School of Mechanical Engineering

Sharif University of Technology

Convection Heat Transfer

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Lecture #3

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Fundamentals and Basic Concepts

Convection Heat Transfer

Summary of second session:

∨ **_Thermodynamics laws_**

∨ **_An introduction to scale analysis_**

This session: Scale analysis-
Introduction to boundary layer flow

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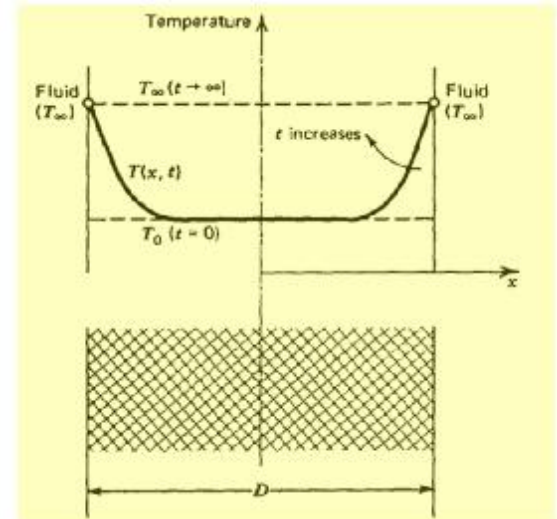
We are interested in estimating the time needed by the thermal front to penetrate the plate, that is, the time until the center plane of the plate "feels" the heating imposed on the outer surfaces.

we focus on a half-plate of thickness D/2 and the energy equation for pure conduction in one direction:

$$\rho c_P \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}$$

(51)

$$\rho c_P \frac{\partial T}{\partial t} \sim \rho c_P \frac{\Delta T}{t}$$

≡

$$k \frac{\partial^2 T}{\partial x^2} = k \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) \sim \frac{k}{D/2}\frac{\Delta T}{D/2} = \frac{k \, \Delta T}{(D/2)^2}$$

(52,53)

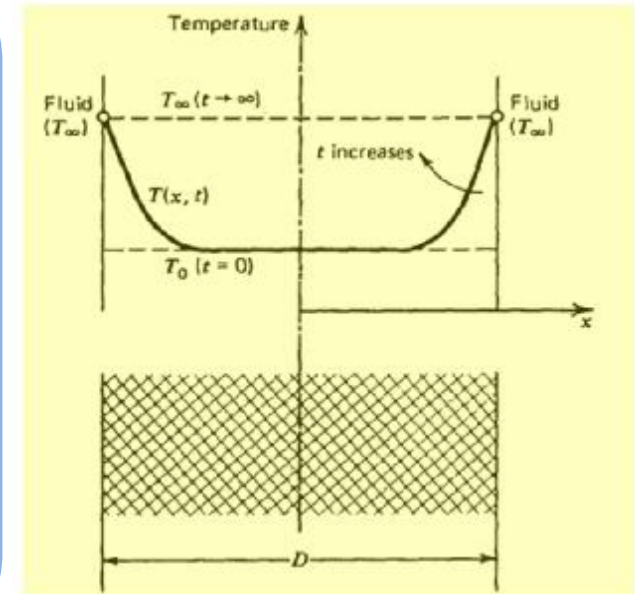We conclude that:

$$t \sim \frac{(D/2)^2}{\alpha}$$

(54)

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RULES OF SCALE ANALYSIS

Rule #1. Always define the spatial extent of the region in which you perform the scale analysis.

❖ In most problems, such as boundary layer flow, the size of the region of interest is unknown.

❖ The scale analysis begins by selecting the region and by labeling the unknown thickness of this region d. Any scale analysis of a flow or a flow region that is not uniquely defined is pure nonsense.
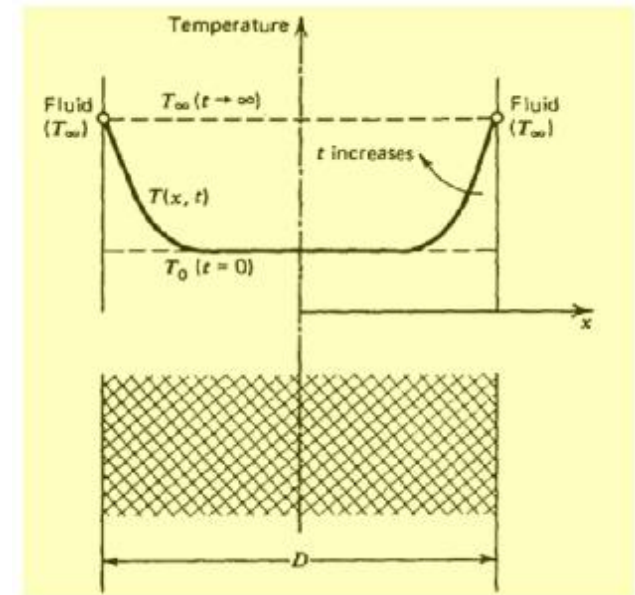

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Rule #2. Specify the dominant terms.

One equation constitutes an equivalence between the scales of two dominant terms appearing in the equation.

❖In the transient conduction example, the left-hand side of Eq. (51) could only be of the same order of magnitude as the right-hand side. The two terms appearing in Eq. (51) are the dominant terms (considering that the discussion referred to pure conduction).

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Rule #3. Order of magnitude of summation(I).

If in the sum of two terms, $\qquad c = a + b$ (55)

the order of magnitude of one term is greater than the order of magnitude

of the other term, $\qquad O(a) > O(b)$ (56)

then the order of magnitude of the sum is dictated by the dominant term:

$$O(c) = O(a) \qquad (57)$$

The same conclusion holds if instead of Eq. (55), we have the difference

$$c = a-b$$

RULES OF SCALE ANALYSIS

Rule #4. Order of magnitude of summation(II).

If in the sum of two terms, $c = a + b$ (55)

the two terms are of the same order of magnitude,

$$O(a) = O(b) \qquad (58)$$

then the sum is also of the same order of magnitude:

$$O(a) \sim O(b) \sim O(c) \qquad (59)$$

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Rule #5. Order of magnitude of products(II).

In any product $\qquad$ $p = ab$ $\qquad$ (60)

The order of magnitude of the product is equal to the product of the orders

$$O(p) = O(a)O(b)$$

(61)

Similarly we have:

$$r = a/b$$

(62)

$$O(p) = O(a)/O(b)$$

(63)

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HEATLINES FOR VISUALIZING CONVECTION

In convection problems it is important to visualize the flow field and isotherm curves.

In the two-dimensional Cartesian configuration, it has been common practice to define a streamfunction $\psi(x,y)$ as:

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}$$

(64)

Such that the mass continuity equation for incompressible flow is satisfied identically.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(65)

For steady-state two-dimensional convection through a constant-property homogeneous fluid, energy equation becomes:

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

(66)

HEATLINES FOR VISUALIZING CONVECTION

The heatfunction is defined as follows:

Net energy flow in the x direction:

$$\frac{\partial H}{\partial y} = \rho c_P u (T - T_{\text{ref}}) - k \frac{\partial T}{\partial x}$$

(68)

Net energy flow in the x direction:

$$-\frac{\partial H}{\partial x} = \rho c_P v (T - T_{\text{ref}}) - k \frac{\partial T}{\partial y}$$

(69)

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Chapter 2:

LAMINAR BOUNDARY LAYER FLOW

Convection Heat Transfer

LAMINAR BOUNDARY LAYER FLOW

As students and researchers, we can learn important lessons from the history of boundary layer theory:

1. No theory is perfect and forever, not even boundary layer theory.

2. It is legal and, indeed, desirable to question any accepted theory.

3. Any theory is better than no theory at all.

4. It is legal to propose a new theory or a new idea in place of any accepted theory.

5. Lack of immediate acceptance of a new theory does not mean that the

new theory is not better.

6. It is crucial to persevere to prove the worth of a new theory.

Convection Heat Transfer

FUNDAMENTAL PROBLEM IN CONVECTIVE HEAT TRANSFER
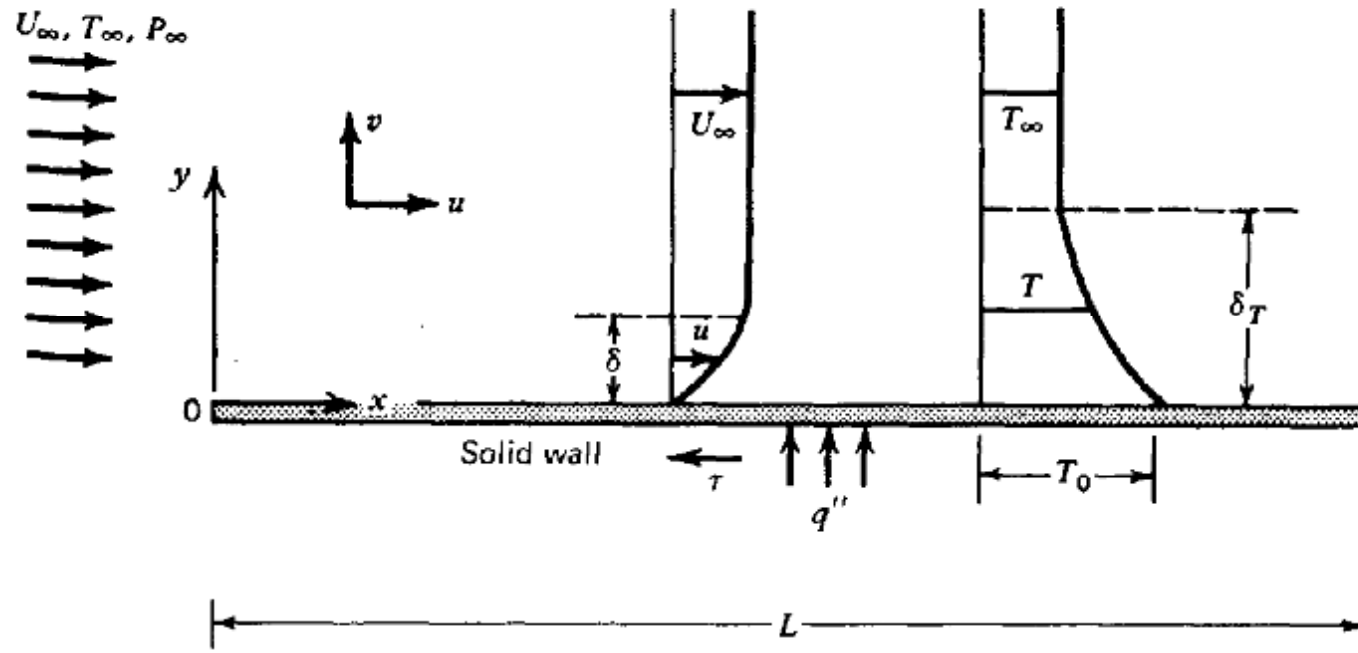
Velocity and temperature boundary layers near a plate parallel to a uniform flow

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FUNDAMENTAL PROBLEM IN CONVECTIVE HEAT TRANSFER

If this flat plate is the plate fin protruding
from a heat exchanger surface into the stream that bathes
it, we want to know:

1. The net force exerted by the stream on the plate
2. The resistance to the transfer of heat from the plate to
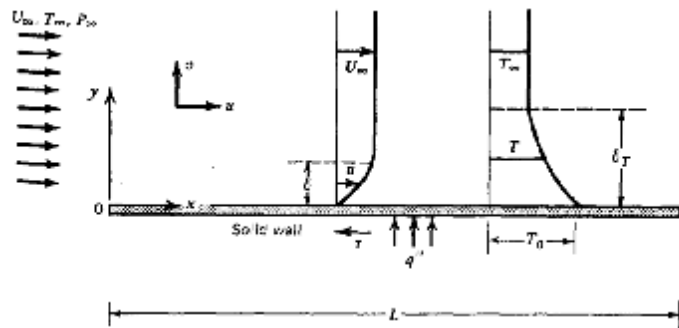the stream

FUNDAMENTAL PROBLEM IN CONVECTIVE HEAT TRANSFER

we are interested in calculating the total force and toal heat transfer as follows:

$$F = \int_0^L \tau W \, dx \qquad (1)$$

$$q = \int_0^L q'' W \, dx \qquad (2)$$

$$\tau = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} \qquad (3)$$

$$q'' = h(T_0 - T_\infty) \qquad (4)$$

Convection Heat Transfer

FUNDAMENTAL PROBLEM IN CONVECTIVE HEAT TRANSFER

The no-slip condition implies that since the $0 < y < \epsilon \longrightarrow 0$ Fluid layer is motionless, **the heat transfer** from the wall to the fluid is first governed by **pure conduction**. Therefore, in place of eq. (4), we can write the statement for pure conduction through the fluid layer immediately adjacent to the wall.
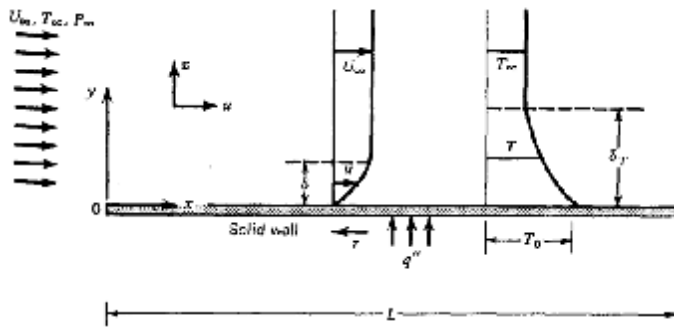
$$q'' = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

(5)

$$h = \frac{-k(\partial T/\partial y)_{y=0}}{T_0 - T_\infty}$$

(6)

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Convection Heat Transfer

FUNDAMENTAL PROBLEM IN CONVECTIVE HEAT TRANSFER

Modeling the flow as incompressible and of constant property (Chapter 1), the complete mathematical statement of this problem consists of the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \tag{8}$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \tag{9}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{10}$$

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FUNDAMENTAL PROBLEM IN CONVECTIVE HEAT TRANSFER

for four unknowns (u,v,P,T), subject to the following boundary conditions:

(i) No slip	$u = 0$	
(ii) Impermeability	$v = 0$	at the solid wall
(iii) Wall temperature	$T = T_0$	
(iv) Uniform flow	$u = U_\infty$	infinitely far from
(v) Uniform flow	$v = 0$	the solid, in both
(vi) Uniform temperature	$T = T_\infty$	the y and x directions

(11)

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Convection Heat Transfer

CONCEPT OF BOUNDARY LAYER

we have the freedom to think that the **velocity change** from u =0 to u =U∝ and the temperature change from T=T0 T=T∝ **occurs** in a space situated **relatively close to the solid wall**.

The free stream is characterized by:

$$u = U_\infty, \qquad v = 0, \qquad P = P_\infty, \qquad T = T_\infty \qquad \text{(12)}$$

Let **d** be the order of magnitude of the distance in which u changes from 0 at the wall to roughly U∝ in the free stream.

$$x \sim L, \qquad y \sim \delta, \qquad u \sim U_\infty \qquad \text{(13)}$$

Convection Heat Transfer

LAMINAR BOUNDARY LAYER FLOW

CONCEPT OF BOUNDARY LAYER

we learn that the inertia terms in eq. (14) are both of order U^2_∞ / L ; hence, neither can be neglected at the expense of the other. However, if the boundary layer region $d \times L$ is slender, such that:

$$\delta \ll L \qquad (16)$$

The last scale in eq. (14) is the scale most representative of the **friction** force. Neglecting the term $\partial^2 U / \partial x^2$ at the expense of the $\partial^2 U / \partial y^2$ term in the **x** momentum equation (8) yields:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \qquad (17)$$

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Next session:

✔ *Laminar Boundary Layer Flow(Velocity and Temperature fields)*