat{k} \cdot H_l = 0. \tag{A6}$$

with $\hat{k}$ a unit vector in the z-direction. Here (A3a), (A3b) stem from the $\nabla \times H$ equations of Maxwell, (A4a), (A4b) from the $\nabla \times E$ equation, and (A5), (A6) from the $\nabla \cdot D$, $\nabla \cdot B$ equations. In addition to (A3)–(A6), one finds by the same separation of variables

$$\frac{dI(z)}{dz} = -\frac{\gamma}{Z_0} V(z) \tag{A7a}$$

$$\frac{dV(z)}{dz} = -\gamma Z_0 I(z) \tag{A7b}$$

which are the transmission line equations. As yet, Z_0 and γ are undefined constants. The general solutions to (A7) are known to be given by

$$V(z) = A\, e^{-\gamma z} + B\, e^{\gamma z} \quad \text{(A8a)}$$

$$I(z) = \frac{1}{Z_0}\,[A\, e^{-\gamma z} - B\, e^{\gamma z}] \quad \text{(A8b)}$$

as can be verified by direct substitution. Equation (A8) shows γ to be the propagation constant for the mode. Here, A and B are the complex amplitudes of the forward and reverse traveling waves whose values are determined by the load and driver of the line. Equation (A8) can satisfy the end conditions on the waveguide at the load and driver only to the extent that a single mode is reflected or excited. Realistic boundary conditions at the ends of the waveguide ordinarily can be satisfied only by a superposition of many modes, most of which die out rapidly with distance. For this reason the physical region containing the interconnection line must exclude at the source and termination a length of waveguide sufficient to allow attenuation to negligible levels of all modes other than the propagating mode [5], [6, ch. 4].

An eigenvalue equation for γ now can be found by eliminating the l-components of the fields from (A3)–(A6). Multiply (A4b) by μ^{-1} and take the *curl*. Then multiply (A5) by ϵ^{-1} and take the *gradient*. Subtract this result from the previous result to find

$$\mu\nabla \times \left[\frac{1}{\mu}(\nabla \times E_t)\right] - \nabla\left[\frac{1}{\epsilon}\nabla \cdot (\epsilon E_t)\right] = j\omega\mu\left[-\frac{\gamma}{Z_0}\hat{k} \times H_t - j\omega\epsilon E_t\right] - \frac{\gamma}{Z_0}\eta\nabla(\hat{k} \cdot E_l). \quad \text{(A9)}$$

Using (A4a) we eliminate H_t from (A9). By taking the cross product of (A4a) with $\hat{k}$ we find

$$j\omega\mu(\hat{k} \times H_t) = -\gamma Z_0 E_t - \eta\nabla(\hat{k} \cdot E_l). \quad \text{(A10)}$$

Putting (A10) in (A9)

$$\mu\nabla \times \left[\frac{1}{\mu}\nabla \times E_t\right] - \left\{\nabla\left[\frac{1}{\epsilon}\nabla \cdot (\epsilon E_t)\right] + (\gamma^2 + \omega^2\epsilon\mu)\, E_t\right\} = 0. \quad \text{(A11)}$$

A similar relation can be found for H_t with μ and ϵ interchanged. Solutions to (A11) satisfying the boundary conditions can be found only when γ^2 is one of the eigenvalues of (A11). These eigenvalues may be discrete, continuous or a combination of both depending upon the functions ϵ and μ.

In (A11) ϵ and μ are arbitrary complex functions of transverse position. In particular, (A11) applies to a lossy layered medium. For example, for the case of infinitely wide layers in the x-direction, all x-derivatives vanish. For a TM mode E has only y- and z-components and H has only an x-component. Then

$$\nabla \times E_t = \hat{k}\left(\frac{\partial E_{ty}}{\partial x} - \frac{\partial E_{tx}}{\partial y}\right) = 0 \quad \text{(A12)}$$

and (A11) becomes

$$\frac{d}{dy}\left\{\frac{1}{\epsilon(y)}\frac{d}{dy}[\epsilon(y)E_{ty}]\right\} + [\gamma^2 + \omega^2\epsilon(y)\,\mu(y)]E_{ty} = 0$$

which implies continuity at the interfaces of a many-layered medium with piecewise constant ϵ and μ of ϵE_{ty} and dE_{ty}/dy [11]. The first of these conditions is the usual requirement of continuity of normal displacement. The second condition can be understood from (A5), which becomes

$$E_l = \left(\frac{Z_0}{\gamma\eta}\right)\frac{1}{\epsilon(y)}\frac{d}{dy}[\epsilon(y)E_{ty}]. \quad \text{(A13)}$$

That is, the continuity of dE_{ty}/dy insures continuity of the tangential component of electric field E_l as also is usual.

B. Power

Having found the basic equations determining the transverse fields, now the power at position "z" will be found using Poynting's vector. The average power is

$$P(z) = \tfrac{1}{2}\int dx \int dy\, (E_{tx}H_{ty}^* - E_{ty}H_{tx}^*)\, I^*(z)\, V(z). \quad \text{(A14)}$$

For the transmission line to carry the same power as the waveguide, we require

$$P(z) = \tfrac{1}{2} I^*(z)\, V(z). \quad \text{(A15)}$$

Hence, E_t must be normalized so

$$\int dx \int dy\, (E_{tx}H_{ty}^* - E_{ty}H_{tx}^*) = 1. \quad \text{(A16)}$$

to make (A16) a condition upon E_t alone, replace H_t using (A9) and (A5)

$$\hat{k} \times H_t = -\frac{1}{j\omega}\left(\frac{Z_0}{\gamma}\right)\left\{\nabla \times \left[\frac{1}{\mu}\nabla \times E_t\right] - \omega^2\epsilon E_t\right\}. \quad \text{(A17)}$$

Now

$$\hat{k} \times H_t = -\hat{\imath}H_{ty} + \hat{\jmath}H_{tx} \quad \text{(A18)}$$

where $\hat{\imath}$ and $\hat{\jmath}$ are unit vectors in the x- and y-directions. Hence, (A16) becomes

$$\int dx \int dy\, E_t \cdot \left\{\frac{1}{j\omega}\left(\frac{Z_0}{\gamma}\right)\left[\nabla \times \left(\frac{1}{\mu}\nabla \times E_t\right) - \omega^2\epsilon E_t\right]\right\}^* = 1. \quad \text{(A19)}$$

If we now integrate by parts and assume the fields vanish at the boundaries of the waveguide (or at infinity) (A19) becomes

$$\int dx \int dy \left\{\frac{1}{\mu}(\nabla \times E_t) \cdot (\nabla \times E_t)^* - \omega^2\epsilon|E_t|^2\right\} = \frac{j\omega\gamma}{Z_0}. \quad \text{(A20)}$$

The condition (A20) normalizes the transverse fields so that the transmission line power and the waveguide power both are given by (A15). Suppose instead of eliminating H_t from (A16), E_t is eliminated using the analog of (A17), namely

$$\hat{k} \times E_t = \frac{1}{j\omega\gamma Z_0}\left[\nabla \times \left(\frac{1}{\epsilon}\nabla \times H_t\right) - \omega^2 \mu H_t\right]. \quad \text{(A21)}$$

Then instead of (A20) one finds

$$\int dx \int dy\, H_t^* \left\{\frac{1}{j\omega\gamma Z_0}\left[\nabla \times \left(\frac{1}{\epsilon}\nabla \times H_t\right) - \omega^2 \mu H_t\right]\right\} = 1 \quad \text{(A22)}$$

or, integrating by parts again

$$\int dx \int dy \left\{\frac{1}{\epsilon}|\nabla \times H_t|^2 - \omega^2 \mu |H_t|^2\right\} = j\omega\gamma Z_0. \quad \text{(A23)}$$

In the special case of a TM mode in a many-layered medium infinite in the x-direction, (A12) holds and (A20) becomes

$$W \int dy\, \epsilon(y)|E_{ty}|^2 = \frac{\gamma}{j\omega Z_0} \quad \text{(A24)}$$

where W is the width of the waveguide (that is, (A24) is a per unit width normalization).

C. Interpretation of $I(z)$, $V(z)$

Using the normalization condition in the form (A19) in conjunction with (A2) it can be shown that $V(z)$ is a weighted average of the transverse electric field over the waveguide cross section. Multiplying (A2a) by the complex factor in (A19) and integrating one obtains

$$V(z) = \int dx \int dy\, E(x, y, z) \cdot \left\{\frac{1}{j\omega}\left(\frac{Z_0}{\gamma}\right)\left[\nabla \times \left(\frac{1}{\mu}\nabla \times E_t\right) - \omega^2 \epsilon E_t\right]\right\}^*. \quad \text{(A25)}$$

A similar result for $I(z)$ can be found using the complex conjugate of (A22) in (A2b)

$$I(z) = \int dx \int dy\, H(x, y, z) \cdot \left\{\frac{1}{j\omega\gamma Z_0}\left[\nabla \times \left(\frac{1}{\epsilon}\nabla \times H_t\right) - \omega^2 \mu H_t\right]\right\}^*. \quad \text{(A26)}$$

Equations (A25) and (A26) show that $V(z)$ and $I(z)$ are weighted averages of the transverse electric and magnetic fields across the cross section of the waveguide.

For the special case of a TM mode in a many-layered medium infinite in the x-direction, (A25) is simplified because the *curl* term vanishes in the integrand. Thus (A25) becomes

$$V(z) = -j\omega\left(\frac{Z_0}{\gamma}\right)^* W \int dy\, E(y, z) \cdot \epsilon^*(y)\, E_t^*(y)$$

and (A26) also simplifies to become

$$I(z) = \left(\frac{\gamma}{j\omega Z_0}\right)^* W \int dy\, H(y, z) \cdot \frac{1}{\epsilon^*(y)} H_t^*(y)$$

where again W is the waveguide width.

D. Formulas for R, L, G, and C

Introduce R, L, G, and C using the definitions

$$G + j\omega C \equiv \frac{\gamma}{Z_0} \quad \text{(A27)}$$

$$R + j\omega L \equiv \gamma Z_0. \quad \text{(A28)}$$

From (A27) and (A28) one obtains the customary results

$$\gamma^2 = (R + j\omega L)(G + j\omega C) \quad \text{(A29)}$$

and

$$Z_0^2 = (R + j\omega L)/(G + j\omega C). \quad \text{(A30)}$$

Up to this point Z_0 is not defined, but γ^2 is determined as an eigenvalue of (A11).

Using (A27) and the normalization condition (A19) in the form (A20) one finds

$$G = \int dx \int dy\, \{\sigma|E_t|^2 - \omega\mu_2|H_l|^2/\eta^2\} \quad \text{(A31)}$$

$$C = \int dx \int dy\, \{\kappa\epsilon_0|E_t|^2 - \mu_1|H_l|^2/\eta^2\}. \quad \text{(A32)}$$

Similarly, using the normalization condition (A19) in the form (A23) in conjunction with (A28) one finds

$$R = \int dx \int dy\, \{\sigma\eta^2|E_l|^2 - \omega\mu_2|H_t|^2\} \quad \text{(A33)}$$

$$L = \int dx \int dy\, \{\mu_1|H_t|^2 - \kappa\epsilon_0\eta^2|E_l|^2\}. \quad \text{(A34)}$$

In these expressions the l-components are obtained by substitution of (A3b) and (A4b) for the *curl* quantities.

For comparison with results for homogeneous media, it is convenient to replace the transverse and longitudinal field quantities with the field components from (A2). Then (A31)–(A34) become

$$G = \frac{1}{|V(z)|^2}\int dx \int dy\, \{\sigma\, |E_n(x, y, z)|^2 - \omega\mu_2|H_z(x, y, z)|^2\} \quad \text{(A35)}$$

$$C = \frac{1}{|V(z)|^2}\int dx \int dy\, \{\kappa\epsilon_0|E_n(x, y, z)|^2 - \mu_1|H_z(x, y, z)|^2\} \quad \text{(A36)}$$

$$R = \frac{1}{|I(z)|^2}\int dx \int dy\, \{\sigma\, |E_z(x, y, z)|^2 - \omega\mu_2|H_n(x, y, z)|^2\} \quad \text{(A37)}$$

$$L = \frac{1}{|I(z)|^2} \int dx \int dy \, \{\mu_1 |H_n(x, y, z)|^2 - \kappa\epsilon_0 |E_z(x, y, z)|^2\}. \tag{A38}$$

Here the subscript "n" refers to the component of the full fields normal to the direction of propagation, including z-dependence from (A2).

Equations (A35), (A36), and (A38) reduce to the results of Collin [6, p. 80] when ϵ and μ are position independent and the z-components of the fields are small. An exception is (A37), which is more general than Collin's result for R, because his result assumes R can be found entirely by a perturbation treatment of the skin effect in metallic boundaries of the waveguide. When losses are important, R, L, C, and G from (A35)–(A38) differ from the usual static or perturbation theory values.

From (A35) and (A36) it appears that the usually assumed static symmetry of G and C under the exchange of σ and $\kappa\epsilon_0$ fails when there is a z-component of H. It also is noteworthy that a z-component of H reduces C, analogous to an inductance in the shunt admittance, and a z-component of E reduces L, analogous to a series capacitance in the series impedance.

A simple check upon (A35)–(A38) is obtained using the vector identity

$$\nabla \cdot (E \times H^*) = H^* \cdot \nabla \times E - E \cdot \nabla \times H^*. \tag{A39}$$

Using Maxwell's relations for a time dependence exp $(j\omega t)$

$$\nabla \times H^* = (-j\omega\kappa\epsilon_0 + \sigma)E^*$$

$$\nabla \times E = -j\omega\mu H$$

one finds

$$\nabla \cdot (E \times H^*) = -j\omega\mu |H|^2 + j\omega\kappa\epsilon_0 |E|^2 - \sigma\, |E|^2. \tag{A40}$$

Now integrate (A40) over a volume which consists of the slice of transmission line between "z" and "$z + \delta z$". Using the divergence theorem to change the left side to a surface integral and ignoring the contribution at infinity of the infinitesimal sides of width "δz" we find (A40) becomes (using (A2))

$$\int dx \int dy \, (E_t \times H_t^*) \left\{ V(z) \frac{dI(z)^*}{dz} + \frac{dV(z)}{dz} I^*(z) \right\} \delta z = \int dx \int dy \, [-j\omega\mu |H|^2 + j\omega\kappa\epsilon_0 |E|^2 - \sigma\, |E|^2]\, \delta z. \tag{A41}$$

Finally, using (A7), (A16), (A27), and (A28), (A41) provides the following identity:

$$(R + j\omega L)\, |I(z)|^2 + (G - j\omega C)\, |V(z)|^2 = \int dx \int dy \, \{ j\omega\mu |H|^2 - j\omega\kappa\epsilon_0 |E|^2 + \sigma\, |E|^2 \}. \tag{A42}$$

Using (A35)–(A38) we find that R, L, G, and C do satisfy the requirement (A42) with

$$|E|^2 = |E_n|^2 + |E_z|^2; \qquad |H|^2 = |H_n|^2 + |H_z|^2. \tag{A43}$$

E. Restrictions on Z_0

Equations (A35)–(A38) determine R, L, G, and C from the condition (A19) that the power flow in the transmission line be the same as that in the waveguide. However, these equations do not fully determine R, L, G, and C. As is more apparent from the expressions (A31)–(A34), the (x, y) field factors are *normalized* by (A31)–(A34)—that is, these equations determine how the field amplitudes are divided between the (x, y) factors and the z factors $V(z)$ and $I(z)$. However, the ratios $G/(\omega C)$ and $R/(\omega L)$ are independent of the normalization of $E_t(x, y)$ and $H_t(x, y)$. For example

$$\frac{R}{\omega L} = \frac{\int dx \int dy \left\{ \frac{\sigma}{|\omega\epsilon|^2} |\nabla \times H_t|^2 - \omega\mu_2 |H_t|^2 \right\}}{\int dx \int dy \left\{ \omega\mu_1 |H_t|^2 - \frac{\kappa\epsilon_0}{\omega |\epsilon|^2} |\nabla \times H_t|^2 \right\}} \tag{A44}$$

is independent of the normalization of H_t because any multiplicative factor in H_t cancels from numerator and denominator. Similarly

$$\frac{G}{\omega C} = \frac{\int dx \int dy \left\{ \sigma\, |E_t|^2 - \frac{\mu_2}{\omega |\mu|^2} |\nabla \times E_t|^2 \right\}}{\int dx \int dy \left\{ \omega\kappa\epsilon_0 |E_t|^2 - \frac{\mu_1}{\omega |\mu|^2} |\nabla \times E_t|^2 \right\}} \tag{A45}$$

is independent of the normalization of E_t.

The normalization independent ratios (A44) and (A45) express a condition on the *phase* of Z_0. Thus Z_0 cannot be chosen arbitrarily, unlike the case for lossless media where frequently Z_0 is set arbitrarily to a real value such as unity. Rather (A44) and (A45) determine the phase of Z_0 through (A27) and (A28), which can be rewritten as

$$Z_0 = \frac{\gamma}{G + j\omega C} \tag{A46}$$

$$Z_0 = \frac{R + j\omega L}{\gamma}. \tag{A47}$$

Because $\gamma = \alpha + j\beta$ is a known eigenvalue, its value is fixed. Consequently, the phase of Z_0 is determined by $G/(\omega C)$ or, equivalently, by $R/(\omega L)$. The magnitude of Z_0 is arbitrary, and can be chosen to match a convenient low-frequency representation of the waveguide in the case of a mode that propagates at low frequencies.

The conditions (A44) and (A45) appear much more complicated than (15) and (16) of the text, where these ratios are expressed in terms of γ and Q. Nonetheless, these conditions are equivalent because they stem from the same relation, namely the power relation (A15) in terms of $I(z)$ and $V(z)$. To show this equivalence directly, one can derive an expression for Q in terms of the transverse field components. For a single traveling wave the transmission line equations (A7) provide (see (A8))

$$E_n(x, y, z) = E_t(x, y) A e^{-\gamma z} \tag{A48}$$

$$H_n(x, y, z) = H_t(x, y) \frac{1}{Z_0} A e^{-\gamma z} \tag{A49}$$

where A is the amplitude of the "voltage" wave. Forming the power flow according to Poynting's vector and taking the quotient of imaginary to real parts one finds

$$Q = \frac{\operatorname{Im}\left\{Z_0 \int dx \int dy\, E_t \times H_t^*\right\}}{\operatorname{Re}\left\{Z_0 \int dx \int dy\, E_t \times H_t^*\right\}}. \tag{A50}$$

Following the steps leading to (A20), (A50) becomes

$$Q = \frac{\operatorname{Im}\left\{(\beta - j\alpha) \int dx \int dy \left[\frac{1}{\mu^*} |\nabla \times E_t|^2 - \omega^2 \epsilon^* |E_t|^2\right]\right\}}{\operatorname{Re}\left\{(\beta - j\alpha) \int dx \int dy \left[\frac{1}{\mu^*} |\nabla \times E_t|^2 - \omega^2 \epsilon^* |E_t|^2\right]\right\}}. \tag{A51}$$

Using (A45) for $G/(\omega C)$, (A51) becomes

$$Q = -\frac{\alpha - \beta \dfrac{G}{\omega C}}{\alpha \dfrac{G}{\omega C} + \beta} \tag{A52}$$

or, rearranging terms

$$\frac{\omega C}{G} = -\frac{\alpha Q - \beta}{\alpha + \beta Q} \tag{A53}$$

which agrees with (16) of the text. Equation (15) of the text follows in the same way using (A45) with the steps leading to (A23). The remaining equations (17) and (18) follow from these two and (A29) for γ^2.

F. Summary

It has been shown that a transmission line equivalent to a lossy, inhomogeneous waveguide can be found such that the same power is found at each position "z" in both structures. Also, the propagation of $I(z)$ and $V(z)$ in the z-direction of the transmission line mimics exactly the corresponding z-dependence of the transverse H and E fields in the waveguide. However, according to (A25) and (A26), $I(z)$ and $V(z)$ are to be interpreted as weighted averages of the transverse magnetic and electric fields across the transverse dimensions of the waveguide, and need bear no relation to the meaning of "current" and "voltage" as might be inferred from line integrals of these fields about some intuitively selected contours.

Also, explicit expressions (A35)–(A38) for the transmission line R, L, G, and C in terms of the E and H fields have been found. Equivalently, these quantities can be determined in terms of the power quotient (A50) of a single traveling wave, as derived differently in the text.

The characteristic impedance of the equivalent transmission line has a magnitude that can be chosen for convenience, but its phase is fixed by the requirement of correct average complex power flow.

Appendix II
Separation of Variables and the Introduction of γ and Z_0

In this appendix it is shown how (A3)–(A7) of Appendix I result from Maxwell's equations and the assumed form (A2) for the fields that is repeated here

$$E(x, y, z) = E_t(x, y)\, V(z) + \eta E_l(x, y)\, I(z) \tag{B1a}$$

$$H(x, y, z) = H_t(x, y)\, I(z) + \eta^{-1} H_l(x, y)\, V(z). \tag{B1b}$$

Substituting (B1) in the Maxwell relation

$$\nabla \times E = -j\omega\mu H \tag{B2}$$

one obtains

$$\begin{aligned} &V(z)\, \nabla \times E_t(x, y) + \frac{dV(z)}{dz} \hat{k} \times E_t(x, y) \\ &\quad + I(z)\, \eta \nabla \times E_l(x, y) \\ &\quad = -j\omega\mu[I(z)\, H_t(x, y) + V(z)\, \eta^{-1} H_l(x, y)]. \end{aligned} \tag{B3}$$

Here $\hat{k}$ is a unit vector in the z-direction.

In (B3) $\nabla \times E_t$ is in the z-direction because E_t depends only on (x, y). Hence, taking the z-components of (B3) one obtains

$$V(z)\, [\nabla \times E_t(x, y) + j\omega\mu\eta^{-1} H_l(x, y)] = 0. \tag{B4}$$

Assuming $V(z)$ is nonzero, (B4) results in (A4b) of Appendix I.

From the transverse components of (B3) one finds

$$\begin{aligned} &\eta \nabla \times E_l(x, y) + j\omega\mu H_t(x, y) \\ &\quad = -\hat{k} \times E_t(x, y) \left[\frac{1}{I(z)} \frac{dV(z)}{dz}\right]. \end{aligned} \tag{B5}$$

Because the left side of (B5) is independent of z, it follows from (B5) that

$$\frac{1}{I(z)} \frac{dV(z)}{dz} = \text{constant} = c_1. \tag{B6}$$

Using (B1) in the Maxwell relation

$$\nabla \times H = j\omega\epsilon E \tag{B7}$$

and following an analogous procedure, one finds from the z-components

$$I(z)\, [\nabla \times H_t(x, y) - j\omega\epsilon\eta E_l] = 0. \tag{B8}$$

If $I(z)$ is nonzero, (B8) leads to (A3b) of Appendix I.

From the transverse components one finds

$$\eta^{-1}\nabla \times H_l(x, y) - j\omega\epsilon E_t(x, y) = -\hat{k} \times H_t(x, y) \left[\frac{1}{V(z)} \frac{dI(z)}{dz}\right]. \quad \text{(B9)}$$

As with (B5), the z-independence of the left side of (B9) implies

$$\frac{1}{V(z)} \frac{dI(z)}{dz} = \text{constant} = c_2. \quad \text{(B10)}$$

At this point, consider the two equations (B6) and (B10), which involve the two separation constants c_1 and c_2. Equations (B6) and (B10) can be cast into the form of the transmission line equations by choosing c_1 and c_2 to satisfy

$$c_1 = -\gamma Z_0 \quad \text{(B11)}$$

$$c_2 = -\gamma / Z_0. \quad \text{(B12)}$$

Equations (B11) and (B12) introduce the two constants γ and Z_0 in place of the constants of separation of variables c_1 and c_2. Because c_1 and c_2 are arbitrary and independent of each other, γ and Z_0 are also arbitrary independent constants at this point.

Using (B11) and (B12) in (B5) and (B9), (B5) becomes (A4a) and (B9) becomes (A3a) of Appendix I. Finally, using (B1) in the Maxwell relation

$$\nabla \cdot [\epsilon(x, y) E(x, y, z)] = 0 \quad \text{(B13)}$$

leads to

$$V(z) \nabla \cdot [\epsilon(x, y) E_t(x, y)] + \frac{dI(z)}{dz} \epsilon(x, y)\eta \hat{k} \cdot E_l(x, y) = 0. \quad \text{(B14)}$$

Using (B10) and (B12) in (B14)

$$V(z)\left\{\nabla \cdot [\epsilon(x, y) E_t(x, y)] - \frac{\gamma}{Z_0} \epsilon(x, y)\eta \hat{k} \cdot E_l(x, y)\right\} = 0 \quad \text{(B15)}$$

which leads to (A5) if $V(z) \neq 0$. In the same way, using (B1) in the Maxwell relation

$$\nabla \cdot [\mu(x, y) H(x, y, z)] = 0 \quad \text{(B16)}$$

leads to

$$I(z) \nabla \cdot [\mu(x, y) H_t(x, y)] + \mu(x, y)\eta^{-1}\hat{k} \cdot H_l(x, y) \frac{dV(z)}{dz} = 0 \quad \text{(B17)}$$

which in combination with (B6) and (B11) leads to

$$I(z)\{\nabla \cdot [\mu(x, y) H_t(x, y)] - \gamma Z_0 \mu(x, y)\eta^{-1}\hat{k} \cdot H_l(x, y)\} = 0. \quad \text{(B18)}$$

For $I(z) \neq 0$, (B18) leads to (A6) of Appendix I.

To summarize, it has been shown that by a judicious choice of the separation constants in separating the variables, (B6) and (B10) governing the z-dependence of the fields can be made the same as the transmission line equations, with γ and Z_0 related to the separation constants. When this choice is made, the Maxwell equations governing the dependence of the fields upon transverse coordinates are given by (A3)–(A6) of Appendix I.

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References

[1] T. Sakurai, "Approximation of wiring delay in MOSFET LSI," *IEEE J. Solid-State Circuits*, vol. SC-18, p. 418, 1983.
[2] D. L. Carter and D. F. Guise, "Analysis of signal propagation delays and chip level performance due to on-chip interconnections," in *Proc. IEEE Int. Conf. Computer Design: VLSI and Computers* (Portchester, NY, Oct. 31–Nov. 3, 1983), pp. 218–221.
[3] I. T. Ho and S. K. Mullick, "Analysis of transmission lines on integrated-circuit chips," *IEEE J. Solid-State Circuits*, vol. SC-2, p. 201, 1967.
[4] M. Cases and D. M. Quinn, "Transient response of uniformly distributed RLC transmission lines," *IEEE Trans. Circuits Syst.*, vol. CAS-27, p. 200, 1980.
[5] C. G. Montgomery, in *Principles of Microwave Circuits*, C. G. Montgomery, R. H. Dicke, and E. M. Purcell, Eds., vol. 8 in *MIT Radiation Laboratory Series*, L. N. Ridenour and G. B. Collins, Eds. New York: McGraw-Hill, 1948, ch. 3.
[6] R. E. Collin, *Foundations of Microwave Engineering.* New York: McGraw-Hill, 1966.
[7] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics.* New York: Wiley, 1965, pp. 397–400.
[8] N. Marcuvitz, *Waveguide Handbook*, vol. 10 in *MIT Radiation Laboratory Series*, L. N. Ridenour and G. B. Collins, Eds. New York: McGraw-Hill, 1951, pp. 8 and 105.
[9] L. G. H. Huxley, *Survey of the Principles and Practice of Waveguides.* New York: Macmillan, 1947, pp. 117–126.
[10] K. Kurokawa, *An Introduction to the Theory of Microwave Circuits.* New York: Academic, 1969, pp. 164–167.
[11] B. Friedman, *Principles and Techniques of Applied Mathematics.* New York: Wiley, 1956, pp. 174–177.

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