Two Phase Flows

(Section 3)

The Basic Model

By: Prof. M. H. Saidi

Center of Excellence in Energy Conversion

School of Mechanical Engineering

Sharif University of Technology

Homework

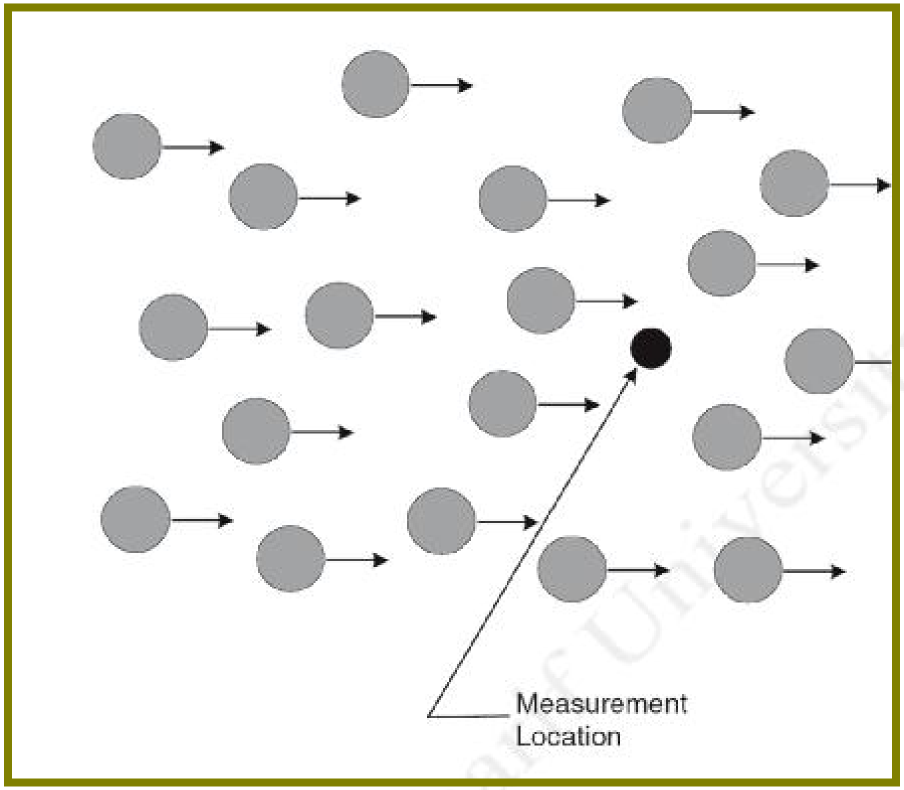
Homework set#1

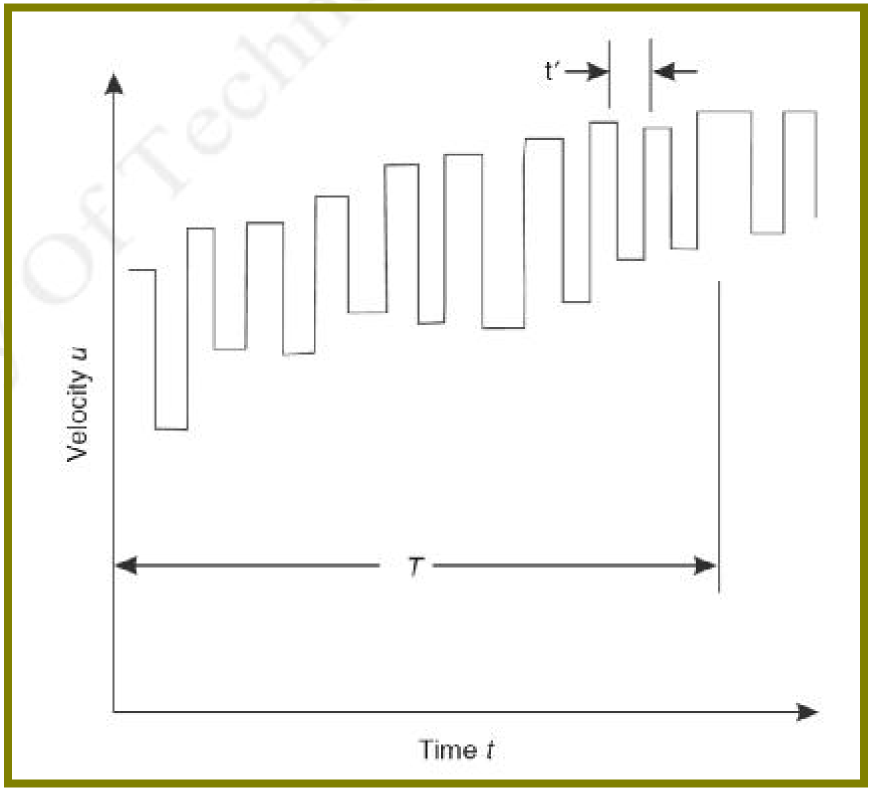
Problems **1,2,3,4**; Chapter **1**, Collier and Thome.

Due to next Tuesday (Mehr, 14th)

Time Averaging

$$\overline{B} = \frac{1}{T}\int_0^T B \, dt$$





Volume Averaging

$$\hat{B} = \frac{1}{V}\int_V B \, dV$$

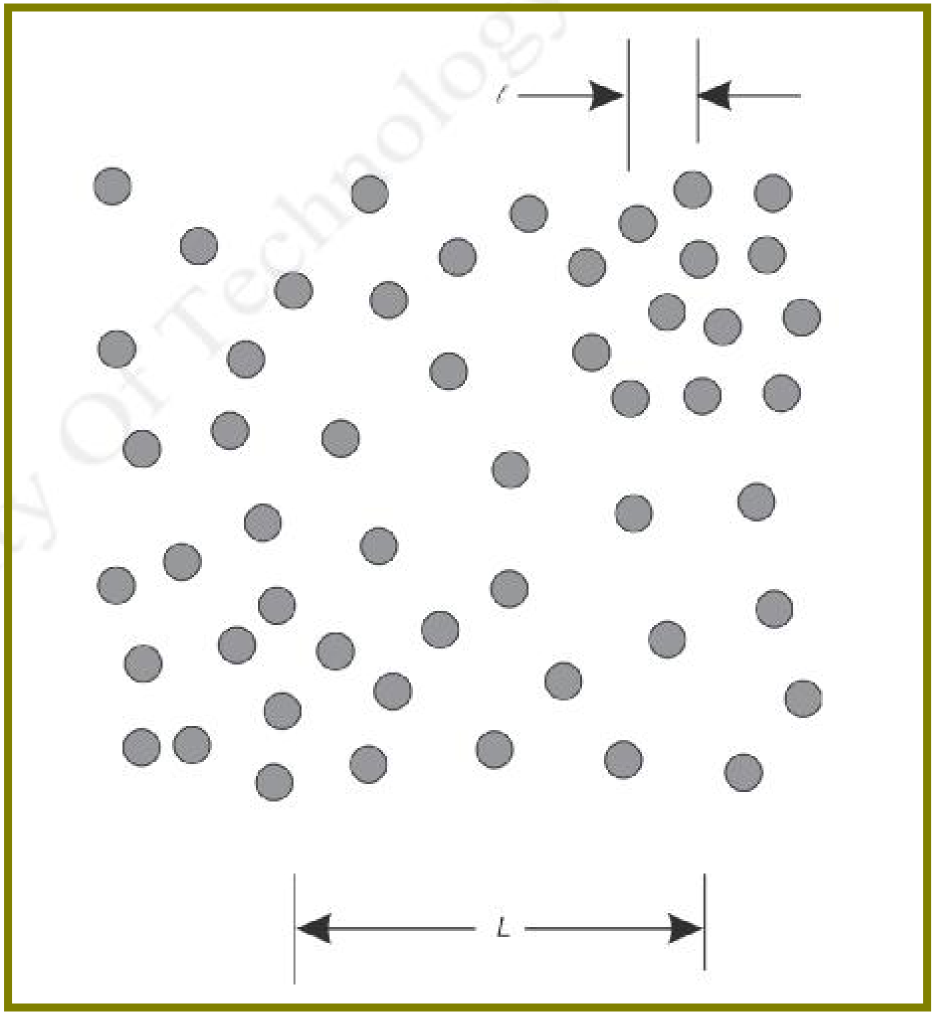
Volume Averaging

$$\{B\} = \frac{1}{V_c}\int_{V_c} B \, dV$$

Phase Averaging

Relation between phase and volume averaging for continues phase

$$\{B\} = a_c \hat{B}$$



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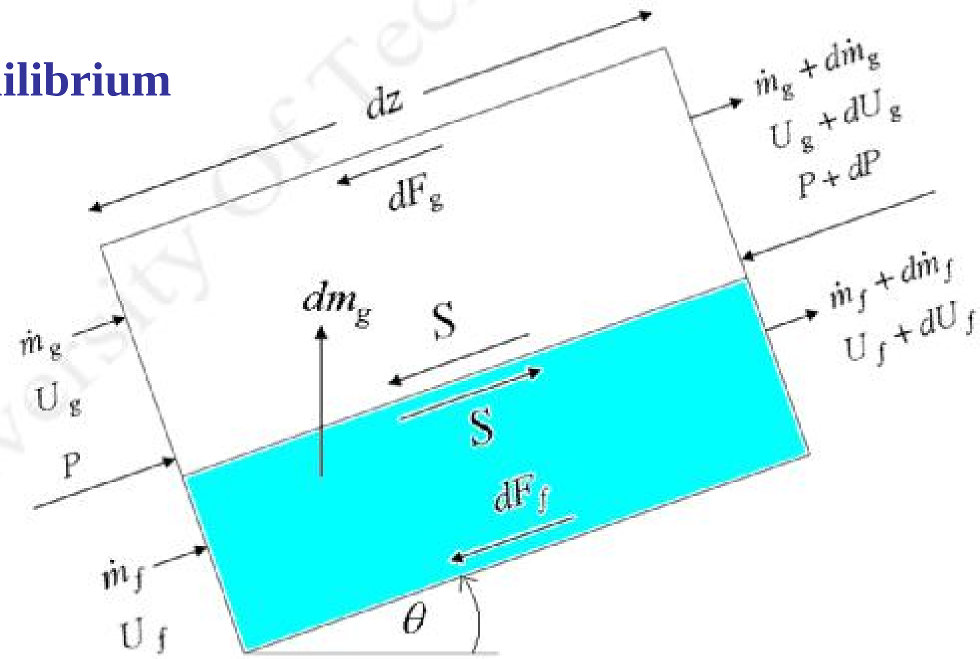
Principal Equation of Two Phase Flow

1. **One dimensional flow**
2. **Steady state**
3. **Constant physical properties**
4. **Existence of Thermodynamic equilibrium**



Conservation of Mass

$$\frac{\partial}{\partial t}(A\alpha_k\rho_k)+\frac{\partial}{\partial Z}(A\alpha_k\rho_k u_k)=\Gamma_k$$

Continuity equation of phase k

α_k — **Void fraction of phase k**

ρ_k — **Density of phase k**

Γ_k — **Mass Generation rate per unit length for phase k**

$$\sum_k \Gamma_k = 0$$

For steady state flow

$$\frac{\partial}{\partial t}(A a_k r_k)=0$$

For gas (g)- liquid (f) two phase flow

$$\frac{d}{dz}(A_g r_g u_g)=\Gamma_g$$

$$\frac{d}{dz}(A_f r_f u_f)=\Gamma_f$$

$$\Gamma_g = -\Gamma_f = \frac{dW_g}{dz} = -\frac{dW_f}{dz}$$

Conservation of Momentum

$$\frac{\partial}{\partial t}(W_k dz) + \left(W_k u_k + dz\frac{\partial}{\partial z}(W_k u_k)\right) - W_k u_k \Rightarrow \frac{\partial}{\partial t}(W_k dz) + dz\frac{\partial}{\partial z}(W_k u_k)$$

- $\frac{\partial}{\partial t}(W_k dz)$: **rate of creation of momentum of phase k**
- $\left(W_k u_k + dz\frac{\partial}{\partial z}(W_k u_k)\right) - W_k u_k$: **rate of inflow of momentum within the control volume**

$$W_k = Aa_k r_k u_k$$

Pressure forces on the element:

$$\left[Aa_k p - \left(Aa_k p + dz\frac{\partial}{\partial z}(Aa_k p)\right) - \left\{p\left(-dz\frac{\partial}{\partial z}(Aa_k)\right)\right\}\right]$$

$$-Aa_k r_k dz\, g \sin q - t_{kw} r_k dz\, g \sin q - t_{kw} P_{kw}\, dz + \sum_{1}^{n} t_{knz} P_{kn} dz + u_k \Gamma_k$$

- $-Aa_k r_k dz\, g \sin q$: **Gravity**
- $-t_{kw} P_{kw}\, dz$: **Wall shear**
- $\sum_{1}^{n} t_{knz} P_{kn} dz$: **Interfacial shear**
- $u_k \Gamma_k$: **Rate of momentum generation**

Conservation of Momentum

∑Force= creation of momentum + inflow of momentum within the control volume

$$-Aa_k \frac{\partial p}{\partial z} dz - t_{kw} P_{kw}\, dz + \sum_{1}^{n} t_{knz} P_{kn} dz - Aa_k r_k dz\, g \sin q + u_k \Gamma_k$$

$$= \frac{\partial}{\partial t}(W_k dz) + dz \frac{\partial}{\partial z}(W_k u_k)$$

Steady gas- liquid two phase flow

$$-A_g dp - t_{gw} P_{gw} dz + t_{gf} P_{gf} dz - A_g r_g dz\, g \sin q + u_g \Gamma_g = W_g du_g \qquad \text{I}$$

$$-A_f dp - t_{fw} P_{fw} dz + t_{fg} P_{fg} dz - A_f r_f dz\, g \sin q + u_f \Gamma_f = W_f du_f \qquad \text{И}$$

Momentum conservation at interface

$$t_{gf} P_{gf} dz + u_g \Gamma_g = t_{fg} P_{fg} dz + u_f \Gamma_f \qquad \text{Ш}$$

Conservation of Momentum

summation of equation I , И & Ш

$$* \quad -A\,dp - t_{gw} P_{gw}\,dz - t_{fw} P_{fw}\,dz - g\ \sin q\,(A_f\, r_f + A_g\, r_g) = d\,(W_g u_g + W_f u_f)$$

Friction force for each phase

$$** \quad (dF_g + S) = -t_{gw} P_{gw}\,dz - t_{gf} P_{gf}\,dz = -A_g\,(\frac{dp}{dz} g\ F)dz$$

$$(dF_f - S) = -t_{fw} P_{fw}\,dz + t_{gf} P_{gf}\,dz = -A_f\,(\frac{dp}{dz} f\ \ F)dz$$

$$(dF_g + dF_f) = -t_{gw} P_{gw}\,dz - t_{fw} P_{fw}\,dz = -A\,(\frac{dp}{dz} F)dz$$

Part of total pressure gradient which is need for prevalence of friction

Conservation of Momentum

substitution equation ******** **in** ******* **yields**

$$\left(\frac{dP}{dz}\right) = \left(\frac{dP}{dz}F\right) + \left(\frac{dP}{dz}a\right) + \left(\frac{dP}{dz}z\right)$$

$$-\left(\frac{dP}{dz}a\right) = \frac{1}{A}\frac{d}{dz}\left(W_g u_g + W_f u_f\right) = G^2 \frac{d}{dz}\left[\frac{x^2 n_g}{a} + \frac{(1-x)^2 n_f}{(1-a)}\right]$$

$$-\left(\frac{dP}{dz}z\right) = g \sin q \left[\frac{A_g}{A} r_g + \frac{A_f}{A} r_f\right] = g \sin q \left[a r_g + (1-a) r_f\right]$$

Total pressure lost

Energy Conservation

$$\frac{\partial}{\partial t}[a_k\, r_k\,(e_k+\frac{u_k^2}{2})A\,dz\,]+W_k\,(e_k+\frac{u_k^2}{2})dz\,-[W_k\,(e_k+\frac{u_k^2}{2})-dz\,\frac{\partial}{\partial z}W_k\,(e_k+\frac{u_k^2}{2})] \quad \textbf{A}$$

rate of increase of total energy in the C.V

rate of entrance of energy within the control volume

e_k → Internal energy per unit mass

Rate of heat entrance to C.V of phase k →

$$f_{kw}P_{kw}\,dz+\sum_{1}^{n}f_{kn}P_{kn}\,dz+f\&_k\,A\,a_k\,dz \quad \textbf{B}$$

Heat flow from channel wall

H.V. via the various interfaces

Internal heat generation within C.V

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Energy Conservation

The work done by pressure forces

Work done by expansion of phase k

$$\left[\frac{W_k p}{r_k} - \left(\frac{W_k p}{r_k} + \left(dz\,\frac{\partial}{\partial z}\left(\frac{W_k p}{r_k}\right)\right)\right)\right] - W_k\, g \sin q\, dz - pA\,dz\,\frac{\partial a_k}{\partial t}$$

$$+\Gamma_k \frac{dzp}{r_k} + u_k \sum_{1}^{n} t_{kn} P_{kn}\, dz$$

C

The work done by body force

Work done by pressure and shear forces at the interface with the other phases

Mass generation rate per unit length

$$\Gamma_k dz\,(e_k + \frac{u_k^2}{2})$$

D

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Energy Conservation

A=B+C+D

$$\frac{\partial}{\partial t} A a_k r_k (e_k + \frac{u_k^2}{2}) + \frac{\partial}{\partial z} W_k (i_k + \frac{u_k^2}{2}) = -W_k \, g \sin q + f_{wk} P_{wk}$$
$$+ \sum_1^n f_{kn} P_{kn} + \dot{f}_k A a_k - pA \frac{\partial a_k}{\partial t} + \Gamma_k (i_k + \frac{u_k^2}{2}) + u_k \sum_1^n t_{kn} P_{kn}$$

Enthalpy of phase k

$$i_k = u_k + \frac{p}{r_k}$$

For steady gas- liquid two phase flow in channel with constant area

$$d\left[W_g (i_g + \frac{u_g^2}{2})\right] + W_g \, g \sin q \, dz =$$
$$f_{wg} P_{wg} dz + f_{gf} P_{gf} dz + u_g t_{gf} P_{gf} dz + \Gamma_g dz (i_g + \frac{u_f^2}{2})$$ †

$$d\left[W_f (i_f + \frac{u_f^2}{2})\right] + W_f \, g \sin q \, dz =$$
$$f_{wf} P_{wf} dz + f_{fg} P_{fg} dz + u_f t_{fg} P_{fg} dz + \Gamma_f dz (i_f + \frac{u_f^2}{2})$$ ‡

Energy Conservation

Energy conservation at interface

$$f_{gf} P_{gf} + u_g t_{gf} P_{gf} dz + \Gamma_g (i_g + \frac{u_g^2}{2}) = f_{wf} P_{wf} dz + f_{fg} P_{fg} dz + u_f t_{fg} P_{fg} dz + \Gamma_f dz (i_f + \frac{u_f^2}{2}) \quad \top$$

with regard the equations †, ‡ and $\top$

$$\frac{d}{dz}[W_g i_g + W_f i_f] + \frac{d}{dz}[\frac{W_g u_g^2}{2} + \frac{W_f u_f^2}{2}] + (W_g + W_f) g \sin q = Q_{w1}$$

Heat transfer to the fluid across the channel wall per unit length

$$Q_{wl} (= f_{wf} p_{wf} + f_{wg} p_{wg})$$

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Energy Conservation

Total pressure gradient

Frictional dissipation

$$-\frac{dp}{dz}[x n_g + (1-x) n_f] = \left\{\frac{dE}{dz} - \frac{Q_{wf}}{W}\right\}$$

$$+\left\{p\frac{d}{dz}[x n_g + (1-x) n_f] + \frac{G^2}{2}\frac{d}{dz}\left[\frac{x^3 v_g^2}{a^2} + \frac{(1-x)^3 v_f^2}{(1-a)^2}\right]\right\} + g\sin q$$

Acceleration head term

Static head term

Internal energy per unit mass ⇒ $E = x e_g + (1-x) e_f$

Using momentum equation

Using void fraction to calculate acceleration term from

$$-\left(\frac{dP}{dz}_{\alpha}\right) = \frac{1}{A}\frac{d}{dz}\left(W_g u_g + W_f u_f\right) = G^2 \frac{d}{dz}\left[\frac{x^2 \nu_g}{\alpha} + \frac{(1-x)^2 \nu_f}{(1-\alpha)}\right]$$

or static head term from

$$-\left(\frac{dP}{dz}_{z}\right) = g\sin\theta\left[\frac{A_g}{A}\rho_g + \frac{A_f}{A}\rho_f\right] = g\sin\theta\left[\alpha\rho_g + (1-\alpha)\rho_f\right]$$

Then calculating friction pressure term from correlation equation in terms of independent variables.

Using energy equation

- Calculation of pressure lost arising from variation of potential energy
- Calculation of pressure lost arising from variation of kinetic energy
- Calculate the friction pressure term from independent variables

Note: in two methods we need to the void fraction but the degree of importance in each method is not the same.