Two Phase Flows

(Section 4)

The Basic Model

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Energy Conservation

$$\frac{\partial}{\partial t}[a_k r_k (e_k + \frac{u_k^2}{2})A dz] + W_k (e_k + \frac{u_k^2}{2}) dz - [W_k (e_k + \frac{u_k^2}{2}) - dz \frac{\partial}{\partial z} W_k (e_k + \frac{u_k^2}{2})] \quad \textbf{A}$$

- First term: **rate of increase of total energy in the C.V**
- Remaining terms: **rate of entrance of energy within the control volume**

e_k → **Internal energy per unit mass**

Rate of heat entrance to C.V of phase k →

$$f_{kw} P_{kw} dz + \sum_{1}^{n} f_{kn} P_{kn} dz + f\&_k A a_k dz \quad \textbf{B}$$

- First term: **Heat flow from channel wall**
- Second term: **H.V. via the various interfaces**
- Third term: **Internal heat generation within C.V**

Energy Conservation

The work done by pressure forces

Work done by expansion of phase k

$$\left[\frac{W_k p}{r_k}-\left(\frac{W_k p}{r_k}+\left(dz\,\frac{\partial}{\partial z}\left(\frac{W_k p}{r_k}\right)\right)\right)\right]-W_k\, g \sin q\, dz - pA\, dz\,\frac{\partial a_k}{\partial t}$$

$$+\Gamma_k \frac{dzp}{r_k}+u_k\sum_{1}^{n} t_{kn} P_{kn} dz$$

C

The work done by body force

Work done by pressure and shear forces at the interface with the other phases

Mass generation rate per unit length

$$\Gamma_k dz\,(e_k+\frac{u_k^2}{2})$$

D

Energy Conservation

A=B+C+D

$$\frac{\partial}{\partial t} A a_k r_k \left(e_k + \frac{u_k^2}{2}\right) + \frac{\partial}{\partial z} W_k \left(i_k + \frac{u_k^2}{2}\right) = -W_k g \sin q + f_{wk} P_{wk}$$

$$+ \sum_1^n f_{kn} P_{kn} + \dot{f}_k A a_k - pA \frac{\partial a_k}{\partial t} + \Gamma_k \left(i_k + \frac{u_k^2}{2}\right) + u_k \sum_1^n t_{kn} P_{kn}$$

Enthalpy of phase k

$$i_k = u_k + \frac{p}{r_k}$$

For steady gas- liquid two phase flow in channel with constant area

$$d\left[W_g \left(i_g + \frac{u_g^2}{2}\right)\right] + W_g g \sin q \, dz =$$

$$f_{wg} P_{wg} dz + f_{gf} P_{gf} dz + u_g t_{gf} P_{gf} dz + \Gamma_g dz \left(i_g + \frac{u_f^2}{2}\right) \quad †$$

$$d\left[W_f \left(i_f + \frac{u_f^2}{2}\right)\right] + W_f g \sin q \, dz =$$

$$f_{wf} P_{wf} dz + f_{fg} P_{fg} dz + u_f t_{fg} P_{fg} dz + \Gamma_f dz \left(i_f + \frac{u_f^2}{2}\right) \quad ‡$$

Energy Conservation

Energy conservation at interface

$$f_{gf}P_{gf} + u_g t_{gf} P_{gf} dz + \Gamma_g \left(i_g + \frac{u_g^2}{2}\right) = f_{wf} P_{wf} dz + f_{fg} P_{fg} dz + u_f t_{fg} P_{fg} dz + \Gamma_f dz \left(i_f + \frac{u_f^2}{2}\right) \quad ┬$$

with regard the equations †, ‡ and ┬

$$\frac{d}{dz}[W_g i_g + W_f i_f] + \frac{d}{dz}\left[\frac{W_g u_g^2}{2} + \frac{W_f u_f^2}{2}\right] + (W_g + W_f) g \sin q = Q_{w1}$$

Heat transfer to the fluid across the channel wall per unit length → $Q_{wl} (= f_{wf} p_{wf} + f_{wg} p_{wg})$

Energy Conservation

Total pressure gradient | **Frictional dissipation**

$$-\frac{dp}{dz}[xn_g+(1-x)n_f]=\left\{\frac{dE}{dz}-\frac{Q_{wf}}{W}\right\}$$

$$+\left\{p\frac{d}{dz}[xn_g+(1-x)n_f]+\frac{G^2}{2}\frac{d}{dz}\left[\frac{x^3v_g^2}{a^2}+\frac{(1-x)^3v_f^2}{(1-a)^2}\right]\right\}+g\sin q$$

Acceleration head term | **Static head term**

Internal energy per unit mass ⇨ $E=xe_g+(1-x)e_f$

Using momentum equation

Using void fraction to calculate acceleration term from

$$-\left(\frac{dP}{dz}_{a}\right)=\frac{1}{A}\frac{d}{dz}\left(W_g u_g + W_f u_f\right)=G^2\frac{d}{dz}\left[\frac{x^2 \nu_g}{\alpha}+\frac{(1-x)^2 \nu_f}{(1-\alpha)}\right]$$

or static head term from

$$-\left(\frac{dP}{dz}_{z}\right)=g\sin\theta\left[\frac{A_g}{A}\rho_g+\frac{A_f}{A}\rho_f\right]=g\sin\theta\left[\alpha\rho_g+(1-\alpha)\rho_f\right]$$

Then calculating friction pressure term from correlation equation in terms of independent variables.

Use of the momentum or energy equation to evaluate the pressure gradient

Using energy equation

- Calculation of pressure lost arising from variation of potential energy
- Calculation of pressure lost arising from variation of kinetic energy
- Calculate the friction pressure term from independent variables

Note: in two methods we need to the void fraction but the degree of importance in each method is not the same.