# Compact Models for Transient Voltage

## Distributed RC Models

- ~~N-Line Solution~~
- ~~1-Line Solution~~
- ➡ 2-Line Example

# Coupled Line Solutions

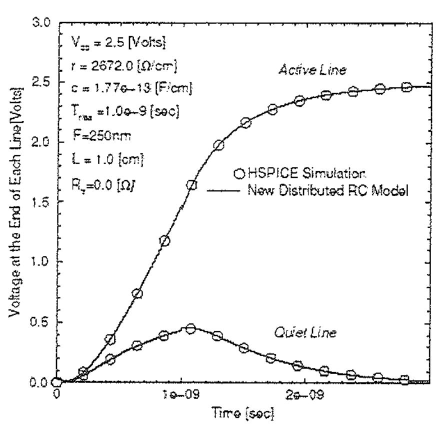
Region I: $t \le T_{rise}$

$$\frac{V_2}{V_{dd}} = \frac{C_m(R+2R_s) + B_0^+ e^{-\frac{\sigma_o^+}{RC^+}t} - B_0^- e^{-\frac{\sigma_o^-}{RC^-}t}}{2T_{rise}}$$

Region II: $t > T_{rise}$

$$\frac{V_2}{V_{dd}} = \frac{-B_0^- e^{-\frac{\sigma_o^-}{RC^-}t}(-e^{\frac{\sigma_o^-}{RC^-}T_{rise}} + 1) + B_0^+ e^{-\frac{\sigma_o^+}{RC^+}t}(1 - e^{\frac{\sigma_o^+}{RC^+}T_{rise}})}{2T_{rise}}$$

# HSPICE Comparison




# Peak Crosstalk Expression

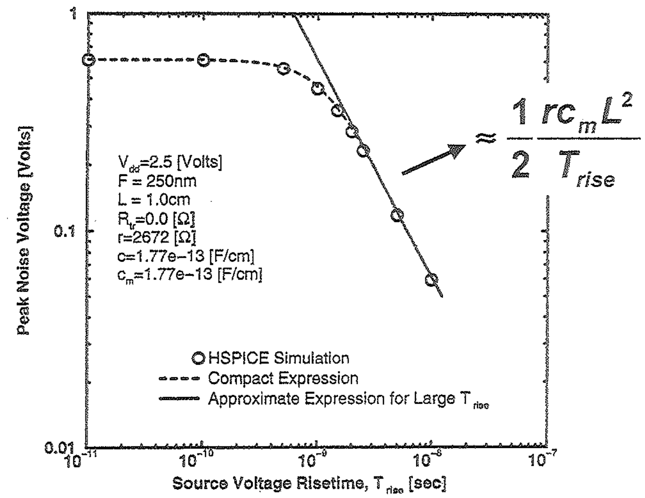
$$\frac{V_{peak}}{V_{dd}} = \frac{\chi B_o^+}{T_{rise}}\left(1 - e^{\frac{\sigma_o^+ T_{rise}}{L^2 rc^+}}\right)\left(1 - \frac{\sigma_o^+ c^-}{\sigma_o^- c^+}\right)\left(\frac{\sigma_o^+ c^- B_o^+\left(1 - e^{\frac{T_{rise}\sigma_o^+}{L^2 rc^+}}\right)}{\sigma_o^- c^+ B_o^-\left(1 - e^{\frac{T_{rise}\sigma_o^-}{L^2 rc^-}}\right)}\right)^{\frac{-\frac{\sigma_o^+}{c^+}}{\left(\frac{\sigma_o^+}{c^+} - \frac{\sigma_o^-}{c^-}\right)}}$$

## Parameter Definitions

$$\sigma_o^{\pm} = \frac{1.04}{\frac{R_{tr}C_L}{rc^{\pm}L^2} + \frac{R_{tr}}{rL} + \frac{C_L}{c^{\pm}L} + \left(\frac{2}{\pi}\right)^2}$$

$$B_o^{\pm} = 1.01\frac{rc^{\pm}L^2}{\sigma_o^{\pm}} \frac{\frac{R_{tr}}{rL} + \frac{C_L}{c^{\pm}L} + 1}{\frac{R_{tr}}{rL} + \frac{C_L}{c^{\pm}L} + \frac{\pi}{4}}$$

## HSPICE Comparison




## Sakurai's key RC result

$$\frac{V_{peak}}{V_{dd}} \approx \frac{1}{2}\frac{c_m}{c + c_m}$$

Parallel plate Approximation

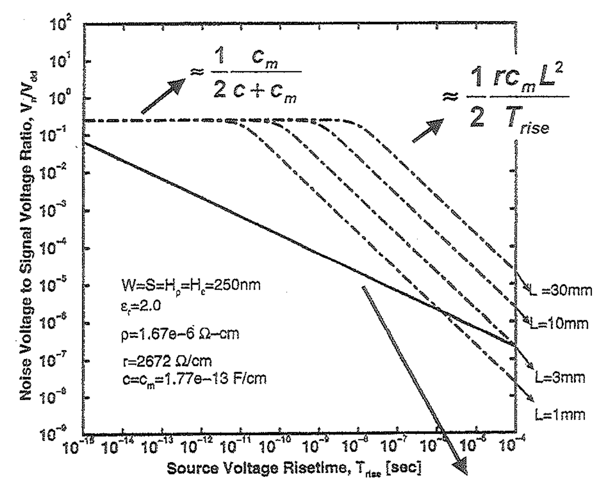
$$\frac{1}{2}\frac{\frac{H_\rho}{S}}{\frac{W}{H_\varepsilon} + \frac{H_\rho}{S}}$$

$R_s = 0$

*Approximately length, material, and scaling independent!!*

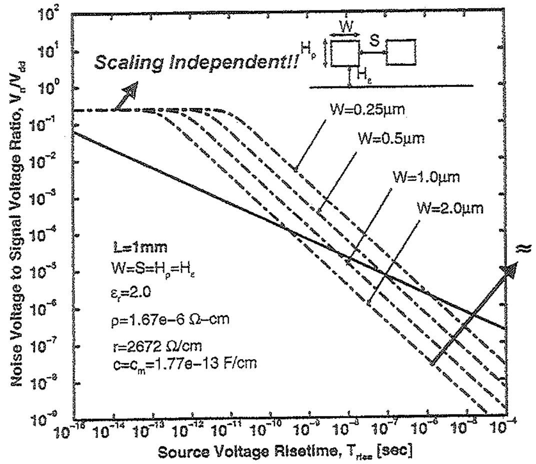
*What can we learn from $T_{rise}$ expressions?*

## Length Dependence



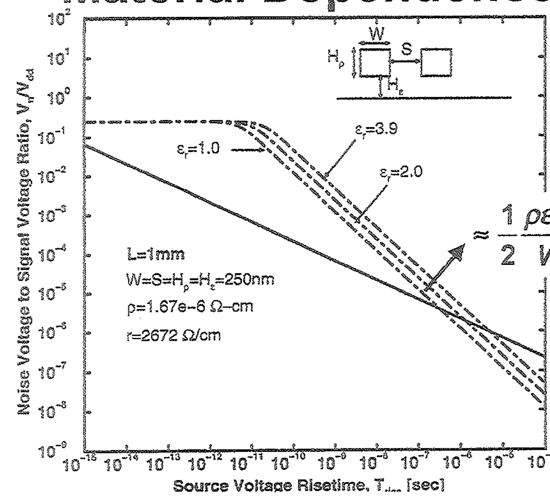

*Fundamental Thermal Noise Limit*

## Scaling Dependence




$$\approx \frac{1}{2}\frac{\rho\varepsilon_r\varepsilon_o}{W^2}\frac{L^2}{T_{rise}}$$

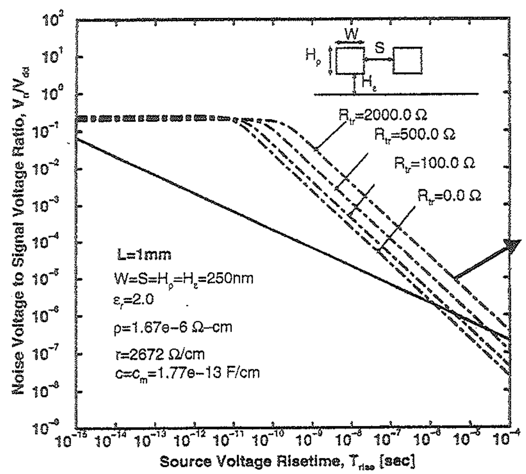
## Material Dependence




$$\approx \frac{1}{2}\frac{\rho\varepsilon_r\varepsilon_o}{W^2}\frac{L^2}{T_{rise}}$$

*50% reduction in crosstalk from $SiO_2$ to low permittivity dielectric!!*

## Driver Resistance Dependence




$\underline{R_{tr} >> rL}$

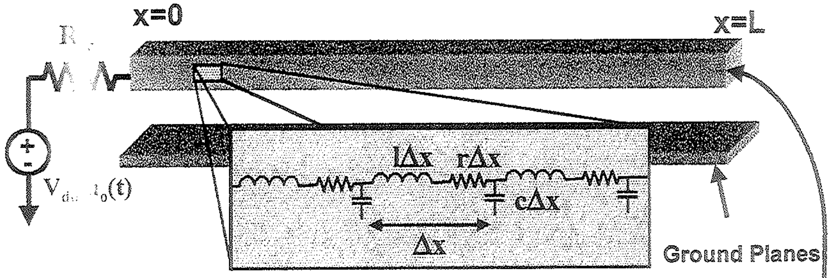
$$\approx \frac{R_{tr}c_mL}{T_{rise}}$$

## Compact Models for Transient Voltage

**Distributed RLC Models**

- **N-Line Solution** ⬅
- **1-Line Solution**
- **2-Line Example**

# Distributed RLC Model

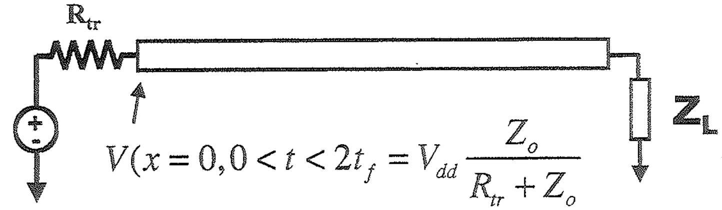



$$\frac{\partial^2}{\partial x^2}V(x,t) = rc\frac{\partial}{\partial t}V(x,t) + lc\frac{\partial^2}{\partial t^2}V(x,t)$$

*Open Circuit Terminations*

$$\left|\frac{1}{jwC_L}\right| >> rL$$

**Review: Lossless Transmission Line Analysis**



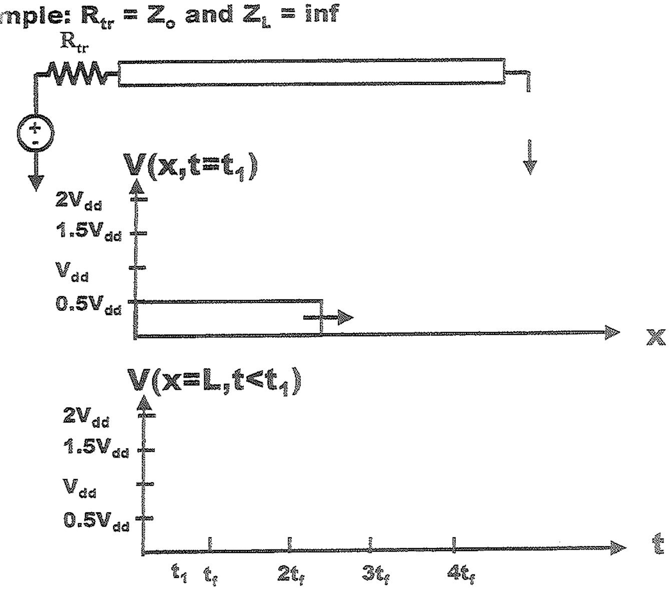

$$V(x=0, 0<t<2t_f) = V_{dd}\frac{Z_o}{R_{tr}+Z_o}$$

$$\Gamma_s = \frac{R_{tr}-Z_o}{R_{tr}+Z_o} \qquad \Gamma_L = \frac{Z_L-Z_o}{Z_L+Z_o}$$

$$Z_o = \sqrt{\frac{l}{c}} \qquad t_f = L/v = \sqrt{lc}L$$

**Review: Lossless Transmission Line Analysis**

**Example: $R_{tr} = Z_o$ and $Z_L$ = inf**

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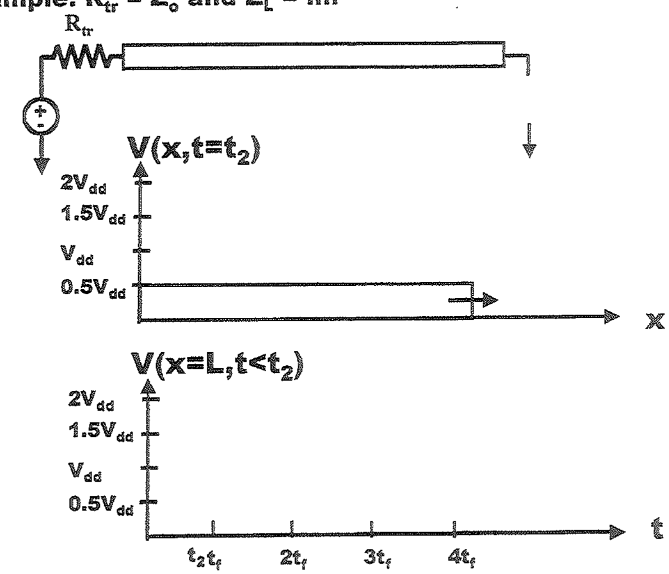

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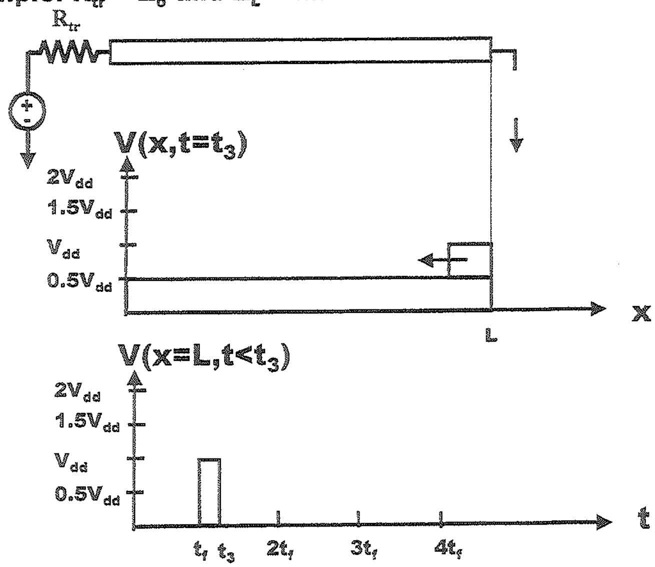
**Example: $R_{tr} = Z_o$ and $Z_L = inf$**

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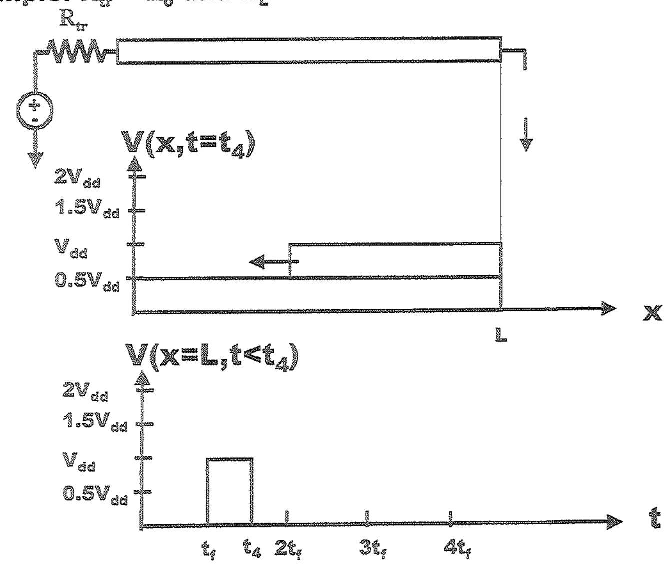


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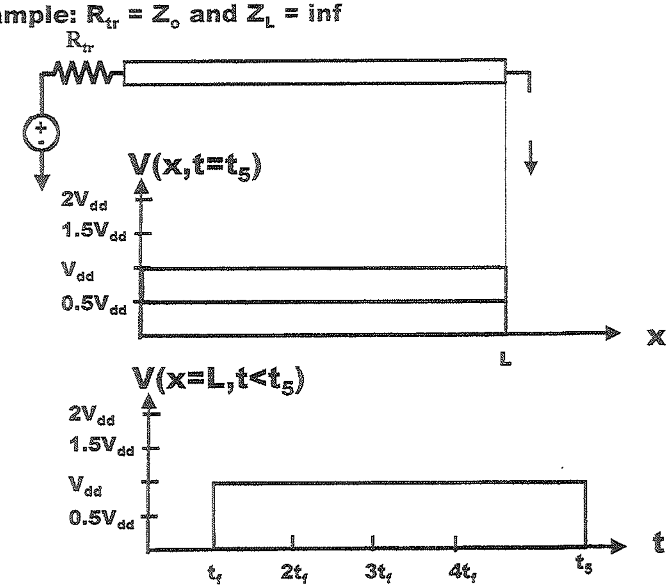
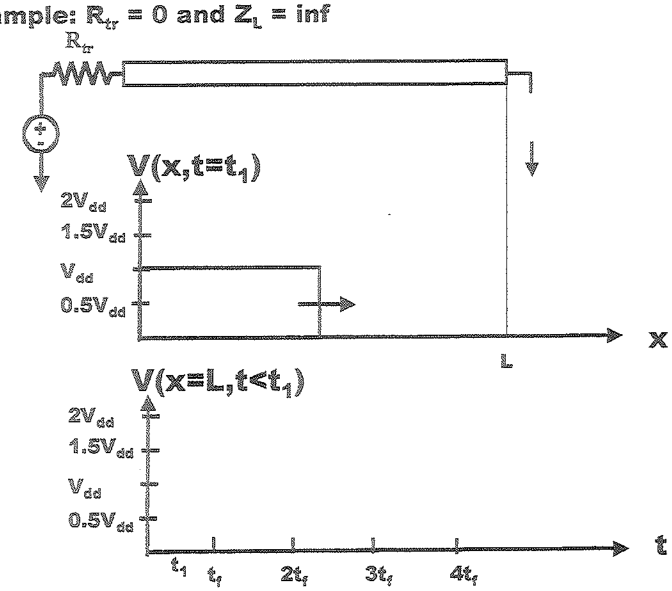



## Review: Lossless Transmission Line Analysis

**Example: $R_{tr} = 0$ and $Z_L = inf$**



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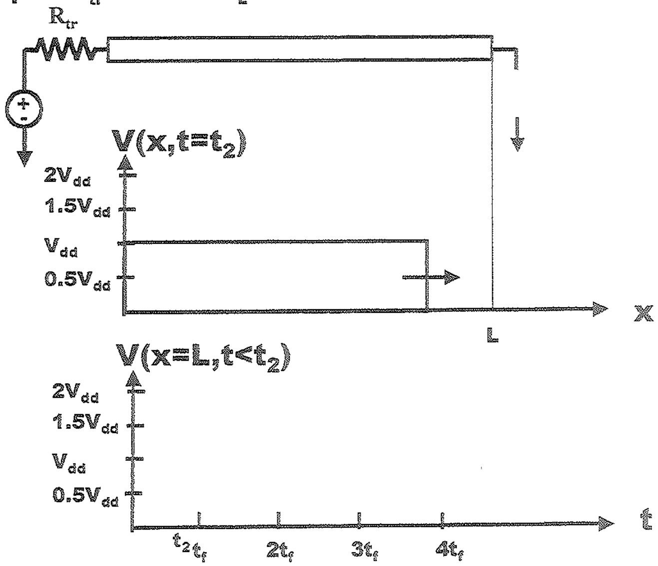


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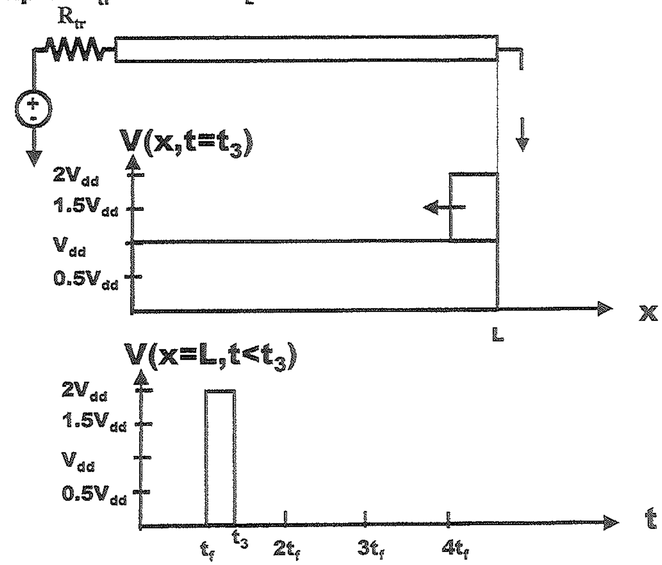


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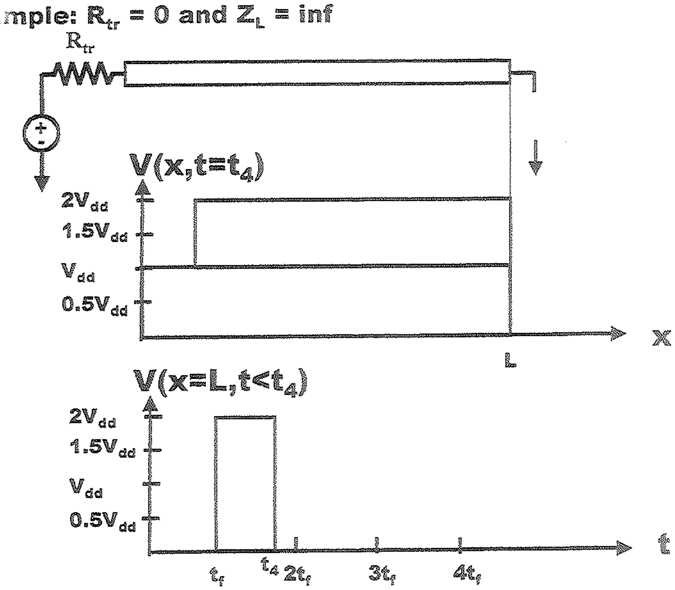


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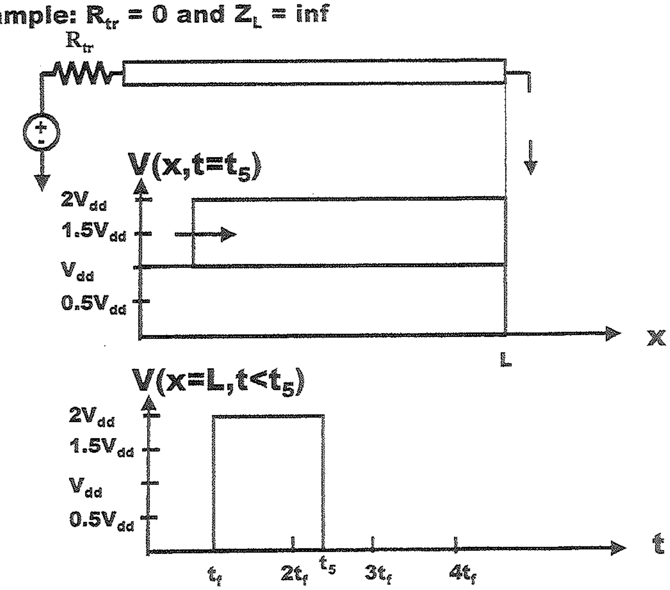
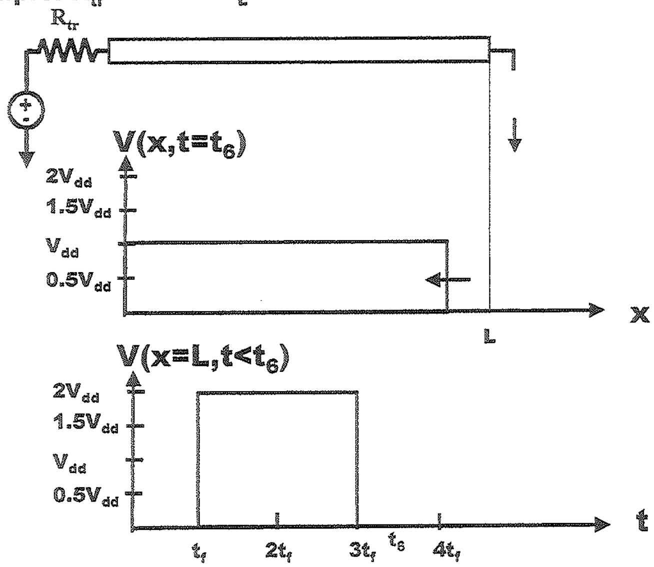


## Review: Lossless Transmission Line Analysis

### Example: $R_{tr}$ = 0 and $Z_L$ = inf

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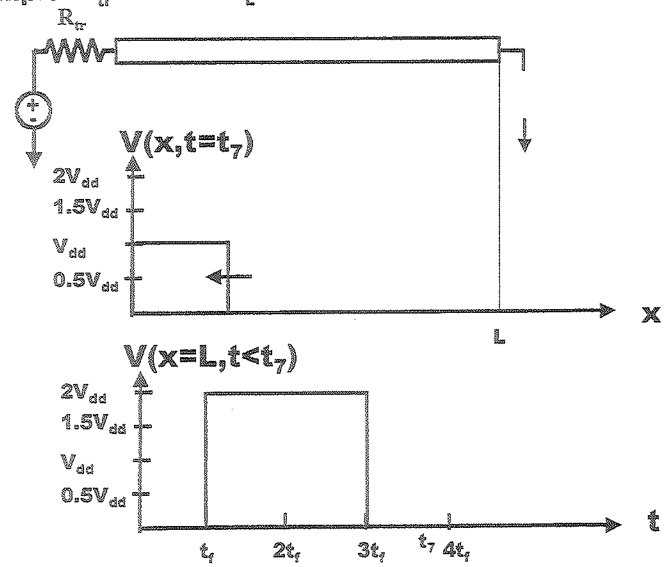


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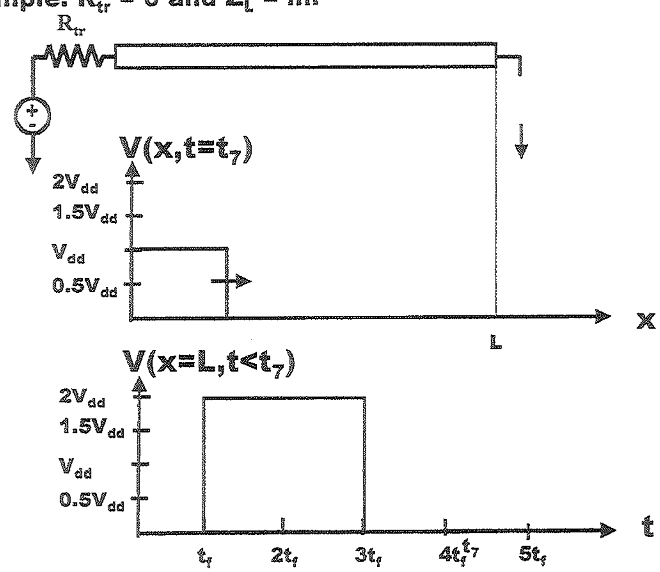


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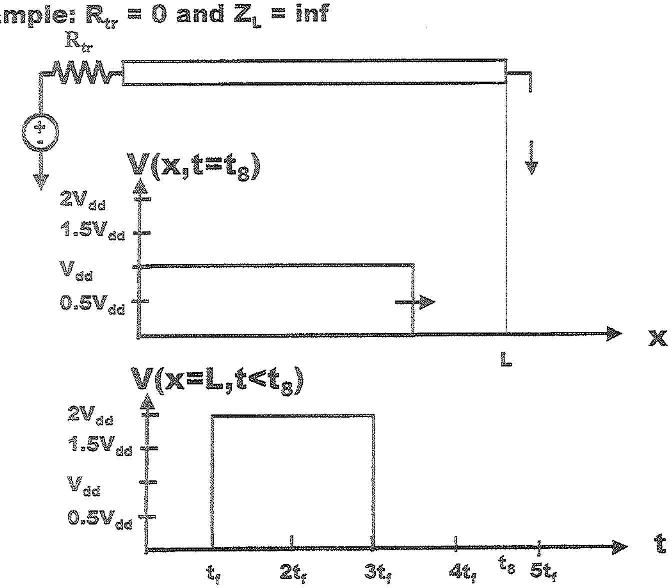
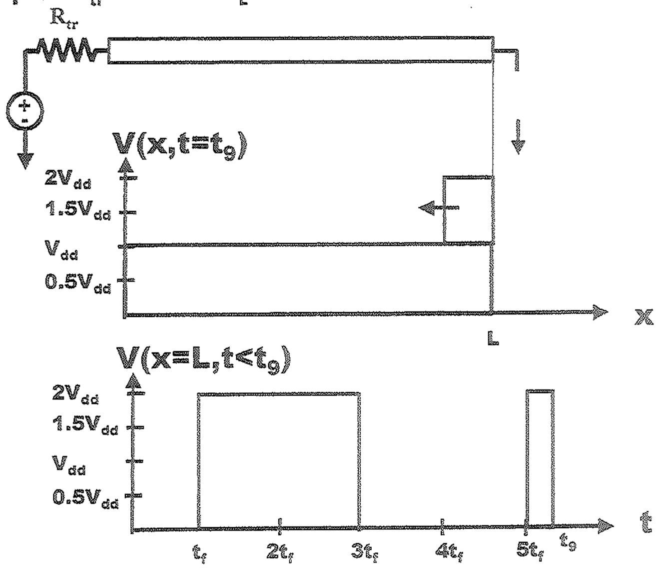
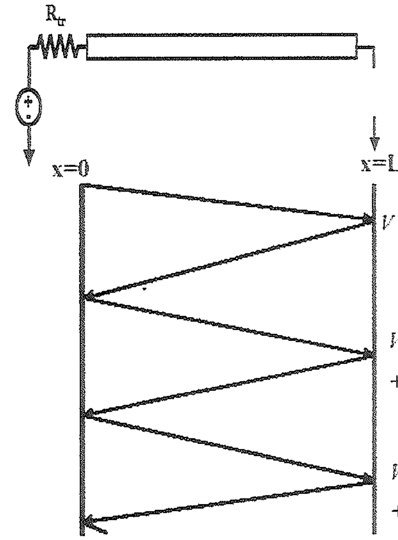


## Review: Lossless Transmission Line Analysis

**Example: $R_{tr} = 0$ and $Z_L = \text{inf}$**




# Reflection Diagram




$t_f \le t < 3t_f$ $\quad$ $q=0$

$V(x=L,s) = 2V_{inf}(x=L,s)$

$3t_f \le t < 5t_f$ $\quad$ $q=1$

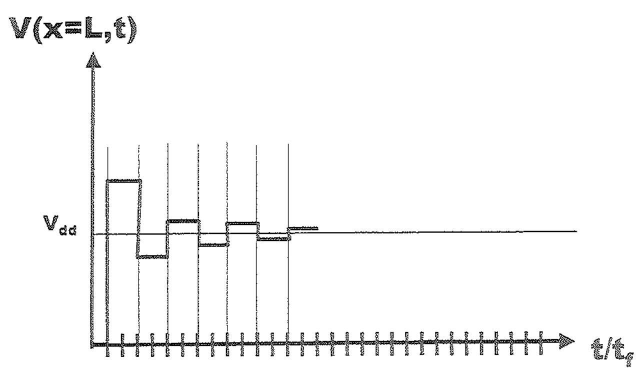
$V(x=L,s) = 2V_{inf}(x=L,s)$
$+2\Gamma V_{inf}(x=3L,s)$

$5t_f \le t < 7t_f$ $\quad$ $q=2$

$V(x=L,s) = 2V_{inf}(x=L,s)$
$+2\Gamma V_{inf}(x=3L,s)$
$+2\Gamma^2 V_{inf}(x=5L,s)$

## Review: Lossless Transmission Line Analysis

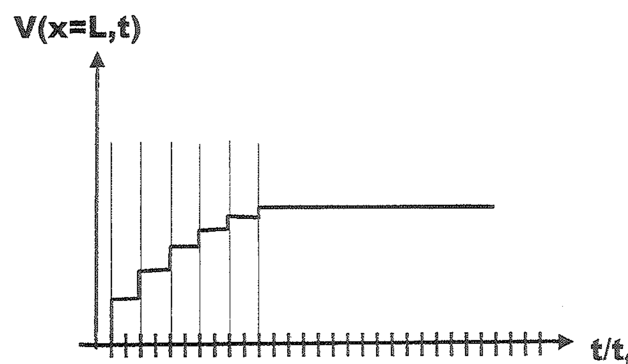
**Example: $0 < R_{tr} < Zo$ and $Z_L = \text{inf}$**




$$\Gamma_s = \frac{R_{tr} - Z_o}{R_{tr} + Z_o} \qquad V(x=0, 0<t<2t_f) = V_{dd}\frac{Z_o}{R_{tr}+Z_o}$$

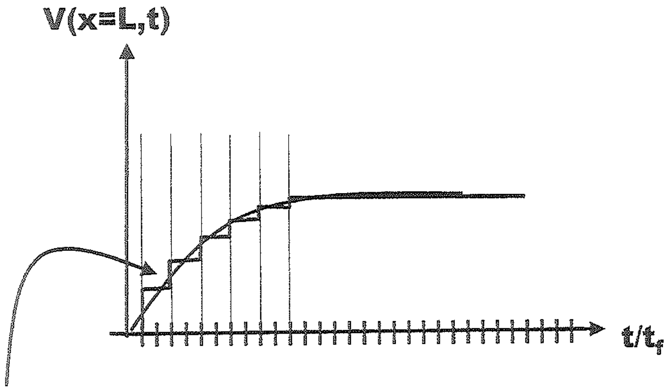
## Review: Lossless Transmission Line Analysis

**Example: $R_{tr} >> Zo$ and $Z_L = \text{inf}$**



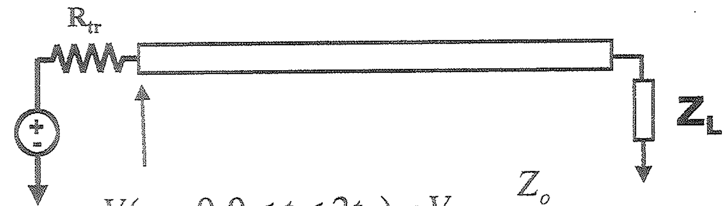
**Review: Lossless Transmission Line Analysis**

**Example: $R_{tr}$ >> Zo and $Z_L$ = inf**




**If $R_{tr}$ is large enough starts to approach $R_{tr}C$ curve!!**

**Review: Low Loss Transmission Line Analysis**

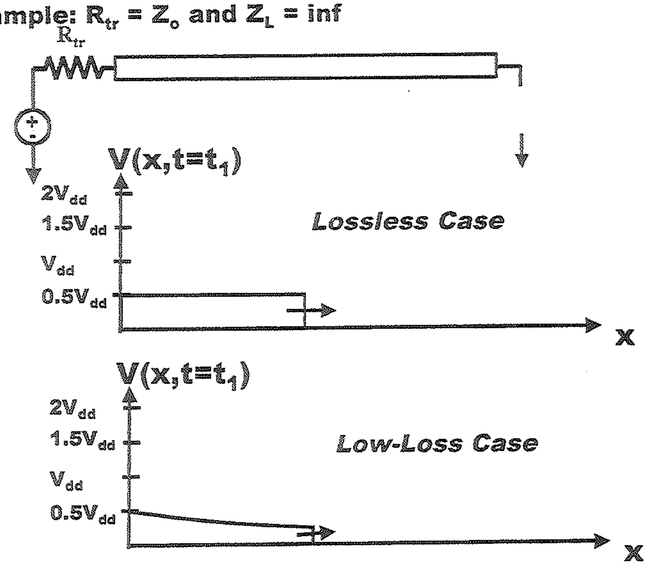



$$V(x=0, 0<t<2t_f) = V_{dd}\frac{Z_o}{R_{tr}+Z_o}$$

$$V(x, 0<t<t_f) = V_{dd}\frac{Z_o}{R_{tr}+Z_o}e^{-\frac{rx}{2Z_o}}$$

**Review: Low Loss Transmission Line Analysis**

**Example: $R_{tr}$ = $Z_o$ and $Z_L$ = inf**




# N-Wire Configuration

$$\frac{\partial^2}{\partial x^2}\tilde{V} = r\left[C\right]\frac{\partial}{\partial t}\tilde{V} + \left[L\right]\left[C\right]\frac{\partial^2}{\partial t^2}\tilde{V}$$

$$\tilde{V} = \begin{bmatrix} V_1 \\ \dots \\ V_n \end{bmatrix}$$

**Assume all cross-sectional area are the same for each conductor**

## Ideal Ground Plane Approximation

$$\frac{\partial^2}{\partial x^2}\tilde{V} = r[C]\frac{\partial}{\partial t}\tilde{V} + [L][C]\frac{\partial^2}{\partial t^2}\tilde{V}$$

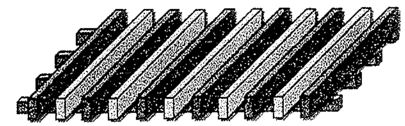
$$[I]\frac{1}{v^2} = [L][C]$$

$$\frac{\partial^2}{\partial x^2}\tilde{V} = r[C]\frac{\partial}{\partial t}\tilde{V} + \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\tilde{V}$$

**igenvectors of [C] decouples solution!**

## Orthogonal Lines makes decoupling difficult!

$$\frac{\partial^2}{\partial x^2}\tilde{V} = r[C]\frac{\partial}{\partial t}\tilde{V} + [L][C]\frac{\partial^2}{\partial t^2}\tilde{V}$$

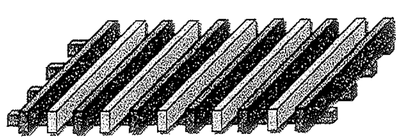


**Except for the 2 line case-- In general this will not decouple!**

## Inductance Approximation

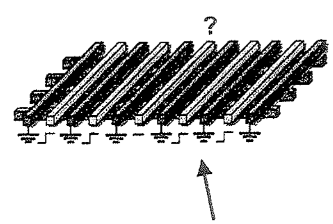
**Ho ever, could approximate with ...**

$$\left[L_{\text{with orthogonal lines}}\right] \approx \left[C_{\text{remove orthogonal}}\right]^{-1}\frac{1}{v^2}$$



## Diagonalized Capacitance Matrix Approximation

$$\frac{\partial^2}{\partial x^2}\tilde{V} = rc\frac{\partial}{\partial t}\tilde{V} + [L]c\frac{\partial^2}{\partial t^2}\tilde{V}$$




***Now eigenvectors of L matrix decouples lines AND single line solutions can be used!***

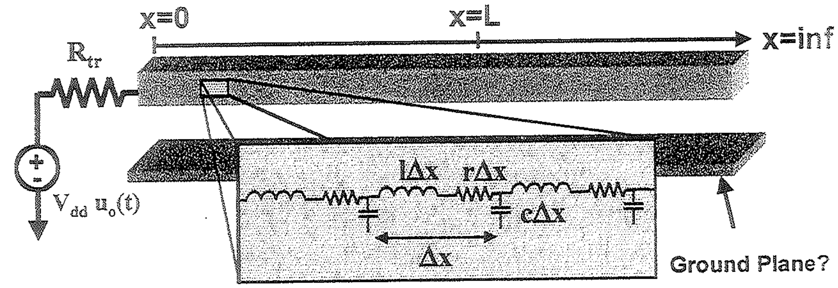
**All identical and all aggressor switching simultaneously!**

# Compact Models for Transient Voltage

## Distributed RLC Models

- ~~N-Line Solution~~
- ➡ 1-Line Solution
- 2-Line Example

# Where to start?




$$\frac{\partial^2}{\partial x^2}V(x,t) = rc\frac{\partial}{\partial t}V(x,t) + lc\frac{\partial^2}{\partial t^2}V(x,t)$$

# Alternate Expansion

**"Distributed RC"**

$$V(x,s) = \frac{K_0}{s} + \frac{K_1}{(s-\delta_1)} + \frac{K_2}{(s-\delta_2)} + \dots + \frac{K_n}{(s-\delta_n)}$$

**"Distributed RLC"**

$$V(x,s) = \frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}}\left(1 + \frac{\sigma K_1}{\left(s+\sqrt{s^2-\sigma^2}\right)} + \frac{\sigma^2 K_2}{\left(s+\sqrt{s^2-\sigma^2}\right)^2} + \dots + \frac{\sigma^n K_n}{\left(s+\sqrt{s^2-\sigma^2}\right)^n}\right)$$

# Inverse Laplace Transform

**"Distributed RC"**

$$\frac{K_n}{(s-\delta_n)} \rightarrow K_n e^{-\delta_n t}$$

**"Distributed RLC"**

$$\frac{e^{-x\sqrt{lc}\sqrt{s^2-\sigma^2}}}{\sqrt{s^2-\sigma^2}} \frac{\sigma^n}{\left(s+\sqrt{s^2-\sigma^2}\right)^n} \rightarrow \left(\frac{t-x\sqrt{lc}}{t+x\sqrt{lc}}\right)^{\frac{n}{2}} I_n\left[\sigma\sqrt{t^2-x^2lc}\right] u_o\left(t-x\sqrt{lc}\right)$$

Modified Bessel Function

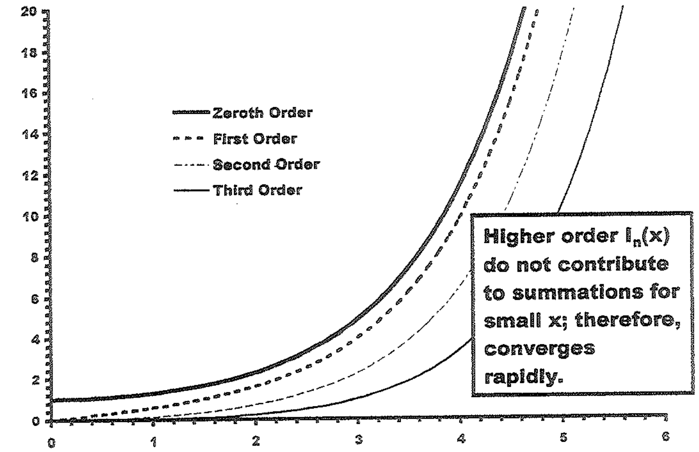
## Infinite Line Solution

### "Distributed RLC"

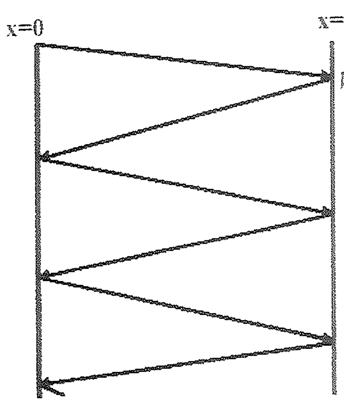
$$V_{INF}(x,t) = V_{dd}\left(\begin{array}{l}\dfrac{Z_o}{R_{tr}+Z_o} e^{-\frac{Rt}{2L}} I_0\left[\dfrac{R}{2L}\sqrt{t^2-\left(x\sqrt{LC}\right)^2}\right] \\ +\dfrac{1}{2}\displaystyle\sum_{k=1}^{\infty}\left(\dfrac{t-x\sqrt{LC}}{t+x\sqrt{LC}}\right)^{\frac{1}{2}k} e^{-\frac{Rt}{2L}} I_k\left[\dfrac{R}{2L}\sqrt{t^2-\left(x\sqrt{LC}\right)^2}\right]\left(4-\Gamma^{k-1}(\Gamma+1)^2\right)\end{array}\right) u\left[t-x\sqrt{LC}\right]$$

Jeff Davis, Azad Naeemi, and James Meindl , "Chapter 4: Transient Models for RC and RLC Interconnects," in ***Interconnect Technology and Design for Gigascale Integration,* 2003.**

## Modified Bessel Functions




## Finite Line Solution

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$V(x=L,s) = 2V_{inf}(x=L,s)$
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$V(x=L,s) = 2V_{inf}(x=L,s)$
$+2\Gamma V_{inf}(x=3L,s)$
$+2\Gamma^2 V_{inf}(x=5L,s)$

## Finite Line Solution

*Infinite Line First Reflection*

$$V_{FIN}(\ell,t) = 2V_{gen}(\ell,t,m=0) +$$
$$2e^{-\frac{R}{2L}t}\sum_{n=1}^{q}\sum_{i=0}^{n}\sum_{j=0}^{\infty}\frac{n(n-1+j)!}{i!\,j!\,(n-i)!}(-1)^i\,\Gamma^{n-i+j}V_{gen}\left(x=(2n+1)\ell,t,m=i+j\right)$$

*Remaining Reflections*

**Where $V_{gen}$ is:**

$$V_{gen}(x,t,m) = V_{dd}\left(\begin{array}{l}\dfrac{Z_o}{R_{tr}+Z_o}\left(\dfrac{t-x\sqrt{LC}}{t+x\sqrt{LC}}\right)^{\frac{m}{2}} e^{-\frac{Rt}{2L}} I_0\left[\dfrac{R}{2L}\sqrt{t^2-\left(x\sqrt{LC}\right)^2}\right] \\ +\dfrac{1}{2}\displaystyle\sum_{k=1}^{\infty}\left(\dfrac{t-x\sqrt{LC}}{t+x\sqrt{LC}}\right)^{\frac{1}{2}(k+m)} e^{-\frac{Rt}{2L}} I_{k+m}\left[\dfrac{R}{2L}\sqrt{t^2-\left(x\sqrt{LC}\right)^2}\right]\left(4-\Gamma^{k-1}(\Gamma+1)^2\right)\end{array}\right) u\left[t-x\sqrt{LC}\right]$$

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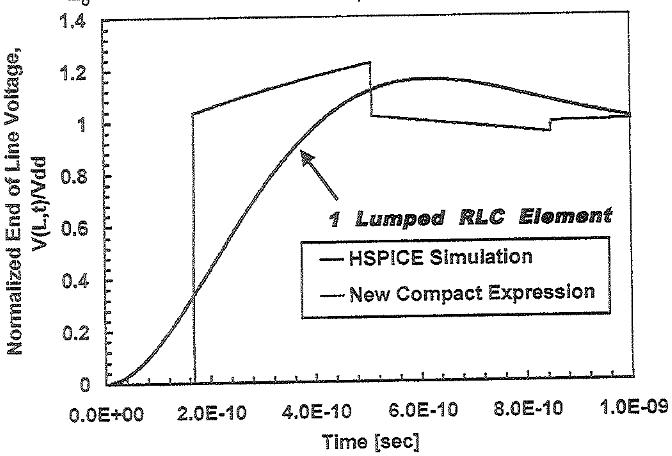
# HSPICE Comparison

$Z_o = 266.4\Omega$ $r = 37.86\ \Omega/cm$ $\varepsilon_r = 2.0$ $l = 3.6cm$ $R_{tr} = 0.5Z_o$

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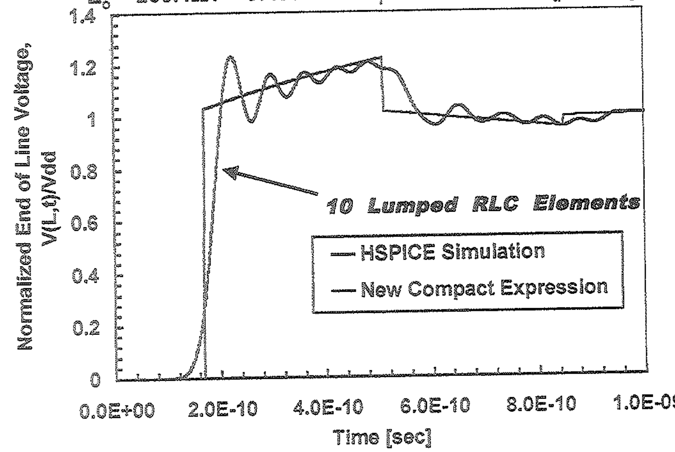
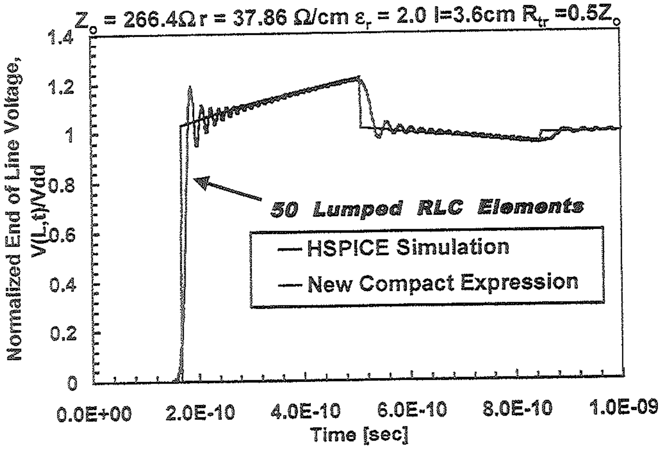


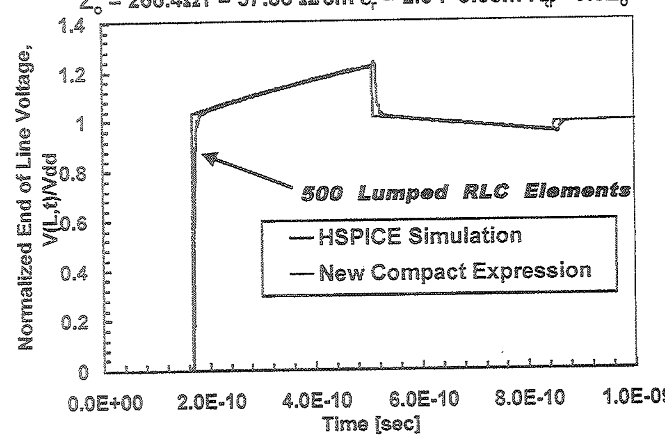

# HSPICE Comparison

$Z_o = 266.4\Omega$ $r = 37.86\ \Omega/cm$ $\varepsilon_r = 2.0$ $l = 3.6cm$ $R_{tr} = 0.5Z_o$

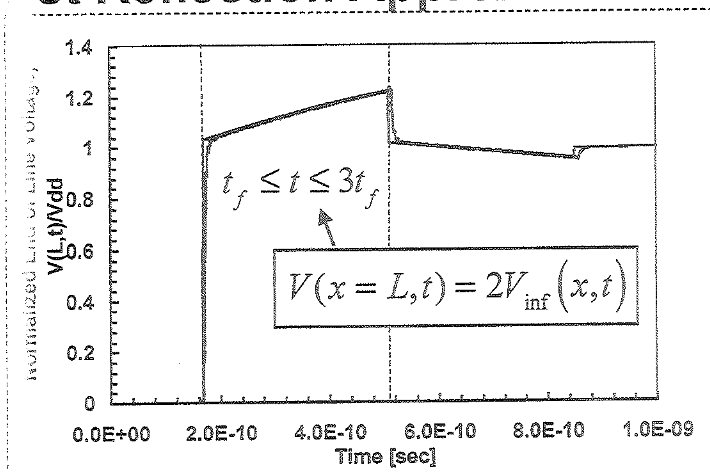



# HSPICE Comparison

$Z_o = 266.4\Omega$ $r = 37.86\ \Omega/cm$ $\varepsilon_r = 2.0$ $l = 3.6cm$ $R_{tr} = 0.5Z_o$



## F st Reflection Approximation



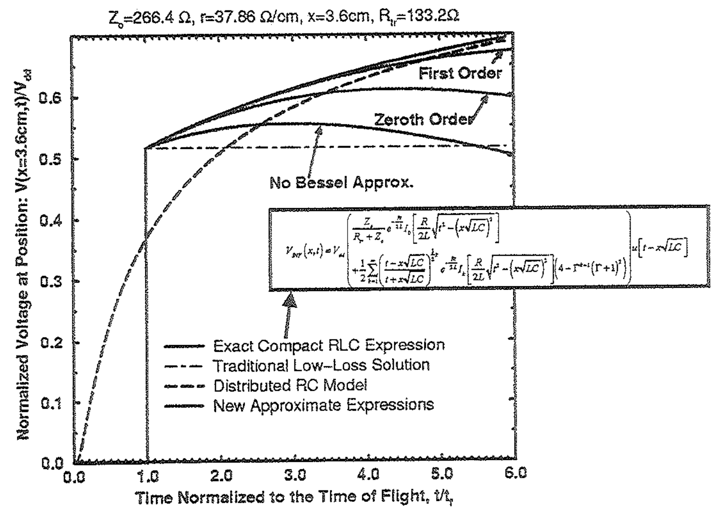

## F rst Reflection Approximation (1 > $\tilde{t}$ > 3)

$$\tilde{t} = \frac{t}{x\sqrt{lc}}$$

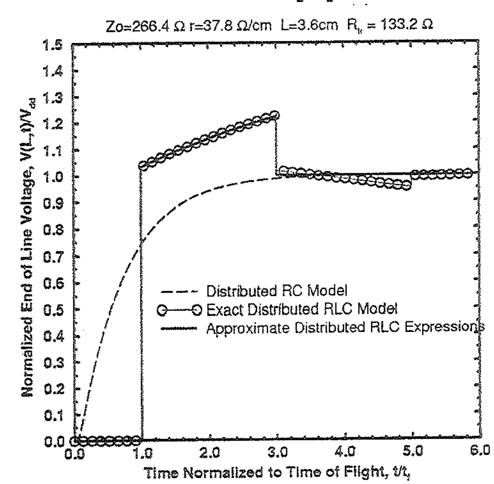
$$V(t) = V_{DD} \cdot e^{-\frac{R}{2Z_o}\tilde{t}} \left( \begin{aligned} &\frac{Z_o}{R_s + Z_o} I_o\left[\frac{R}{2Z_o}\sqrt{\tilde{t}^2 - 1}\right] + \frac{1}{2}\left(\frac{\tilde{t}-1}{\tilde{t}+1}\right)\left(4 - (1+\Gamma^2)\right) I_1\left[\frac{R}{2Z_o}\sqrt{\tilde{t}^2 - 1}\right] \\ &+ \frac{1}{2}\left( (1-\tilde{t})\frac{R}{4Z_o}\left(4 - (1+\Gamma)^2\right) - 4(1 - e^{\frac{R}{4Z_o}(\tilde{t}-1)}) + \frac{(1+\Gamma)^2}{\Gamma}(1 - e^{\frac{R}{4Z_o}\Gamma(\tilde{t}-1)}) \right) \end{aligned} \right)$$

| | |
|---|---|
| **max 5% error @ 3t$_f$ as long as** | $R < \left(3/\sqrt{8}\right) Z_o \approx Z_o$ |
| **max 10% error @ 3t$_f$ as long as** | $R < \sqrt{2} Z_o \approx 1.4 Z_o$ |

## Series Convergence Example

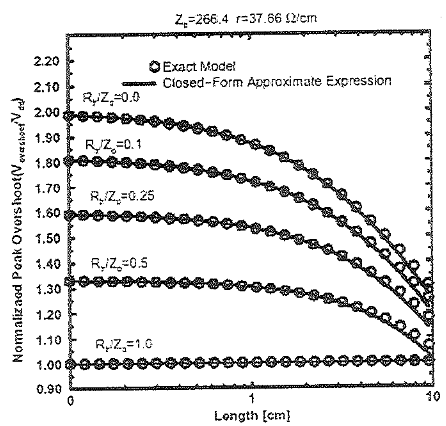



## 1st Reflection Approximation

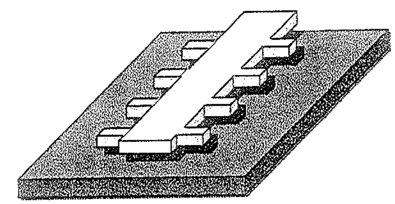


## Simplified Overshoot Validation

$$\frac{V_{overshoot}}{V_{dd}} = \max(1, \frac{V_{1R\_approx}(L, t = 3L\sqrt{lc})}{V_{dd}})$$



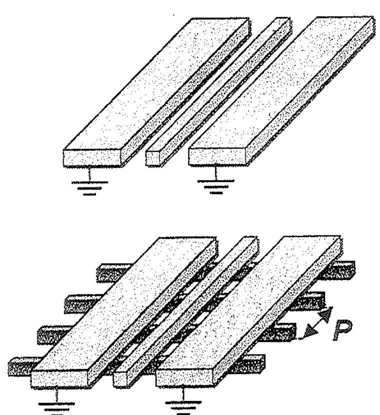

## Periodic Structures Microwave



For large wavelengths, it behaves like a smooth transmission line but with a lower speed of wave:

$$v = \frac{1}{\sqrt{l(c + \frac{C_0}{d})}} < \frac{c_0}{\sqrt{\varepsilon_r}}$$

## Single Line Structure

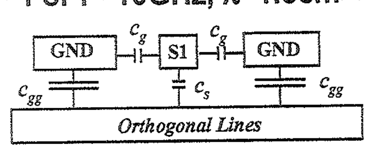



**Ideal transmission line:**

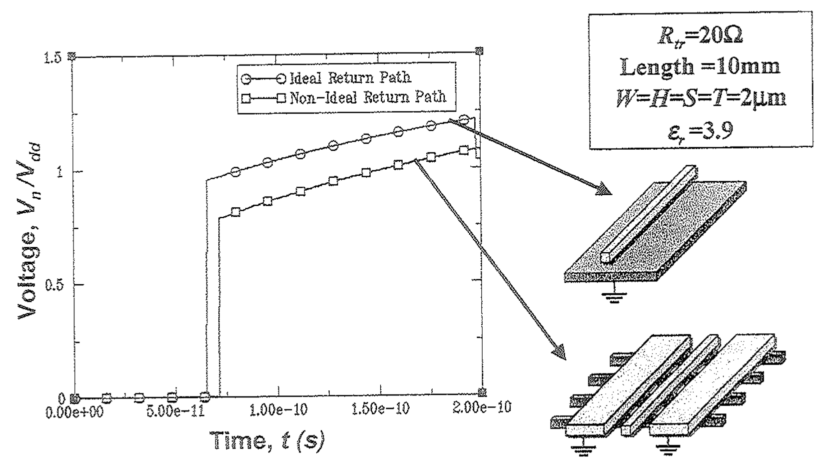
$$\frac{c_0}{\varepsilon_r} = \frac{1}{\sqrt{lc}}$$

**Non-ideal return path:**

- For F=10GHz, λ=1.5cm>>P




## Time of Flight Variation

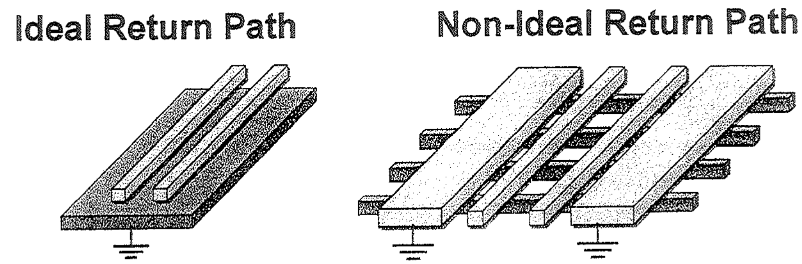


## Compact Models for Transient Voltage

### Distributed RLC Models

- ~~N-Line Solution~~
- ~~1-Line Solution~~
- ➡ 2,3,&5-Line Example

## Impact of Return Path on Crosstalk




## 1&3 RLC Lines Over a Ground Plane

- **Single RLC Line**

$$\frac{\partial^2}{\partial x^2}V(x,t) = rc\frac{\partial}{\partial t}V(x,t) + \frac{1}{v^2}\frac{\partial^2}{\partial t^2}V(x,t)$$

- **3 Coupled RLC Lines**


| | |
|---|---|
| $V_{sum} = V_{fin}(x,t,c=c_g, l=\frac{1}{c_g v^2})$ | $V_{victim} = \frac{1}{3}V_{sum} - \frac{2}{3}V_{dif}$ |
| $V_{dif} = V_{fin}(x,t,c=c_g+3c_m, l=\frac{1}{(c_g+3c_m)v^2})$ | $V_{Active} = \frac{1}{3}V_{sum} + \frac{1}{3}V_{dif}$ |

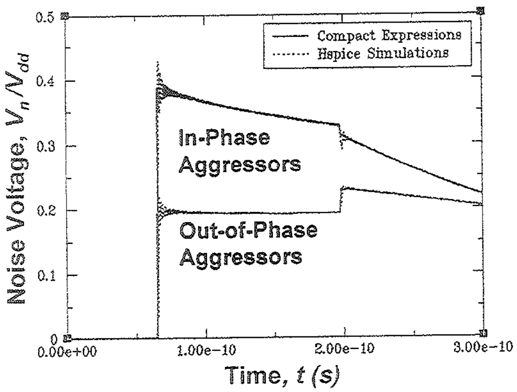
## 5-Conductor RLC Line

$$\frac{\partial^2}{\partial x^2}[V(x,t)] = r[C]\frac{\partial}{\partial t}[V(x,t)] + \frac{1}{v^2}\frac{\partial^2}{\partial t^2}[V(x,t)]$$

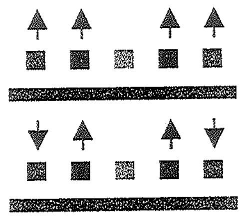

| | |
|---|---|
| $V_{sum} = V_{fin}(x,t,c=c_g, l=\frac{1}{cv^2})$ | $V_{victim} = 0.2V_{sum} - 0.32V_{dif1} + 0.124V_{dif2}$ |
| $V_{dif1} = V_{fin}(x,t,c=c_g+\frac{5+\sqrt{5}}{2}c_m, l=\frac{1}{cv^2})$ | $V_{near} = 0.2V_{sum} - 0.262V_{dif1} + 0.04V_{dif2}$ |
| $V_{dif2} = V_{fin}(x,t,c=c_g+\frac{5-\sqrt{5}}{2}c_m, l=\frac{1}{cv^2})$ | $V_{far} = 0.2V_{sum} - 0.1V_{dif1} + 0.1V_{dif2}$ |

***With ground plane, all modes travel @ the speed of light!***

# HSPICE Verification

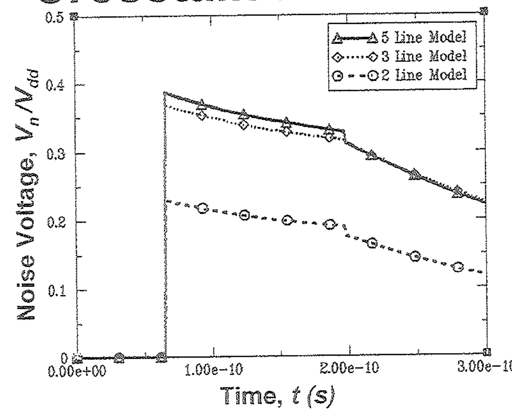


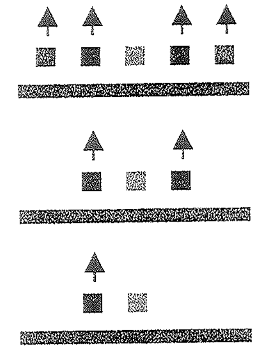

$R_{tr}$=100Ω
Length =10mm
$W$=$H$=$S$=$T$=2μm
$\varepsilon_r$=3.9



***The worst case crosstalk is when all aggressors switch in phase!!***

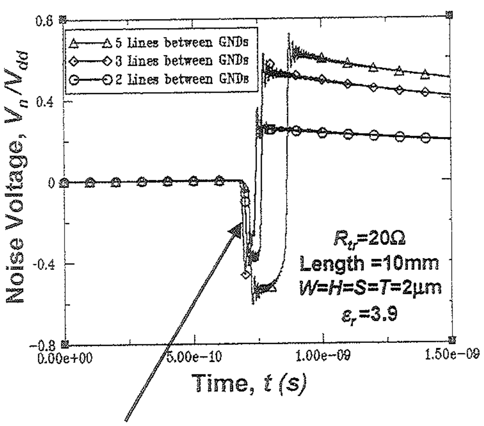
# Crosstalk With Ground Plane

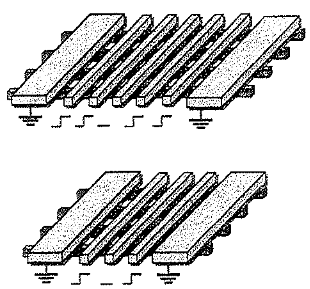


**WITH GROUND PLANE far aggressor have smaller IMPACT!**

# Crosstalk With Orthogonal Lines



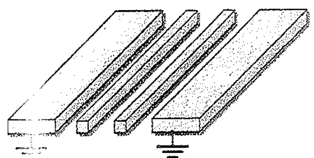
**modal dispersion with orthogonal lines**

# Examples of Using Compact Interconnect Models

- **Coplanar Optimization****
- **Maximum Bit-Rate Calculation**
- **Repeaters to Reduce Inductive Crosstalk**

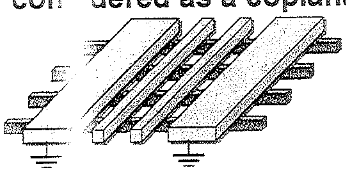
**Azad Naeemi, Jeff Davis, and Jim Meindl, "Compact physical models for multilevel interconnect crosstalk in a Gigascale SoC", Proceedings ofr the SOC Conference, pp. 199-202, 2003.

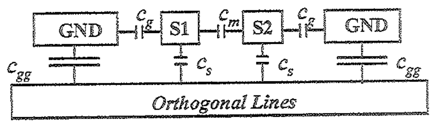
# Two Line Case



$$\begin{bmatrix} l_s & l_m \\ l_m & l_s \end{bmatrix}\begin{bmatrix} c_g + c_m & -c_m \\ -c_m & c_g + c_m \end{bmatrix} = (\varepsilon_r / c_0^2)[I]$$

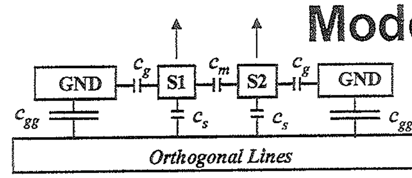
- Eac signal line and its nearby ground line can be con dered as a coplanar transmission line.



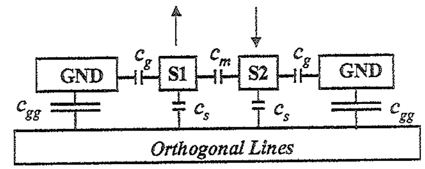



- Ind ctance values are not affected by orthogonal lines.
- Ort ogonal lines just increase the capacitance.

# Common and Differential Modes



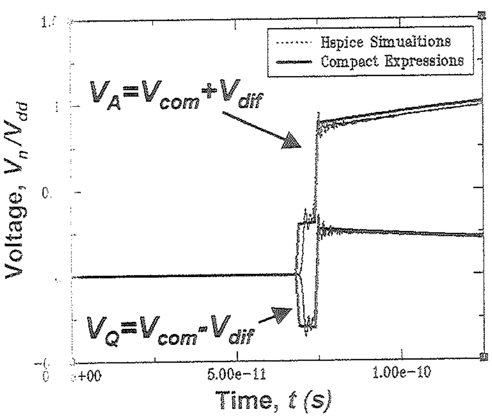

$$c_{com} = c_g + c_s$$
$$l_{com} = l_s + l_m$$
$$v_{com} = 1/\sqrt{l_{com} c_{com}}$$



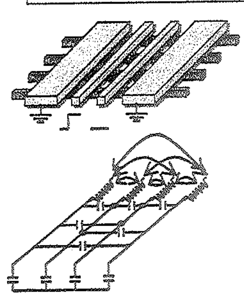

$$c_{dif} = c_g + 2c_m + c_s$$
$$l_{dif} = l_s - l_m$$
$$v_{dif} = 1/\sqrt{l_{dif} c_{dif}} > v_{com}$$

***Differential and common modes can be solved by the single RLC line model.***

# HSPICE Comparison

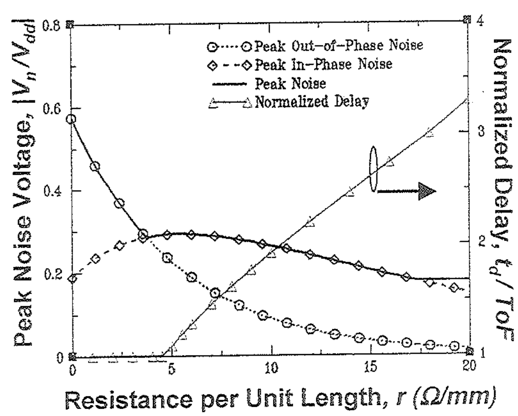



$R_{tr}$=20Ω
Length=10mm
$W$=$H$=$S$=$T$=2μm
$\varepsilon_r$=3.9



***An out-of-phase noise due to different speeds***

# Noise, Delay, and Resistance

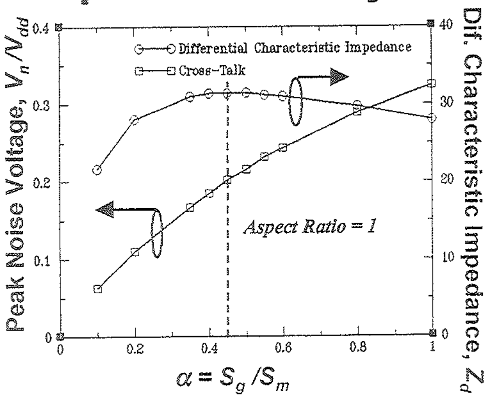



$R_{tr}$=20Ω
Length=10mm
$W$=$H$=$S$=$T$
$\varepsilon_r$=3.9

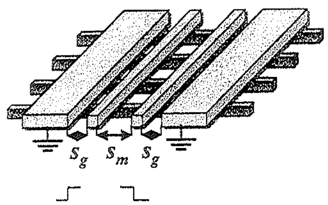
$$V_n^- = -\frac{Z_0^-}{Z_0^- + R_{tr}} e^{\frac{rl}{2Z_0^-}}$$

$$V_n^+ = \frac{Z_0^+}{Z_0^+ + R_{tr}} e^{\frac{rl}{2Z_0^+}} - \frac{Z_0^-}{Z_0^- + R_{tr}} e^{\frac{rl}{2Z_0^-}}$$

## Optimize Delay and Noise

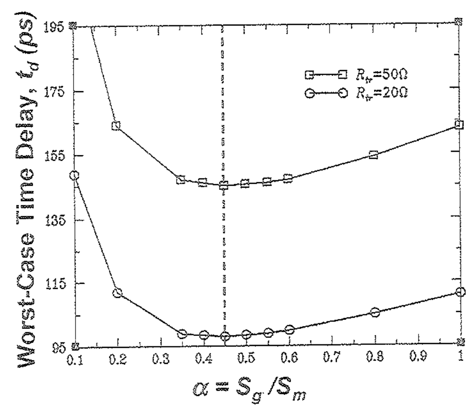



$W=H=T$
$S_g+S_m+S_g=Const.$
$\varepsilon_r=3.9$

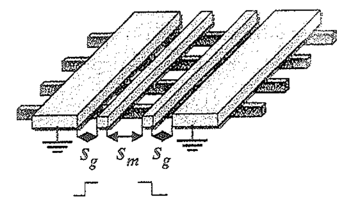



- Characteristic impedance is maximum at $S_g/S_m$=0.45.
- 38% crosstalk reduction by changing $S_g/S_m$ from 1 to 0.45.

## Optimize Noise & Delay




Length =10mm
$W=H=T$=1.5μm
$\varepsilon_r$=3.9



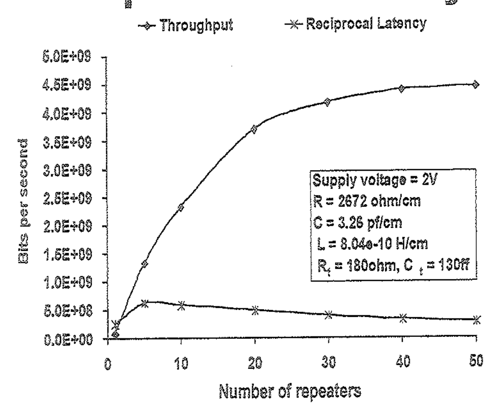

**10% reduction in delay by changing $S_g/S_m$ from 1 to 0.45.**

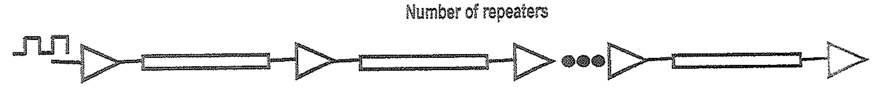
## Examples of Using Compact Interconnect Models

- ~~Coplanar Optimization~~
- **Maximum Bit-Rate Calculation****
- **Repeaters to Reduce Inductive Crosstalk**

**V. Deodhar and J. Davis, "Voltage Scaling and Repeater Insertion for High-Throughput Low-Power Interconnect Networks", Proceedings of the 2003 ISCAS, Bangkok, Thailand, pp. 349-353, June 2003.

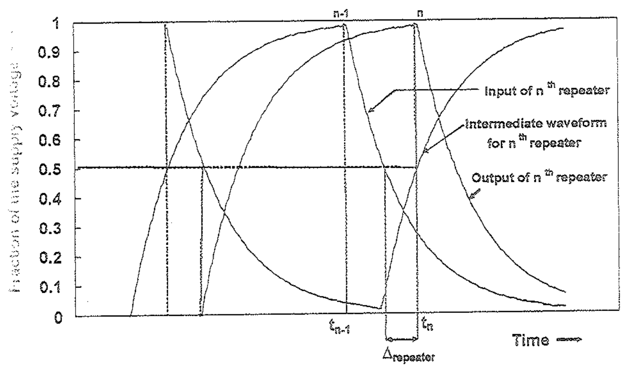
## Maximum Bit-Rate vs. Reciprocal Latency



## Maximum Bit-Rate Criteria

**When can I switch the input and still obtain a 90% swing on last inverter?**




## Compact Bit-Rate Expressions

$$\nu = 1 - \sum_{k=1}^{\infty} k_1 e^{-t/\sigma_k} \approx 1 - k_1 e^{-t/\sigma_1}$$

$$t_\nu = \sigma_{RCseg} \ln\left(\frac{k_1}{1-\nu}\right)$$

$$\Delta_{repeater} = 0.693 R_t C_t$$

$$\sigma_{RCseg} = R_t C_t + R_t C_{seg} + C_t R_{seg} + 0.4 R_{seg} C_{seg}$$

$$k_1 = 1.01 \left[\frac{R_t C_{seg} + R_{seg} C_t + R_{seg} C_{seg}}{R_t C_{seg} + R_{seg} C_t + \frac{\pi}{4} R_{seg} C_{seg}}\right]$$

## Voltages on Repeaters

$$t_n = \sigma_{RCseg} \ln\left(\frac{k_1}{1-\nu_n}\right) + (n-1)\sigma_{RCseg} \ln\left(\frac{k_1}{1-0.5}\right) + n\Delta_{repeater}$$

$$t_{n-1} = \sigma_{RCseg} \ln\left(\frac{k_1}{1-\nu_{n-1}}\right) + (n-2)\sigma_{RCseg} \ln\left(\frac{k_1}{1-0.5}\right) + (n-1)\Delta_{repeater}$$

$$\nu_{n-1} k_1 e^{-(t_n - t_{n-1} - \Delta_{repeater})/\sigma_{RCseg}} = 0.5$$

$$\nu_{n-1} k_1 e^{-\left(\ln 2k_1 + \ln\left(\frac{1-\nu_{n-1}}{1-\nu_n}\right)\right)} = 0.5$$

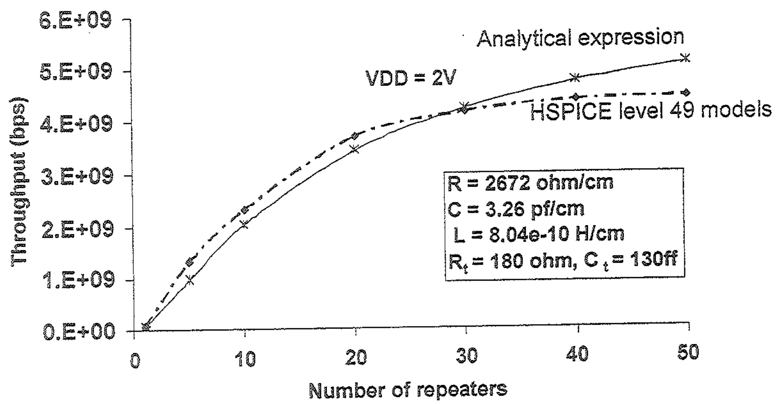
$$\nu_{n-1} = \frac{1}{2-\nu_n}$$

## Compact Maximum Bit-Rate Expression

$$PW_{\min} = \sigma_{RCseg} \ln\left(\frac{k_1}{1-\nu_1}\right) + \Delta_{repeater}$$

$$T_{\max} = \frac{1}{\sigma_{RCseg} \ln\left(\frac{k_1}{1-\nu_1}\right) + \Delta_{repeater}}$$

## HSPICE vs. Compact Model




## Throughput Saturation

$$\left[\ln\left(\frac{k_1}{1-v_1}\right)+0.693\right]R_0C_0=\ln\left(\frac{k_1}{1-v_1}\right)\left(0.4\frac{rl}{n}\frac{cl}{n}+\frac{R_0}{h}\frac{cl}{n}+C_0h\frac{rl}{n}\right)$$

$$T_{sat}\approx\frac{1}{2\left[\ln\left(\frac{1}{1-v_1}\right)+0.693\right]R_tC_t}$$

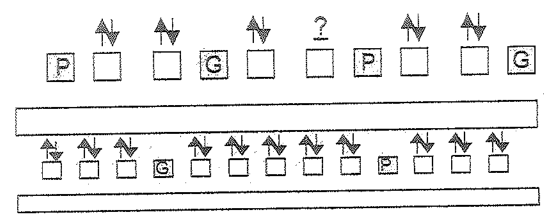
***Saturation throughput is independent of interconnect parameters !***

## Examples of Using Compact Interconnect Models

- ~~Coplanar Optimization~~
- ~~Maximum Bit-Rate Calculation~~
- ➡ **Repeaters to Reduce Inductive Crosstalk****

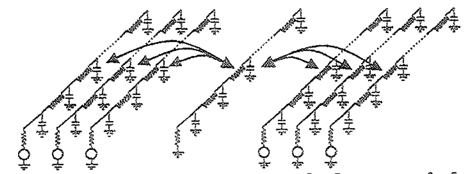
****A. Naeemi, J.A. Davis, and J.D. Meindl, "Compact Physical Models for the Worst Case Crosstalk Induced by Near and Far Aggressors in a SOC," IEEE International ASIC/SoC Conference, Sept 2003, pp. 199-202.**

## Apply Compact Model Here?




$$\frac{\partial}{\partial x^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\ V_n(x,t)\end{bmatrix}=\begin{bmatrix}l_{11}&l_{12}&\cdots&l_{1n}\\l_{21}&l_{22}&\cdots&l_{2n}\\ \vdots&&&\vdots\\ l_{n1}&l_{n2}&\cdots&l_{nn}\end{bmatrix}\begin{bmatrix}c_{11}&c_{12}&\cdots&c_{1n}\\c_{21}&c_{22}&\cdots&c_{2n}\\ \vdots&&&\vdots\\ c_{n1}&c_{n2}&\cdots&c_{nn}\end{bmatrix}\frac{\partial}{\partial t^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\ V_n(x,t)\end{bmatrix}+\begin{bmatrix}r_1&r_1&\cdots&r_1\\r_2&r_2&\cdots&r_2\\ \vdots&&&\vdots\\ r_n&r_n&\cdots&r_n\end{bmatrix}\begin{bmatrix}c_{11}&c_{12}&\cdots&c_{1n}\\c_{21}&c_{22}&\cdots&c_{2n}\\ \vdots&&&\vdots\\ c_{n1}&c_{n2}&\cdots&c_{nn}\end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\ V_n(x,t)\end{bmatrix}$$

## The simplification ..



$$\frac{\partial}{\partial x^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}=\begin{bmatrix}l_{11}&l_{12}&\cdots&l_{1n}\\l_{21}&l_{22}&\cdots&l_{2n}\\ \vdots&&&\vdots\\l_{n1}&l_{n2}&\cdots&l_{nn}\end{bmatrix}\begin{bmatrix}c_{11}&c_{12}&\cdots&c_{1n}\\c_{21}&c_{22}&\cdots&c_{2n}\\ \vdots&&&\vdots\\c_{n1}&c_{n2}&\cdots&c_{nn}\end{bmatrix}\frac{\partial}{\partial t^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}+r\begin{bmatrix}c_{11}&c_{12}&\cdots&c_{1n}\\c_{21}&c_{22}&\cdots&c_{2n}\\ \vdots&&&\vdots\\c_{n1}&c_{n2}&\cdots&c_{nn}\end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}$$

- Mutual capacitances between the far lines and the victim line are negligible.
- For the worst-case scenario, far aggressors switch in-phase; mutual capacitances between far lines can be ignored, too.

## The simplification

$$\frac{\partial}{\partial x^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}=\begin{bmatrix}l_{11}&l_{12}&\cdots&l_{1n}\\l_{21}&l_{22}&\cdots&l_{2n}\\ \vdots&&&\vdots\\l_{n1}&l_{n2}&\cdots&l_{nn}\end{bmatrix}\begin{bmatrix}c_{11}&c_{12}&\cdots&c_{1n}\\c_{21}&c_{22}&\cdots&c_{2n}\\ \vdots&&&\vdots\\c_{n1}&c_{n2}&\cdots&c_{nn}\end{bmatrix}\frac{\partial}{\partial t^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}+r\begin{bmatrix}c_{11}&c_{12}&\cdots&c_{1n}\\c_{21}&c_{22}&\cdots&c_{2n}\\ \vdots&&&\vdots\\c_{n1}&c_{n2}&\cdots&c_{nn}\end{bmatrix}\frac{\partial}{\partial t}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}$$

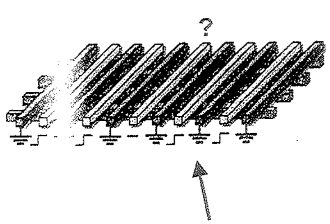
$$\Downarrow$$

$$\frac{\partial}{\partial x^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}=\begin{bmatrix}l_{11}&l_{12}&\cdots&l_{1n}\\l_{21}&l_{22}&\cdots&l_{2n}\\ \vdots&&&\vdots\\l_{n1}&l_{n2}&\cdots&l_{nn}\end{bmatrix}c\frac{\partial}{\partial t^2}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}+rc\frac{\partial}{\partial t}\begin{bmatrix}V_1(x,t)\\V_2(x,t)\\ \vdots\\V_n(x,t)\end{bmatrix}$$

## Diagonalized Capacitance Matrix Approximation

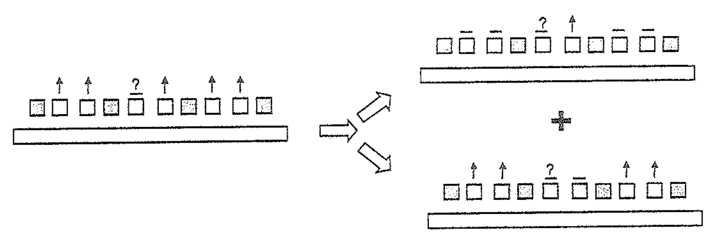
$$\frac{\partial^2}{\partial x^2}\tilde{V}=rc\frac{\partial}{\partial t}\tilde{V}+\left[L\right]c\frac{\partial^2}{\partial t^2}\tilde{V}$$

***Now eigenvectors of L matrix decouples lines***



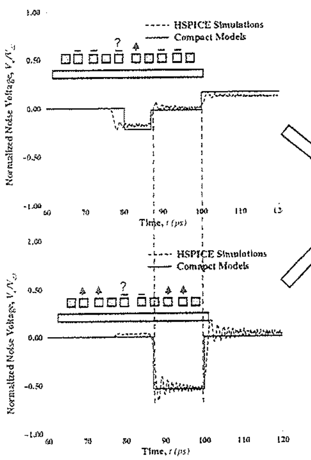

**All identical and all aggressor switching simultaneously!**

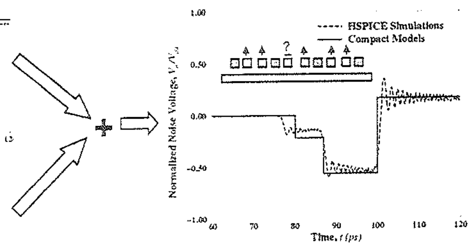
## Superposition




Using superposition theorem, total crosstalk caused by near and far aggressors can be calculated.

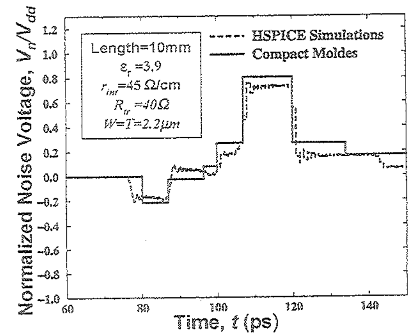
# Superposition

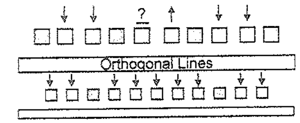



The worst case is when the far aggressors and the near aggressor switch in opposite directions with a phase shift.

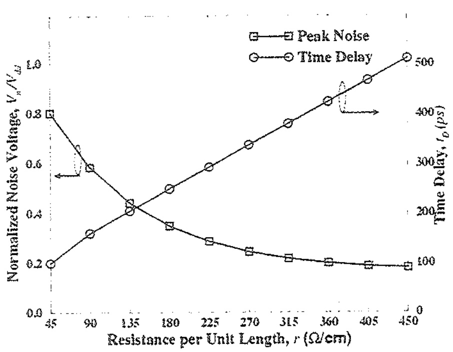
# Worst-Case Crosstalk

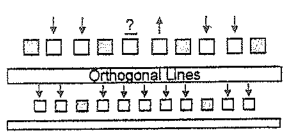



$$V_{peak} = V_{dd}\left[\frac{Z_{com}}{R_{tr}+Z_{com}}e^{-\frac{rl}{2Z_{com}}} - \frac{Z_{dif}}{R_{tr}+Z_{dif}}e^{-\frac{rl}{2Z_{dif}}} + \left(\frac{\sum_{Intra-level\ Far\ Pairs} l_{pi}}{\sqrt{\sum_{Intra-level\ Far\ Pairs} l_{pi}^2}}\right)\frac{Z_{com}}{R_{tr}+Z_{com}}e^{-\frac{rl}{2Z_{op}}} + \frac{2Z_{0f}}{Z_{0f}+R_{tr}}V_{dd}\frac{\sum_{Inter-level\ Far\ Lines} l_{vi}c_{f\text{-}eq}}{(l_{f\text{-}eq}c_{f\text{-}eq}-l_pc_p)}e^{-\frac{rl}{2Z_{com}}}\right]$$

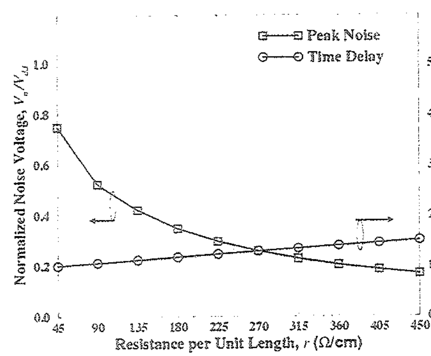
# Latency and Crosstalk Tradeoff

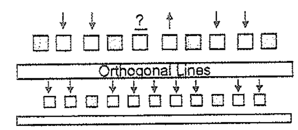



***There is a trade-off between crosstalk and interconnect latency.***

# Repeater Insertion




***Repeater insertion is an effective way to lower crosstalk and increase wiring density with a small latency penalty.***

# What have we learned?

**Compact Models For R,L, &C**

- **Basic empirical formulas for L&C**

**Compact Distributed RC Models**

- **Single conductor problem can be used to model *regular* n-wire configurations**
- **Finite rise-time model increases accuracy of non-linear driver operation**

# What have we learned?

**Compact Distributed RLC Models**

- **Modified Bessel series solutions**
- **First reflection approximation for delay and crosstalk approximations**
- **Orthogonal ground lines slows time-of-flight and causes modal dispersion**
- **Orthogonal conductors cause far aggressors to have a larger impact on crosstalk**

# What have we learned?

**Compact Model Analysis Examples**

- **Coplanar global wire dimensions are optimized to reduce the delay and crosstalk by 10% and 38%**
- **Maximum bit rate can be much greater than reciprocal latency**
- **Use of single line solutions in complex configurations**
- **Repeater insertion is an effective way to lower inductive crosstalk**

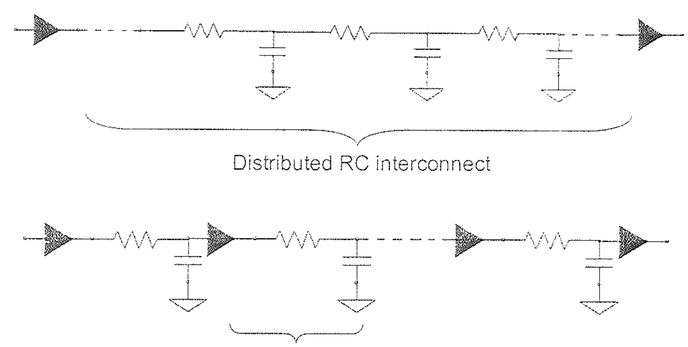
# Wave Pipelining using Repeaters for VLSI Global Interconnects

Vinita V. Deodhar

ECE 8823
Advanced VLSI Interconnect Design
Instructor: Dr. Jeffrey A. Davis

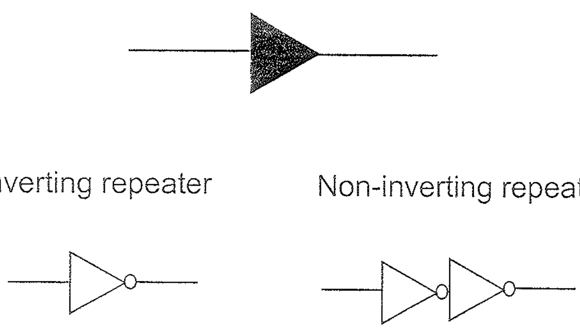
March 15, 2005

# Repeater Insertion



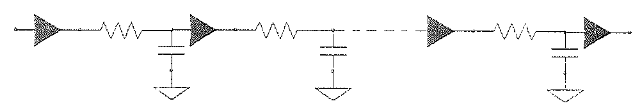



# Repeater Circuits






# Bakoglu Repeater Insertion



*Optimal number of repeaters

$$n_{opt} = \sqrt{\frac{RC}{2.3R_0C_0}}$$

*Optimal size of repeaters

$$h_{opt} = \sqrt{\frac{R_0C}{C_0R}}$$

*H. Bakoglu and J. Meindl, *IEEE Trans. Electron Devices*, vol. ED-32,no.5, pp. 903-909, May 1985.

# Propagation Delay

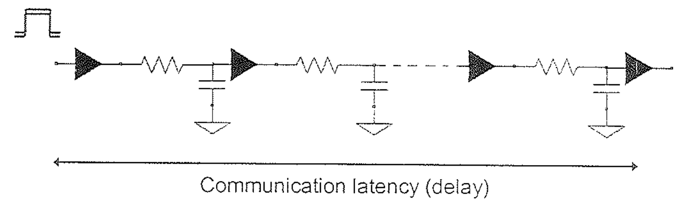
- For a single driver interconnect,
  T $\alpha$ (Interconnect length)$^2$

- For interconnect with repeaters,
  * $T = 7.6\sqrt{R_0 C_0 R C}$
  T $\alpha$ Interconnect length

*H. Bakoglu and J. Meindl, *IEEE Trans. Electron Devices*, vol. ED-32,no.5, pp. 903-909, May 1985.



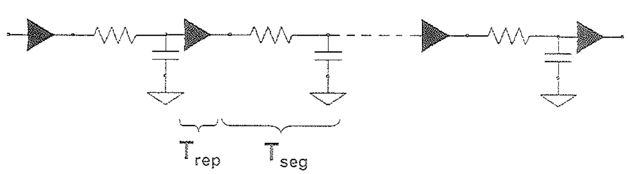
# Data Propagation




$$\text{Bit-rate} = \frac{1}{\text{Communication latency}}$$



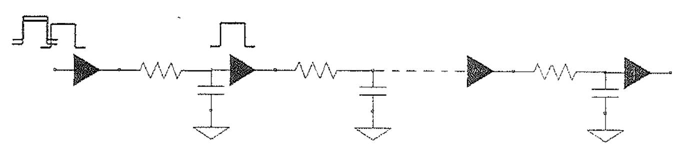
# Concept of Wave Pipelining




Segment Delay ~ $T_{rep} + T_{seg}$



# Data Propagation using Wave Pipelining

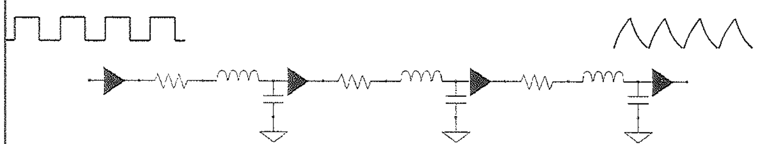


$$\text{Throughput Bit-rate} \approx \frac{1}{\text{Segment delay}}$$

## Calculating Throughput of Wave-Pipelined Interconnect in HSPICE

Requirements:

- ➢ Every data bit is captured at the output.
- ➢ The last segment reaches 90% $V_{dd}$.





## HSPICE Analysis

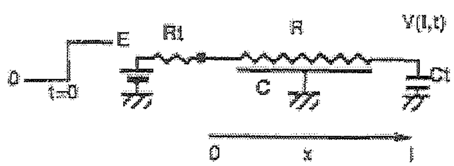
Accurate but not efficient

- ➢ Tedious
- ➢ Time-consuming
- ➢ Trial and error

**Need for a simple analytical throughput model**



## Sakurai Interconnect Model




*A* model of an interconnection driven by a resistive voltage source and loaded with a capacitor

T. Sakurai, *IEEE Trans. Electron Devices*, vol. 40, no. 1, pp. 118-124, Jan. 1993.



## Voltage at the Far End

$$v = \frac{V(l,t)}{E} = 1 + \sum_{k=1}^{\infty} K_k e^{-\sigma_k t/RC} \approx 1 + K_1 e^{-\sigma_1 t/RC}$$

$$K_1 = -1.01\,\frac{R_T + C_T + 1}{R_T + C_T + \pi/4} \qquad R_T = \frac{R_t}{R}$$

$$\sigma_1 = \frac{1.04}{R_T C_T + R_T + C_T + (2/\pi)^2} \qquad C_T = \frac{C_t}{C}$$

## Sakurai Delay Model for Interconnect Segment

$$t_\upsilon = \sigma_{RCseg} \ln\left(\frac{K_1}{1-\nu}\right)$$

$$\sigma_{RCseg} = R_t C_t + R_t C_{seg} + C_t R_{seg} + 0.4 R_{seg} C_{seg}$$

$$K_1 = 1.01\left[\frac{R_t C_{seg} + R_{seg} C_t + R_{seg} C_{seg}}{R_t C_{seg} + R_{seg} C_t + \frac{\pi}{4} R_{seg} C_{seg}}\right]$$



## Analytical Throughput Expression

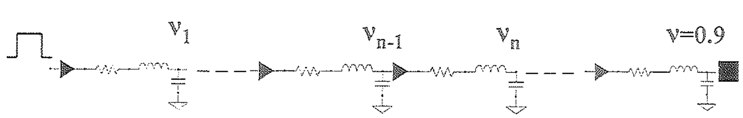
Assumptions:

- The repeater consists of two coupled inverters.
- The inverter turns on at 50% $V_{dd}$.
- The last segment reaches 90% $V_{dd}$.



## Problem Definition

$v_n$ = Normalized maximum voltage swing on n[th] segment




For $\nu = 0.9$ on the last segment, $\nu_1 = ??$



## Design Equations

$$t_n = \sigma_{RCseg} \ln\left(\frac{K_1}{1-\nu_n}\right) + (n-1)\sigma_{RCseg} \ln\left(\frac{K_1}{1-0.5}\right) + n\Delta_{repeater}$$

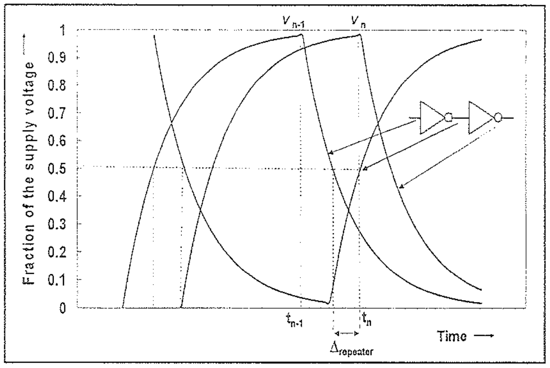
$$t_{n-1} = \sigma_{RCseg} \ln\left(\frac{K_1}{1-\nu_{n-1}}\right) + (n-2)\sigma_{RCseg} \ln\left(\frac{K_1}{1-0.5}\right) + (n-1)\Delta_{repeater}$$

where

$$\Delta_{repeater} = 0.693 R_t C_t$$

*T. Sakurai, *IEEE Trans. Electron Devices*, vol. 40, no. 1, Jan. 1993.

## Repeater Waveforms




$$v_{n-1}K_1 e^{-(t_n - t_{n-1} - \Delta_{repeater})/\sigma_{RCseg}} = 0.5$$



## Intermediate Equations

$$v_{n-1}K_1 e^{-\left(\ln 2K_1 + \ln\left(\frac{1-v_{n-1}}{1-v_n}\right)\right)} = 0.5$$

$$v_{n-1}K_1 e^{-\left(\ln(2K_1)\left(\frac{1-v_{n-1}}{1-v_n}\right)\right)} = 0.5$$

$$v_{n-1}K_1 \frac{1-v_n}{(2K_1)(1-v_{n-1})} = 0.5$$

$$\frac{(v_{n-1})(1-v_n)}{(1-v_{n-1})} = 1$$



## Recursive Relation

$$v_{n-1} = \frac{1}{2 - v_n}$$

By setting $v_n$ for last segment to 0.9, $v_1$ for the first segment can be recursively calculated.

These values are independent of all technology attributes of the interconnect circuit and are fundamental to multi-bit transmission on VLSI interconnects with repeaters.



## Table of Recursive Values

| n # | $v_n$ | n # | $v_n$ | n # | $v_n$ | n # | $v_n$ | n # | $v_n$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.983 | 11 | 0.980 | 21 | 0.974 | 31 | 0.966 | 41 | 0.947 |
| 2 | 0.983 | 12 | 0.979 | 22 | 0.974 | 32 | 0.964 | 42 | 0.944 |
| 3 | 0.982 | 13 | 0.979 | 23 | 0.973 | 33 | 0.963 | 43 | 0.941 |
| 4 | 0.982 | 14 | 0.978 | 24 | 0.972 | 34 | 0.962 | 44 | 0.937 |
| 5 | 0.982 | 15 | 0.978 | 25 | 0.971 | 35 | 0.960 | 45 | 0.933 |
| 6 | 0.981 | 16 | 0.977 | 26 | 0.971 | 36 | 0.958 | 46 | 0.929 |
| 7 | 0.981 | 17 | 0.977 | 27 | 0.970 | 37 | 0.957 | 47 | 0.923 |
| 8 | 0.981 | 18 | 0.976 | 28 | 0.969 | 38 | 0.955 | 48 | 0.916 |
| 9 | 0.980 | 19 | 0.976 | 29 | 0.968 | 39 | 0.952 | 49 | 0.909 |
| 10 | 0.980 | 20 | 0.975 | 30 | 0.967 | 40 | 0.950 | 50 | 0.900 |

n # = repeater id in a chain of 50 repeaters

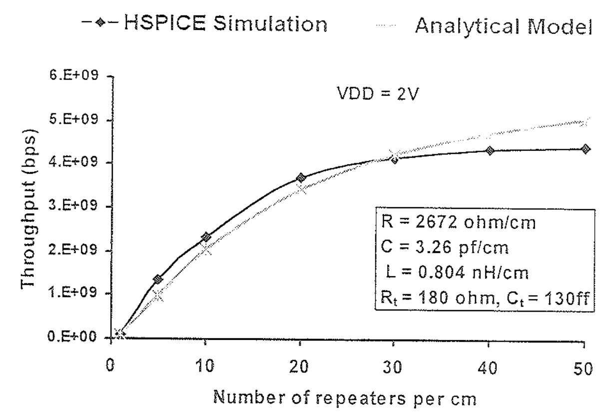
## Expressions for Pulsewidth and Throughput

$$PW_{min} = \sigma_{RCseg} \ln\left(\frac{K_1}{1-\nu_1}\right) + \Delta_{repeater}$$

$$T_{max} = \frac{1}{\underbrace{\sigma_{RCseg} \ln\left(\frac{K_1}{1-\nu_1}\right)}_{\text{Delay of a repeated wire segment}} + \underbrace{\Delta_{repeater}}_{\text{Internal delay of a repeater}}}$$

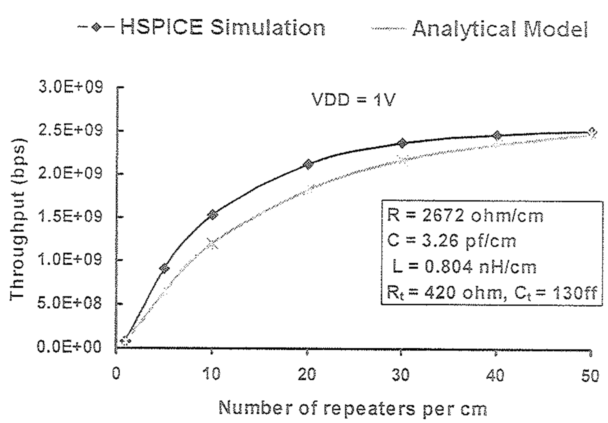


## 180 nm HSPICE Verification: 2V



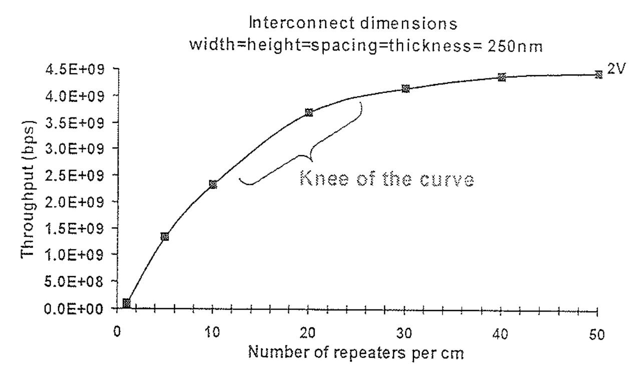



## 180 nm HSPICE Verification: 1V

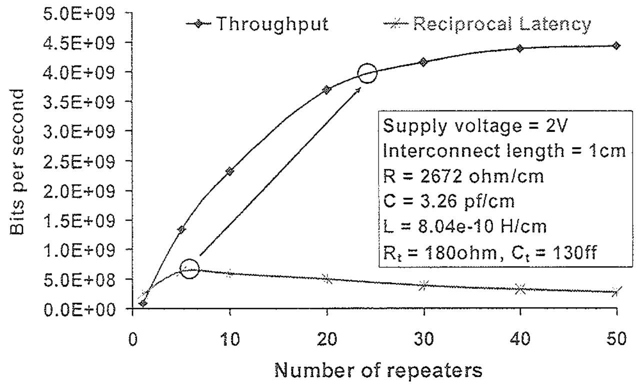





## Operational Region



## Throughput-Centric vs. Latency-Centric Approach






## Physical Understanding using Analytical Throughput Expression

- Impact of circuit parameters on throughput
  - Interconnect length
  - Repeater scaling factor
- Effect of scaling on throughput
  - Constant field scaling
  - Global wire scaling



## Throughput Model

$$T_{\max} = \frac{1}{\sigma_{RCseg} \ln\left(\frac{K_1}{1-\nu_1}\right) + \Delta_{repeater}}$$

⇓

$$T_{\max} = \frac{1}{\left(\frac{R_0}{h}\frac{cl}{n} + C_0 h \frac{rl}{n} + R_0 C_0 + 0.4 \frac{rl}{n}\frac{cl}{n}\right) \ln\left(\frac{K_1}{1-\nu_1}\right) + 0.693 R_0 C_0}$$



## Wire length dependence of throughput

$$T_{\max} = \frac{1}{\left(R_t C_t + R_t \frac{c}{\left(\frac{n}{l}\right)} + C_t \frac{r}{\left(\frac{n}{l}\right)} + 0.4 \frac{rc}{\left(\frac{n}{l}\right)^2}\right) \ln\left(\frac{K_1}{1-\nu_1}\right) + 0.693 R_t C_t}$$

CONSTANT

**Throughput does not vary with number of repeaters or interconnect length alone but it is a function of their ratio**