Sharif University of Technology
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Center of Excellence in Energy Conversion

Advanced Thermodynamics

Lecture 11

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- Heat engine performance index:

$$h_{th} = \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_H}{Q_L}$$

$$\left(S_{gen} = \frac{Q_L}{T_L} - \frac{Q_H}{T_H} \right) \rightarrow h_{th} = 1 - \frac{T_L}{T_H} - \frac{T_L S_{gen}}{Q_H}$$

- Observations:

$$S_{gen} = 0 \qquad \text{(reversible, internally)}$$

$$S_{gen} \neq 0 \qquad h < h_c \quad \text{(reversible heat engine)}$$

- Rewrite 2nd law:

$$Q_L = \frac{T_L}{T_H} Q_H + T_L . S_{gen}$$

- For fixed $Q_H,\ T_L,\ and\ T_H,$, entropy production causes an increase in Q_L , from the 1st law, a decrease in *W* then follows.

- For fixed $Q_H,\ Q_L,\ and\ T_H,$, an increase in entropy production must be accomplished by a decrease in the low temperature, T_L.

- Influence of temperature on performance

$$h_{th} = 1 - \frac{T_L}{T_H} \rightarrow \frac{\partial h_c}{\partial T_H} = \frac{T_L}{T_H^{\ 2}}$$

$$\frac{\partial h_c}{\partial T_L} = -\frac{1}{T_H}$$

- Since $\frac{\partial h}{\partial T_H} > 0 \Rightarrow h \uparrow \text{ when } T_H \uparrow$

- since $\frac{\partial h}{\partial T_L} < 0 \Rightarrow h \downarrow \text{ when } T_L \uparrow$

Heat Engine …

- Key to performance for any heat engine
  - Minimize entropy production (generation), S_{gen}
  - Raising T_H
  - Lowering T_L
- Optimization techniques
  - Objective function: including all system parameters
  - Constraints, for example $S_{gen} \geq 0$ and T_H , $T_L \geq 0$
  - 2nd law optimization or entropy generation minimization

- Heat engine with variable temperature heat transfer:

$$\eta_{th} = 1 - \frac{\dot{Q}_L}{\dot{Q}_H} = 1 - \frac{1}{\dot{Q}_H}\left[\dot{m}_L C_L \left(T_{L2} - T_{L1}\right)\right] =$$

$$1 - \frac{1}{\dot{Q}_H}\left\{\dot{m}_L C_L \left[T_{L1} \exp\left(\frac{1}{\dot{m}_L C_L}\left(\dot{S}_{gen} - \dot{m}_H C_H \ln\left(T_{H2}/T_{H1}\right)\right)\right) - T_{L1}\right]\right\}$$

$$\Rightarrow \eta_{th} = 1 + \frac{\dot{m}_L C_L T_{L1}}{\dot{Q}_H} - \frac{\dot{m}_L C_L T_{L1}}{\dot{Q}_H}\left[1 - \left(\frac{T_{H2}}{T_{H1}}\right)^{-\frac{\dot{m}_H C_H}{\dot{m}_L C_L}}\right]$$

$$\boxed{\eta_{th} = 1 + \frac{\dot{m}_L C_L T_{L1}}{\dot{Q}_H}\left[1 - \left(\frac{T_{H2}}{T_{H1}}\right)^{-\frac{\dot{m}_H C_H}{\dot{m}_L C_L}}\right]}$$