Where the story begins……

Under-determined System of Linear Equations (USLE)

As=x , Unknowns > Equations

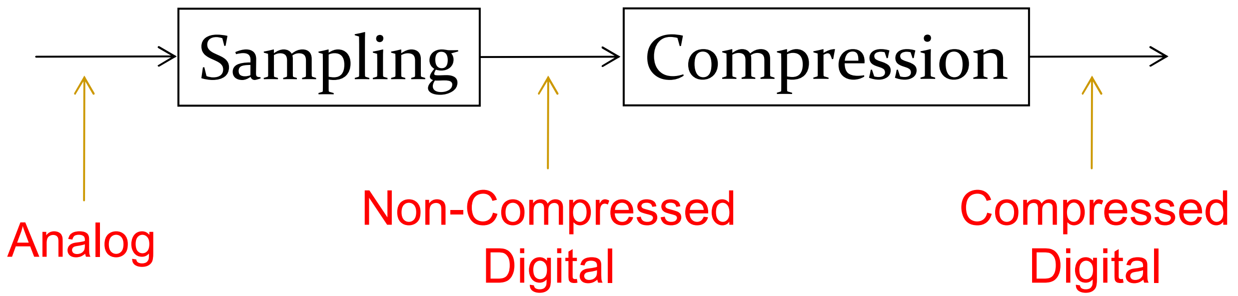
- Generally non-unique solution (infinite number of solutions)
- However, Sparse solution is unique, under some mild conditions
- $\Rightarrow$ Many many applications!

Application 1:

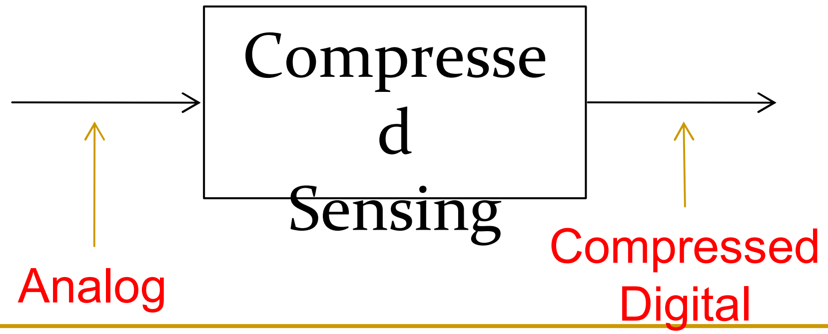
Compressed Sensing

Traditional Sampling vs. Compressed Sensing

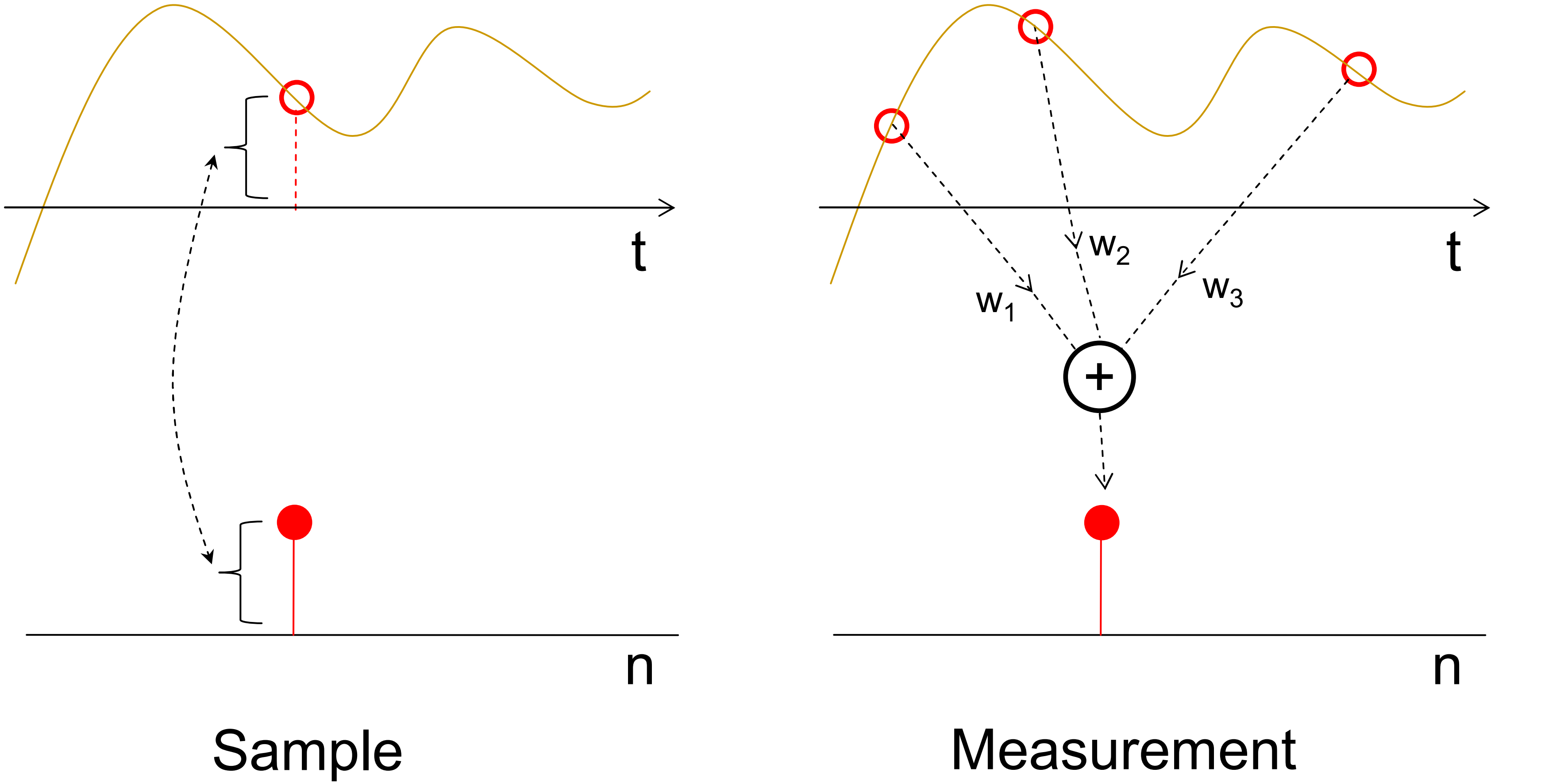
- Traditional Signal Acquisition:



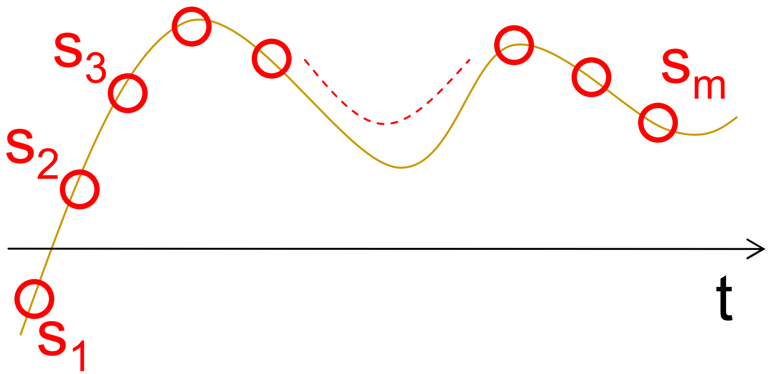
- Compressed Sensing (CS)



CS: Sample → Measurement



CS: A (smaller) set of random measurements



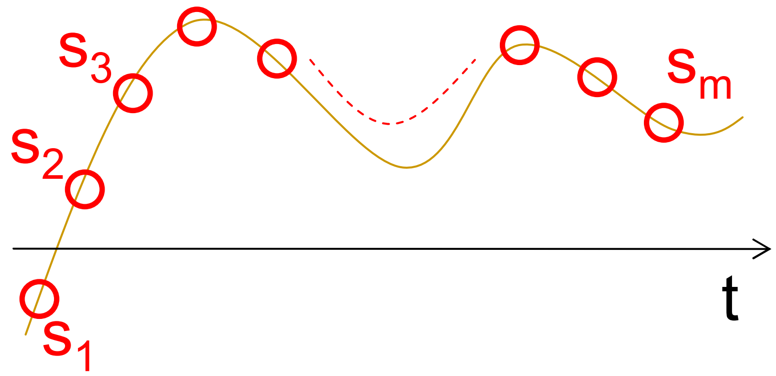
- 1^{st} measurement $\rightarrow$ $x_1 = \varphi_{11} s_1 + \varphi_{12} s_2 + \ldots + \varphi_{1n} s_m$
- 2^{nd} measurement $\rightarrow$ $x_2 = \varphi_{21} s_1 + \varphi_{22} s_2 + \ldots + \varphi_{2n} s_m$

:

- n^{th} measurement $\rightarrow$ $x_n = \varphi_{n1} s_1 + \varphi_{n2} s_2 + \ldots + \varphi_{nm} s_m$

$n < m \quad \Rightarrow$ USLE

CS: A (smaller) set of random measurements



$$\Phi \mathbf{s} = \mathbf{x}$$

Measurement matrix

?

Measurement vector

CS: A (smaller) set of random measurements

$$\Phi \mathbf{s} = \mathbf{x}$$

$\downarrow$

?

- $\Psi_{m \times m} \rightarrow$ sparsifying transform:

$$\mathbf{s} = \Psi\, \theta,$$

where θ is sparse

$$\Downarrow$$

$$(\Phi\, \Psi)\, \theta = \mathbf{x}$$

(USLE with sparsity)

Application 2 (of USLE):

Error Correcting Codes

(Real-field coding)

Coding Terminology

- **u**=(u_1,…, u_k) → the message to be sent (k symbols)
- **G** → Code Generator matrix ($n \times k$, $n>k$)
- **v** =(v_1,…, v_n) → Codeword:

  v=G.u

  (adding n-k "parity" symbols)

- **H** → Parity check matrix ($(n-k) \times n$):

  HG=0

- **v** is a codeword if and only if: **H.v=0**

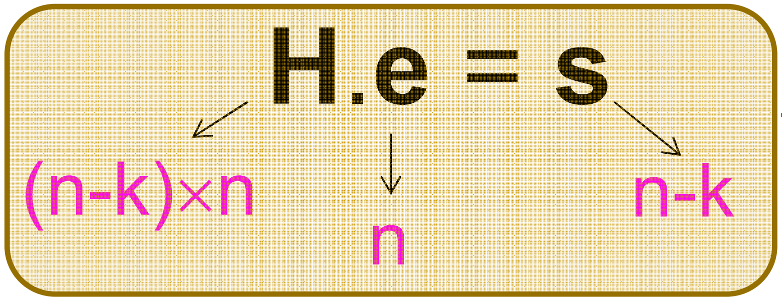
Error Correction

- **v** sent, **r = v + e** received

  (**e** is the error → assumed sparse)

- Syndrome of **r** → **s = H.r**

  ⇒ **s = H.(v+e) = H.e**



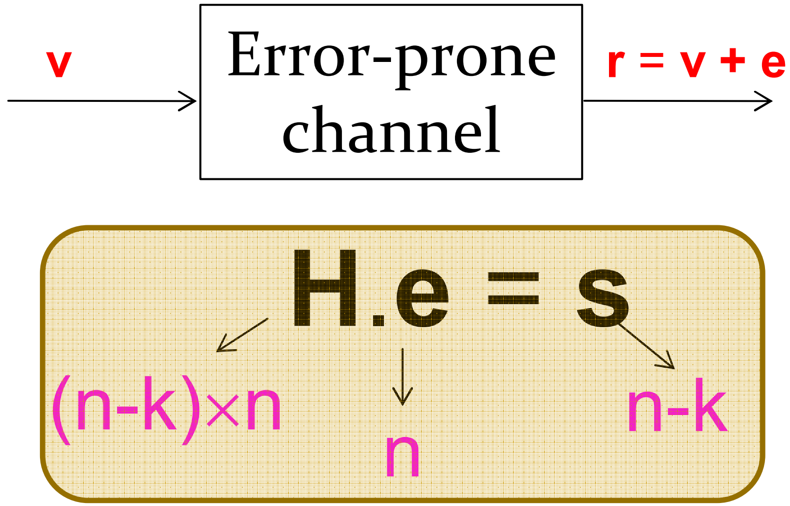
→ **USLE**

Error Correction

The receiver:

- Receives **r** = **v**+**e**
- Computes **s** = **H**.**r**
- Finds sparse solution of USLE **H.e=s**
- Error Correction

Sparsity of **e**?



- Galois fields (binary) codes ⇔ small probability of error

- Real-field codes ⇔ Impulsive noise, Laplace noise