Two Phase Flows

(Section 7)

Empirical Treatments of Two Phase Flow

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Drift Flux Model

Zuber and Findlay(1965)

Wallis (1969)

Ishii et al. (1975 ,1977)

Developed DFM model by:

$$j_g = a\, u_g = b\, j = \frac{Gx}{r_g} \quad , j_f = (1-a)\, u_f = (1-b)\, j = \frac{G(1-x)}{r_f}$$

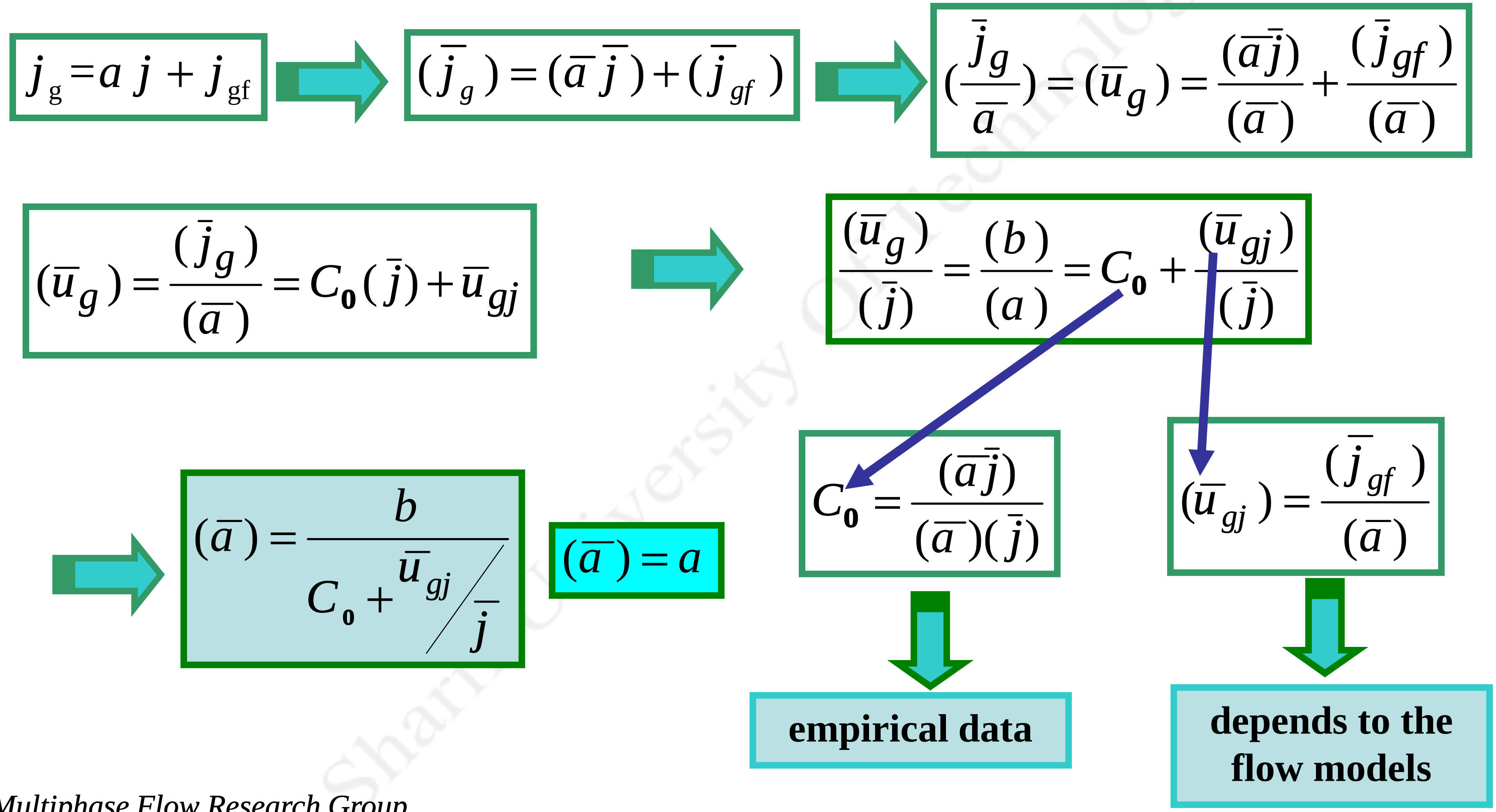
$$u_{gf} = (u_g - u_f) = \frac{j_g}{a} - \frac{j_f}{1-a}$$

Relative velocity between phases.

$$j_{gf} = a\,(1-a)\,u_{gf} = (1-a)\,j_g - a\,j_f = j_g - a\,j$$

With simplifying, the drift flux can be defined

Drift Flux Model



$$j_g = a\,j + j_{gf}$$

$$(\bar{j}_g) = (\bar{a}\,\bar{j}) + (\bar{j}_{gf})$$

$$\left(\frac{\bar{j}_g}{\bar{a}}\right) = (\bar{u}_g) = \frac{(\overline{a\bar{j}})}{(\bar{a})} + \frac{(\bar{j}_{gf})}{(\bar{a})}$$

$$(\bar{u}_g) = \frac{(\bar{j}_g)}{(\bar{a})} = C_0(\bar{j}) + \bar{u}_{gj}$$

$$\frac{(\bar{u}_g)}{(\bar{j})} = \frac{(b)}{(a)} = C_0 + \frac{(\bar{u}_{gj})}{(\bar{j})}$$

$$(\bar{a}) = \frac{b}{C_0 + \bar{u}_{gj}/\bar{j}}$$

$$(\bar{a}) = a$$

$$C_0 = \frac{(\overline{a\bar{j}})}{(\bar{a})(\bar{j})}$$

empirical data

$$(\bar{u}_{gj}) = \frac{(\bar{j}_{gf})}{(\bar{a})}$$

depends to the flow models

Drift Flux Model

$if \quad \bar{u}_{gj} = 0 \ then$ ⇒ $a = \frac{b}{C_0}$

for 1-D homogeneous flow ⇒ $a = b$

C_0 is the empirical coefficient which is stated that the velocity and concentration profiles can be varied independently

Zuber et al. (1967) for water- steam flow

$$C_0 = 1.13 \quad , \quad \bar{u}_{gj} = 1.41\left[\frac{s\ g\,(r_g - r_g)}{r_f^{\ 2}}\right]^{0.25}$$

This model is valuable when $U_{gj} > 0.05J$ so this model is appropriate for Bubbly, Slug and Churn flow patterns.

Bubbly Flow

Bankoff (1960) variable density model

Homogeneous model with corrections for two dimensional effects

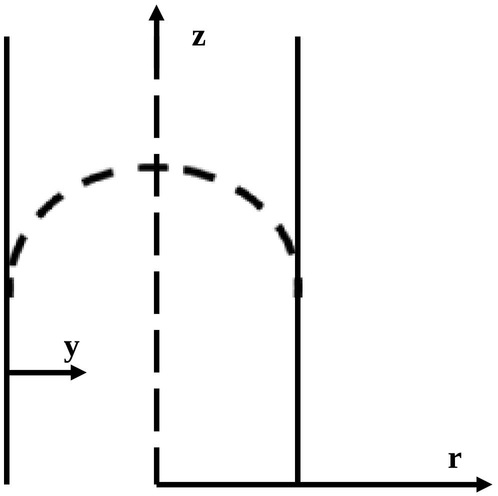
Concentration of bubbles are high at the center of channel.

Radial relative velocity between bubbles and liquid are negligible.

Power distribution law is supposed for velocity and void fraction.

$$\boxed{\frac{u}{u_{\max}} = \left(\frac{y}{r}\right)^{\frac{1}{m}}}$$

$$\boxed{\frac{a}{a_{\max}} = \left(\frac{y}{r}\right)^{\frac{1}{n}}}$$



Bubbly Flow

$$a = C_A \, \bar{b}$$

Such a this relation are greatly reported in the Russian documents.

$$C_A = \frac{2(m+n+mn)(m+n+2mn)}{(n+1)(2n+1)(m+1)(2m+1)}$$

for $0.5 < n < 1$ and $2 < m < 7$

$0.5 < C_A < 1$

Bankoff find a good agreement with Martinelli-Nelson model for $C_A=0.89$

Armand(1946) ؛Armand and Treschev (1947) and Armand (1954) proposed $C_A=0.833$ for $\beta < 0.9$

Treschev and Armand (1947) and Bankoff (1960) proposed increasing C_A with pressure

Slip Ratio Correlations

Smith(1969) Chisholm(1983) find the simple and useful relation for slip ratio (SR)

A

$$\frac{u_g}{u_f} = e + (1-e)\left[\frac{\frac{r_f}{r_g} + e\left(\frac{1}{x}-1\right)}{1+e\left(\frac{1}{x}-1\right)}\right]^{1/2}$$

Chisholm(1983) find

B

$$\frac{u_g}{u_f} = \left(\frac{r_f}{\bar{r}}\right)^{1/2} = \left(\frac{\bar{v}}{n_f}\right)^{1/2} = \left[1 + x\left(\frac{r_f}{r_g}-1\right)\right]^{1/2}$$

Equations A and B anticipate the same value for S.R but in the high quality B propose greater value

$$\frac{u_g}{u_f} = \left(\frac{n_g}{n_f}\right)^{1/4} = \left(\frac{r_f}{r_g}\right)^{1/4}$$

for X<1

for X>1

$$\frac{u_g}{u_f} = \left[1 - b\left(1 - \frac{n_f}{n_g}\right)\right]^{-1/2}$$

Slip Ratio Correlations

$$\boxed{\frac{1}{C_A} = b + \frac{1-b}{\left[1 - b\left(1 - \frac{n_f}{n_g}\right)\right]^{1/2}}}$$

for X>1

$$\boxed{if\ \frac{v_f}{v_g} >> 1}$$

$$\boxed{C_A = \frac{1}{b + (1-b)^{1/2}}}$$

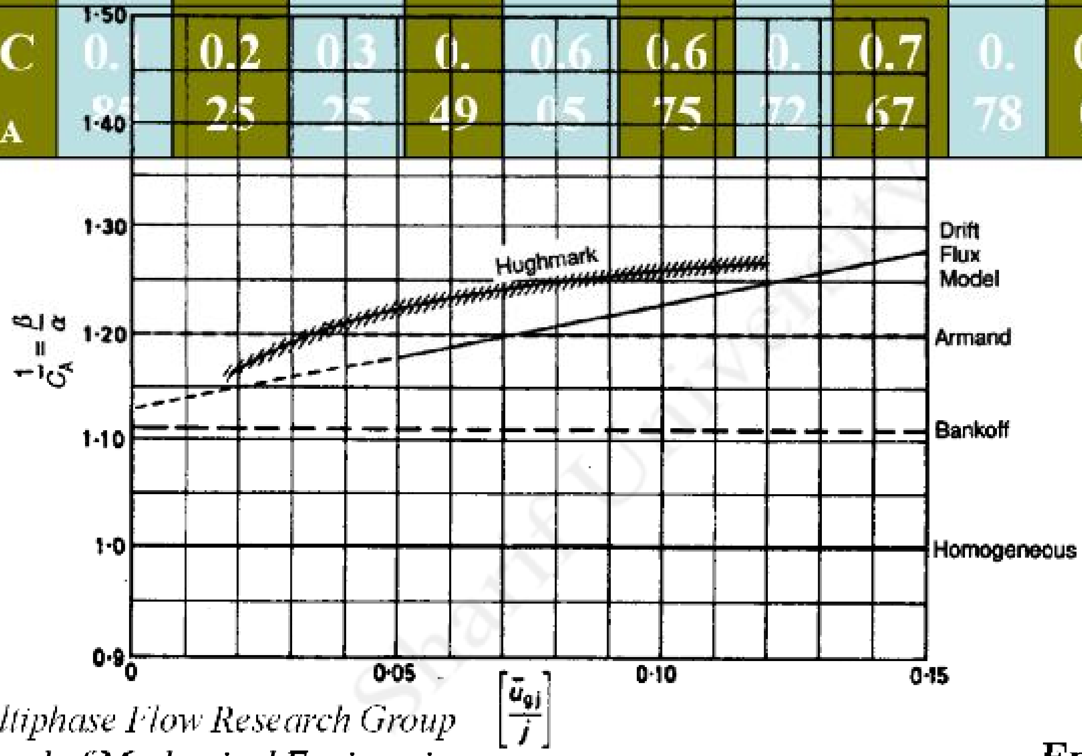
$$\boxed{a = \frac{b}{b + (1-b)^{1/2}}}$$

The Hughmark correlation

Hughmark (1962) proposed C_A as a function of z

$$Z = \left[\frac{GD}{(1-a)m_f + a m_g}\right]^{1/6} \left(\frac{j^2}{gD}\right)^{1/8} (1-b)^{-1/4}$$

z	1.3	1.5	2	3.0	4.0	5.0	6.0	8.0	10.0	15.0	20.0	40.0	70.0	130.0
C_A	0.185	0.225	0.325	0.49	0.605	0.675	0.72	0.767	0.78	0.808	0.83	0.88	0.93	0.98



Comparison of different models

Drift Flux Model

$$(\bar{a}) = \frac{b}{C_0 + \bar{u}_{gj}/\bar{j}}$$

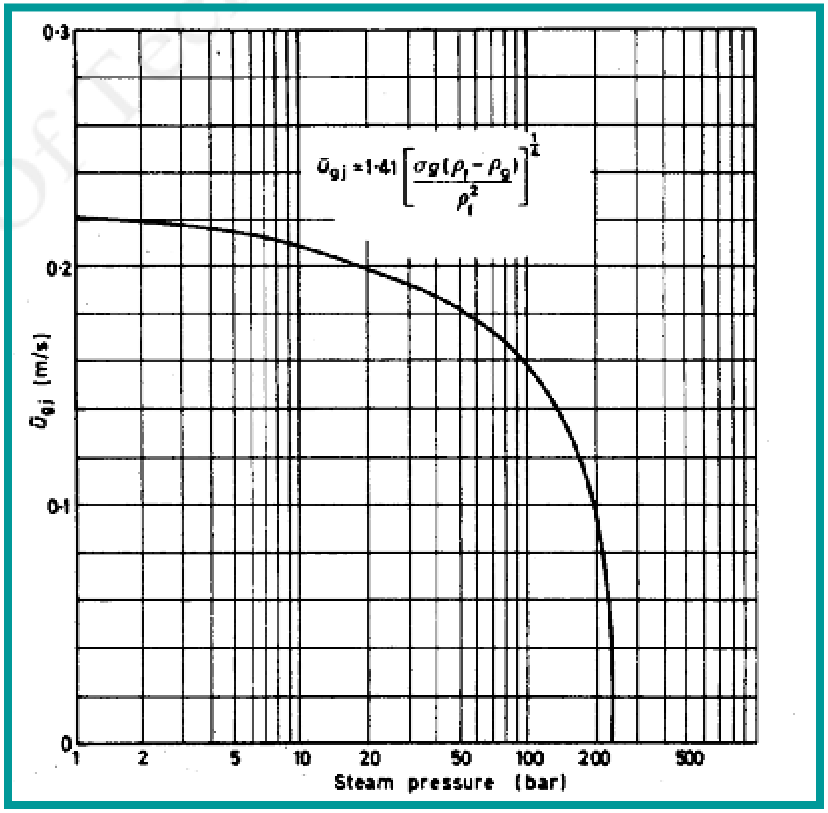
Wallis 1969

$$C_0 = 1.0\ ,\ \bar{u}_{gj} = 1.53(1-a)^2\left[\frac{s\,g.(r_f - r_g)}{r_f^{\,2}}\right]^{1/4}$$

Zuber et al.
1967 proposed
1.41

in vertical upward bubbly- slug flow

$$\bar{u}_{gj} = 1.41\left[\frac{s\ g(r_f - r_g)}{r_f^{\,2}}\right]^{1/4}$$



Slug Flow

Drift Flux Model

$$(\bar{a}) = \frac{b}{C_0 + \bar{u}_{gj}/\bar{j}}$$

$$for \quad jDr_f / m_f > 8000$$

for vertical turbulence slug flow

$$C_0 = 1.2,\ \bar{u}_{gj} = 0.35\left[\frac{g(r_f - r_g)D}{r_f}\right]^{1/2}$$

for horizontal turbulence slug flow

$$for \quad jDr_f / m_f > 3000$$

$$\bar{u}_{gj} = 0 \ and \ C_0 = 1.2$$

$$a = 0.833b$$

Arman (1946)

If wall's shear stresses for vapor bubbles be negligible

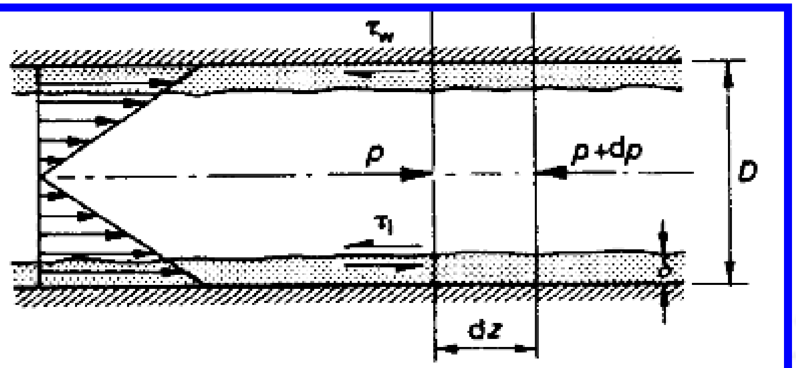
$$-\left(\frac{dp}{dz}F\right) = \frac{2f_{fo}G^2 n_f}{D}(1-a)$$

for slug flow

Annular Flow

Liquid Film

- **Total of liquid flow are supposed be in a thin film**
- **Interface of phases is smooth**
- **Gravitation and acceleration forces are negligible**



$$-(\frac{dp}{dz}F)_{TP} = \frac{4t_w}{D}$$

$$-(\frac{dp}{dz}F)_g = \frac{4t_i}{(D-2d)}$$

$$(\frac{dp}{dz}F)_f$$

$$\div$$

$$f_f^2 = \frac{t_w}{t_f}$$

$$u_f = \frac{j_f}{(1-a)}$$

$$t_w = f_{TP}(\frac{r_f u_f^2}{2}) \quad ; \quad t_f = f_f(\frac{r_f j_f^2}{2})$$

$$f_f^2 = \frac{1}{(1-a)^2}\frac{f_{TP}}{f_f}$$