Session 5: Solid State Physics

Charge Mobility
Drift
Diffusion
Recombination-Generation

Outline

- A
  - B
  - C
  - D
  - E
- F
  - G
- H
- I
- J

Mobile Charge Carriers in Semiconductors

Three primary types of carrier action occur inside a semiconductor:

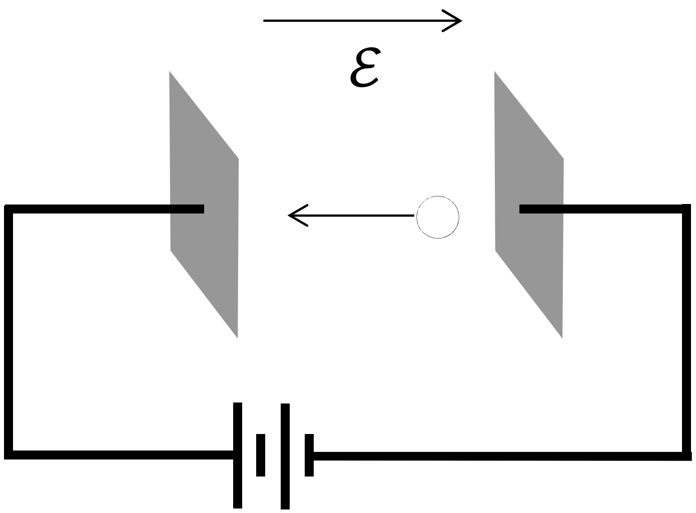
Drift: charged particle motion under the influence of an electric field.

Diffusion: particle motion due to concentration gradient or temperature gradient.

Recombination-generation (R-G)

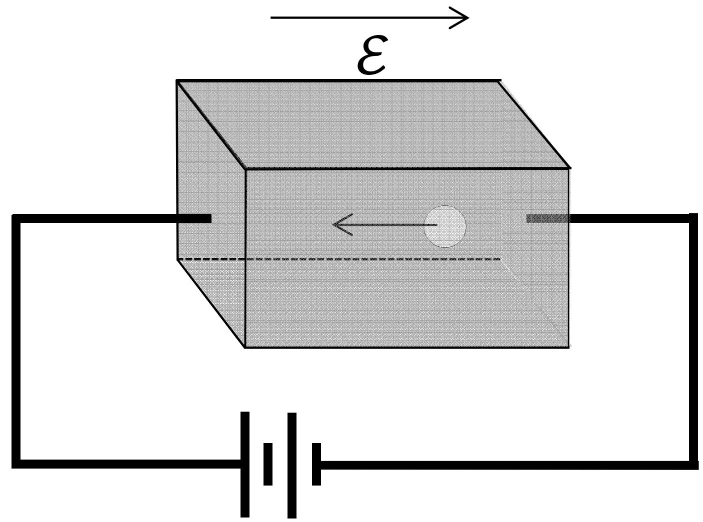
Electrons as Moving Particles

In vacuum



$$F = (-q)\mathcal{E} = m_0 a$$

In semiconductor



$$F = (-q)\mathcal{E} = m_n^* a$$

where m_n^* is the electron effective mass

Thermal Velocity

We saw that:
In an electric field, $\mathcal{E}$, an electron or a hole accelerates

electrons: $$a = \frac{-q\mathcal{E}}{m_n^*}$$

holes: $$a = \frac{q\mathcal{E}}{m_p^*}$$

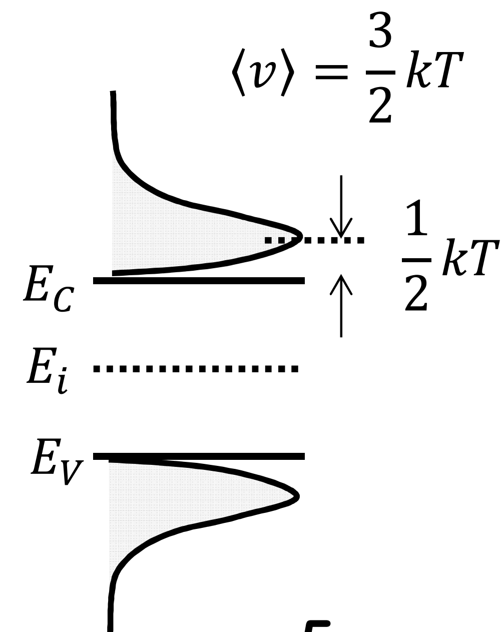
electron and hole effective masses

	Si	Ge	GaAs
m_n^*/m_0	0.26	0.12	0.068
m_p^*/m_0	0.39	0.3	0.5

Average electron kinetic energy= $\frac{3}{2}kT = \frac{1}{2}m_n^* v_{th}^2$

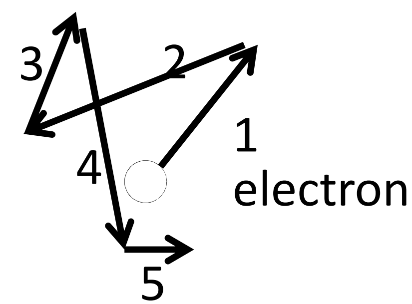
$$v_{th} = \sqrt{\frac{3kT}{m_n^*}} = \sqrt{\frac{3 \times 0.026eV \times 1.6 \times 10^{-19} J/eV}{0.26 \times 9.1 \times 10^{-31} Kg}}$$

$$= 2.3 \times 10^5\ m/s = 2.3 \times 10^7\ cm/s$$



Carrier Scattering

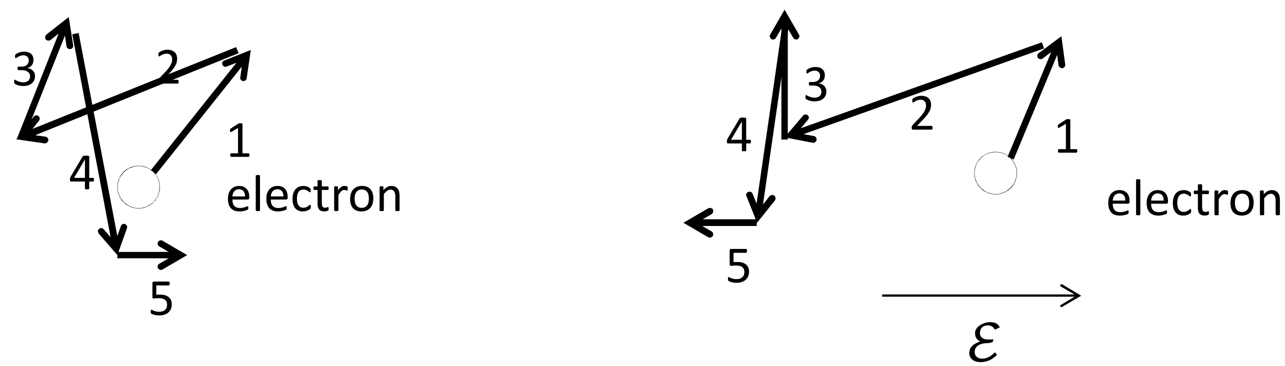
Mobile electrons and atoms in the Si lattice are always in random thermal motion. Electrons make frequent collisions with the vibrating atoms called “lattice scattering” or “phonon scattering” (increases with increasing temperature)
Average velocity of thermal motion for electrons: ~10^7 cm/s @ 300K



Other scattering mechanisms:
deflection by ionized impurity atoms
deflection due to Columbic force between carriers (carrier-carrier scattering)
only significant at high carrier concentrations
The net current in any direction is zero, if no electric field is applied.

Carrier Drift

When an electric field (e.g. due to an externally applied voltage) is applied to a semiconductor, mobile charge carriers will be accelerated by the electrostatic force. This force superimposes on the random motion of electrons:



Electrons drift in the direction opposite to the electric field

→ current flows

Because of scattering, electrons in a semiconductor do not achieve constant acceleration. However, they can be viewed as quasi-classical particles moving at a constant average drift velocity v_d

Carrier Mobility

With every collision, the electron loses momentum = $m_n^* v_d$

Between collisions, the electron gains momentum = $(-q)\mathcal{E}\,\overline{\tau}$

$\overline{\tau}$ is the average time between electron scattering events (mean time between collisions)

In steady state

$$m_n^* v_d = (-q)\mathcal{E}\,\overline{\tau} \quad \rightarrow |v_d| = q\mathcal{E}\,\overline{\tau}\,/m_n^* \equiv \mu_n \mathcal{E}$$

$\mu_n = \dfrac{q\,\overline{\tau}}{m_n^*}$ is the electron mobility $\mu_p = \dfrac{q\,\overline{\tau}}{m_p^*}$ is the electron mobility

Electron and hole mobilities of selected intrinsic semiconductors (T=300K)

	Si	Ge	GaAs	InAs
$\mu_n[cm^2/Vs]$	1400	3900	8500	30000
$\mu_p[cm^2/Vs]$	470	1900	400	500

Mean Free Path

Average distance traveled between collisions is called mean free path

$$\lambda = v_{th}\,\overline{\tau}$$

This is an important length, structures at the order or smaller that m.f.p. show different performance.

Mechanisms of Carrier Scattering

Dominant scattering mechanisms:

Phonon scattering (lattice scattering)

Impurity (dopant) ion scattering

Phonon scattering mobility decreases when T increases:

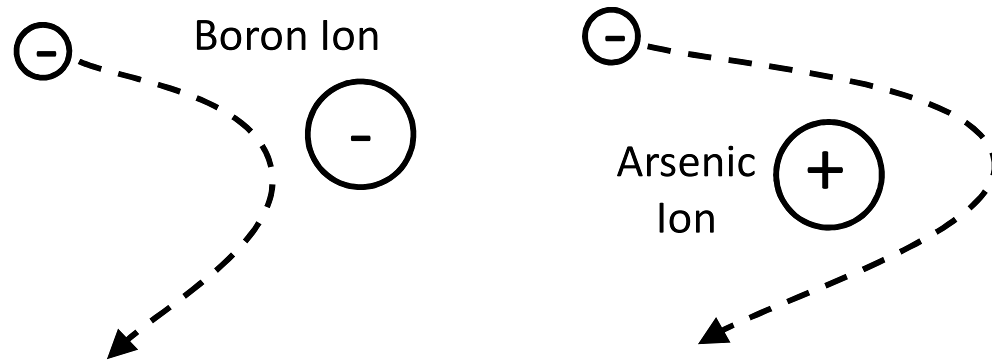
$$\mu_{phonon} \propto \tau_{phonon} \propto$$

$$\frac{1}{\text{phonon density } \times \text{carrier thermal velocity}}$$

$$\propto \frac{1}{T \times T^{\frac{1}{2}}} \propto T^{-3/2}$$

$$v_{th} \propto \sqrt{T}$$

There is less change in the electron's direction of travel if the electron zips by the ion at a higher speed.



$$\mu_{phonon} \propto \frac{v_{th}^3}{N_A + N_D} \propto \frac{T^{3/2}}{N_A + N_D}$$

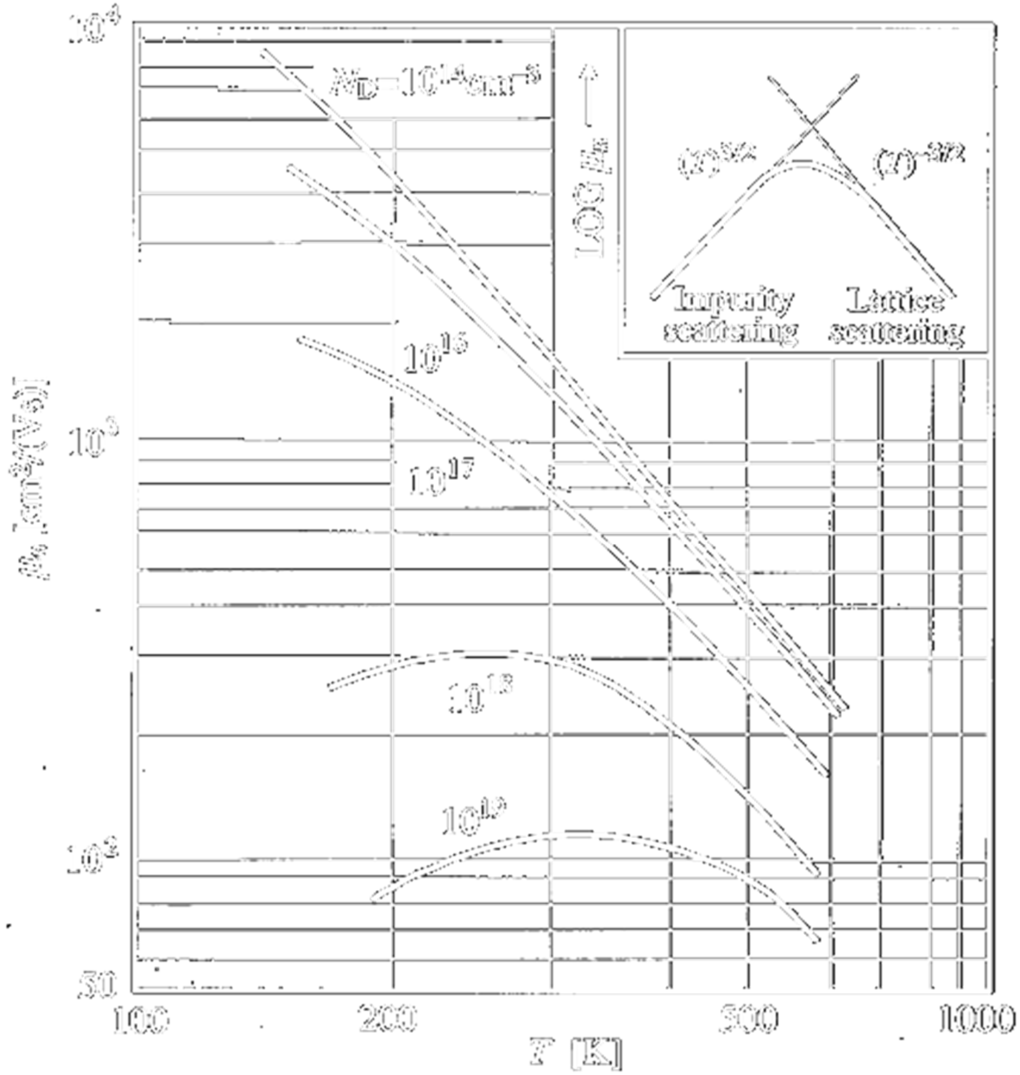
Matthiessen's Rule

The probability that a carrier will be scattered by mechanism i within a time period dt is dt/τ_i , where τ_i is the mean time between scattering events due to mechanism i.
Hence, The probability that a carrier will be scattered within a time period dt is $\sum dt/\tau_i$

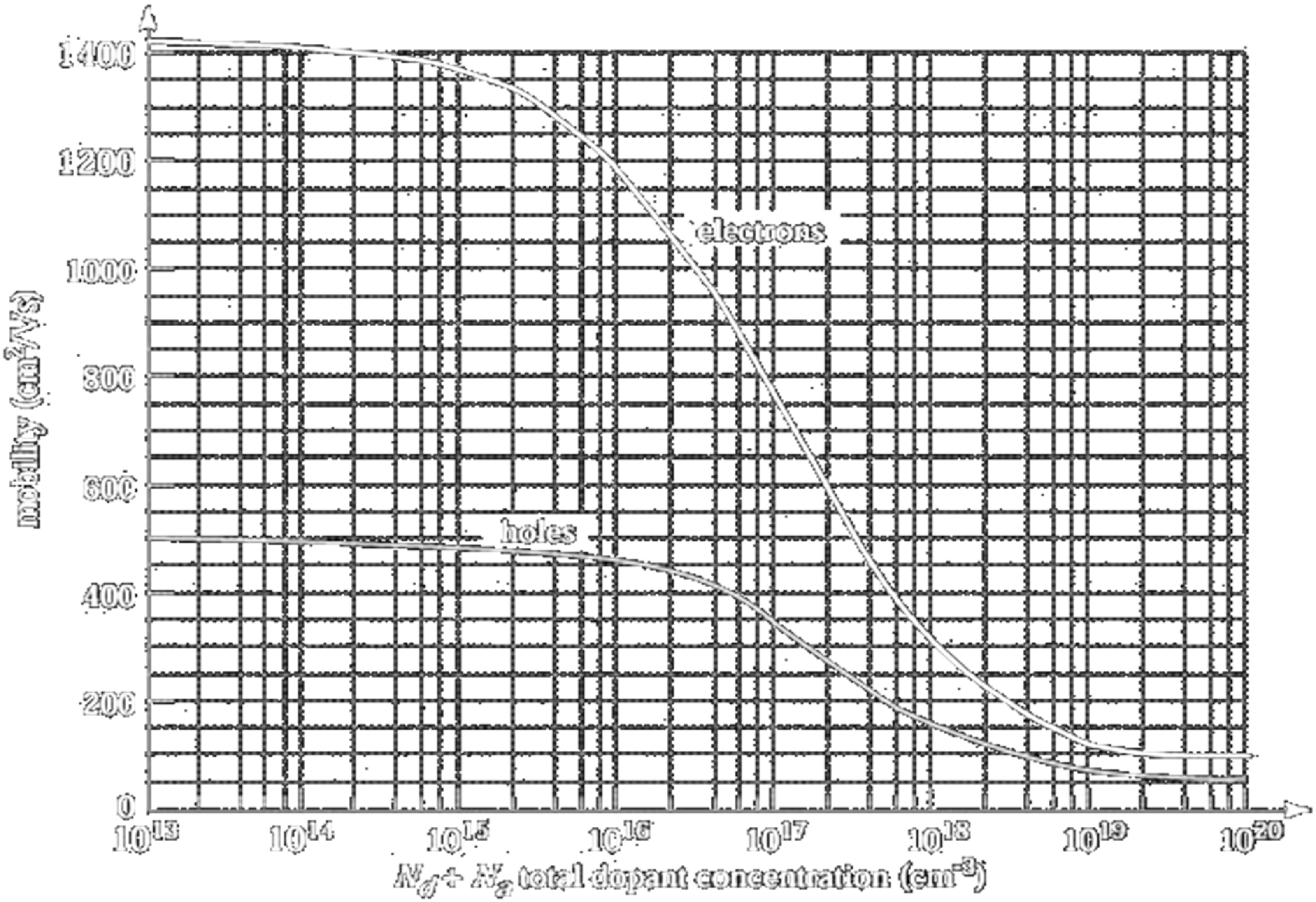
$$\frac{1}{\tau} = \frac{1}{\tau_{phonon}} + \frac{1}{\tau_{imp}}$$

$$\rightarrow \quad \frac{1}{\mu} = \frac{1}{\mu_{phonon}} + \frac{1}{\mu_{imp}}$$

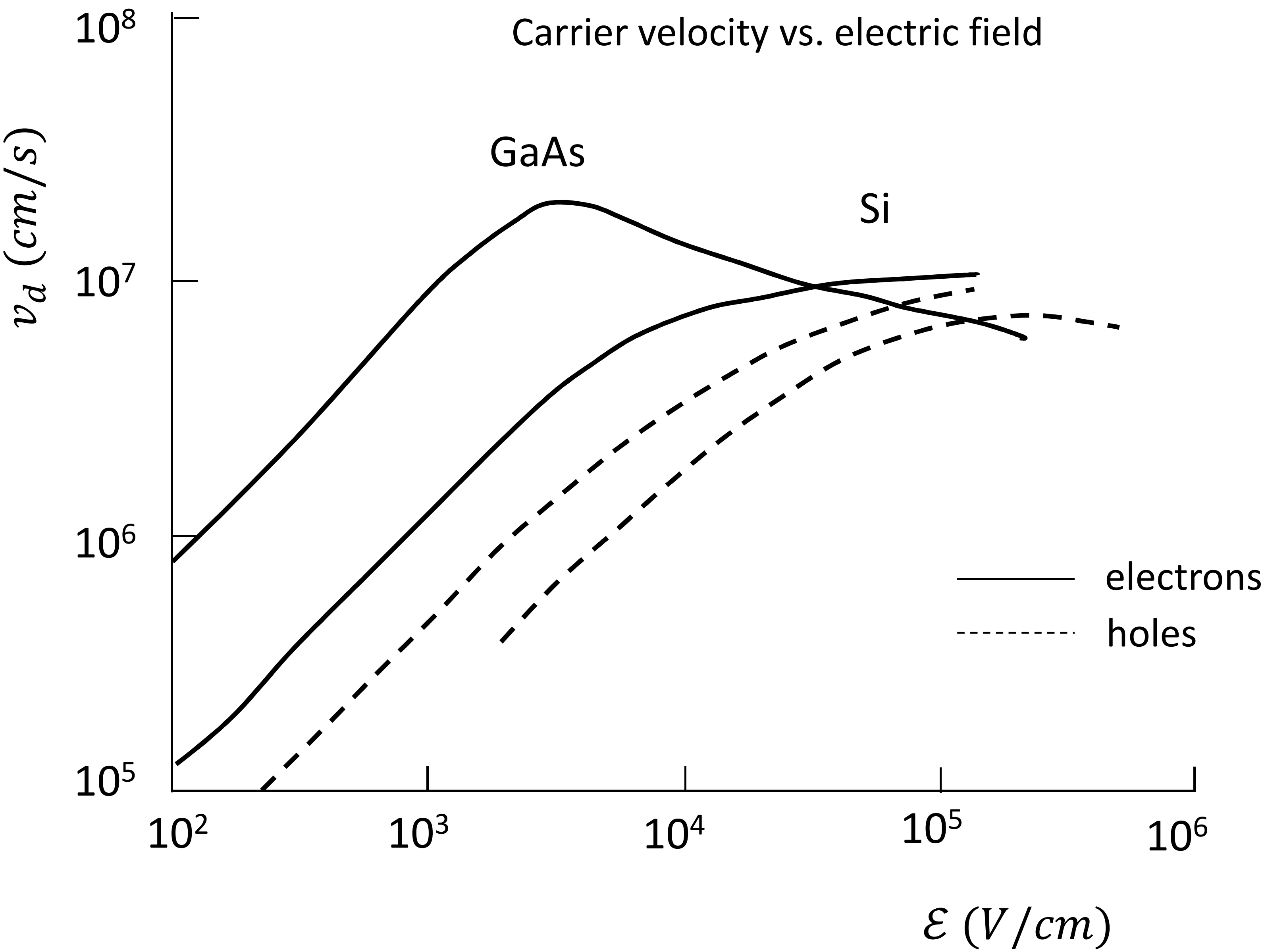
Temperature Effect on Mobility:



Mobility Dependence on Doping (Si)



Velocity Saturation



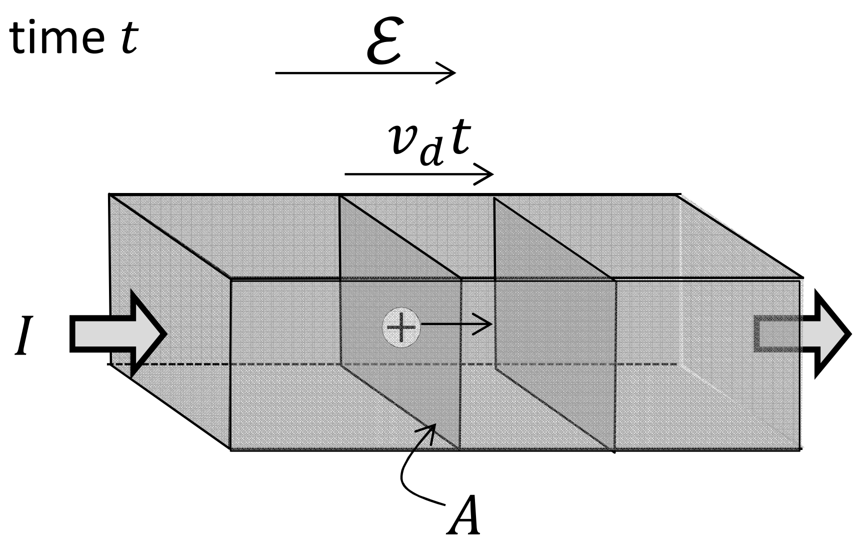
Drift Velocity

$v_d tA$ = volume from which all holes cross plane in time t

$pv_d tA$ = # of holes crossing plane in time t

$qpv_d tA$ = charge crossing plane in time t

$qpv_d A$ = charge crossing plane per unit time
= hole current



→ Hole current per unit area = $J_p = \dfrac{I}{A} = qpv_d$

$$J_{n\_drift} = -qnv_{dn} = qn\mu_n \mathcal{E}$$

$$J_{p\_drift} = -qpv_{dp} = qp\mu_p \mathcal{E}$$

$$J_{drift} = J_{n\_drift} + J_{p\_drift} = (qn\mu_n + qp\mu_p)\mathcal{E} = \sigma\mathcal{E}$$

Conductivity of a semiconductor is $\sigma \equiv (qn\mu_n + qp\mu_p)$

Resistivity $\rho \equiv 1/\sigma$ (Unit: ohm-cm)

Resistivity vs. Doping

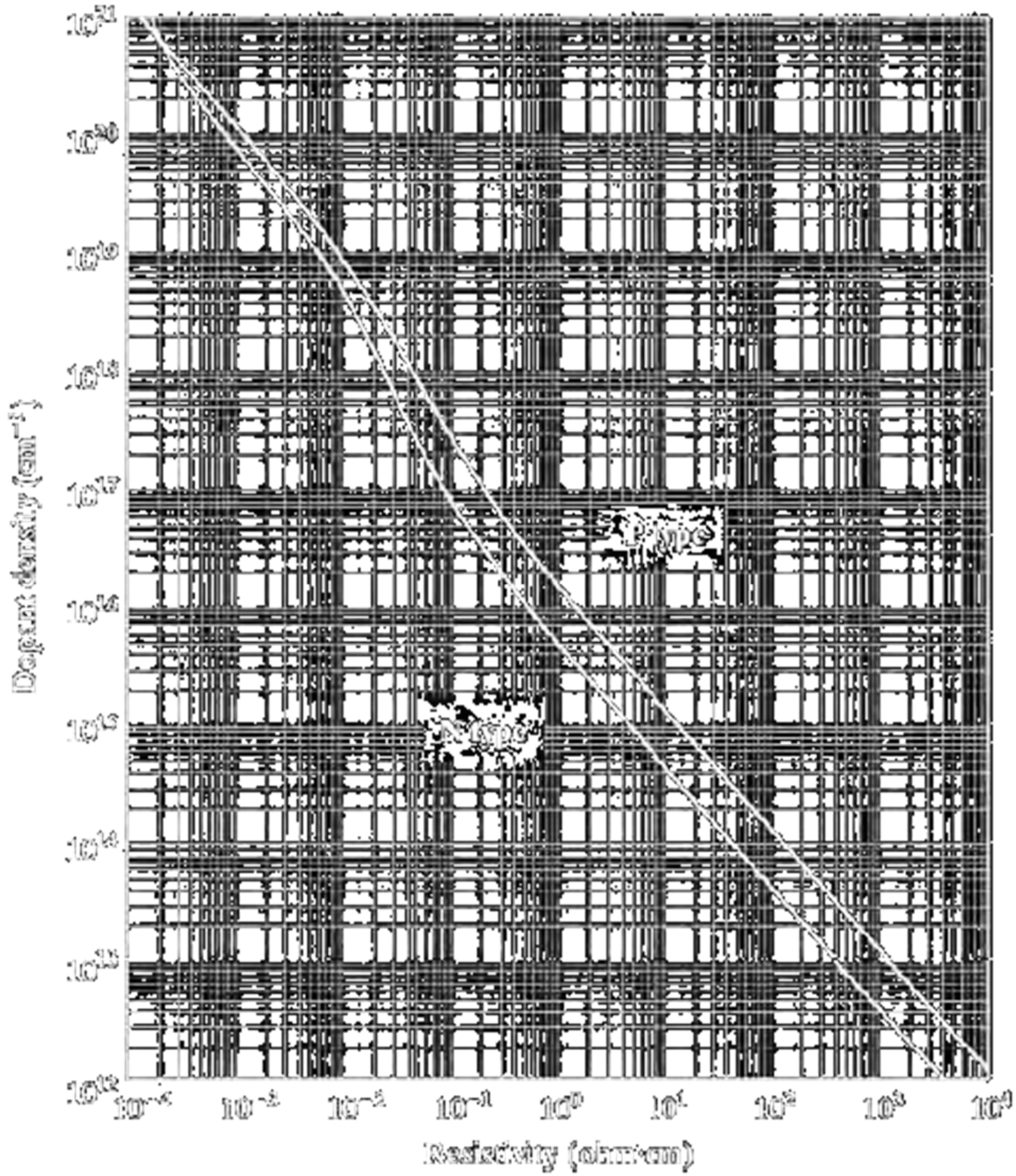
For n-type material:

$$\rho \cong \frac{1}{qn\mu_n}$$

For p-type material:

$$\rho \cong \frac{1}{qp\mu_p}$$

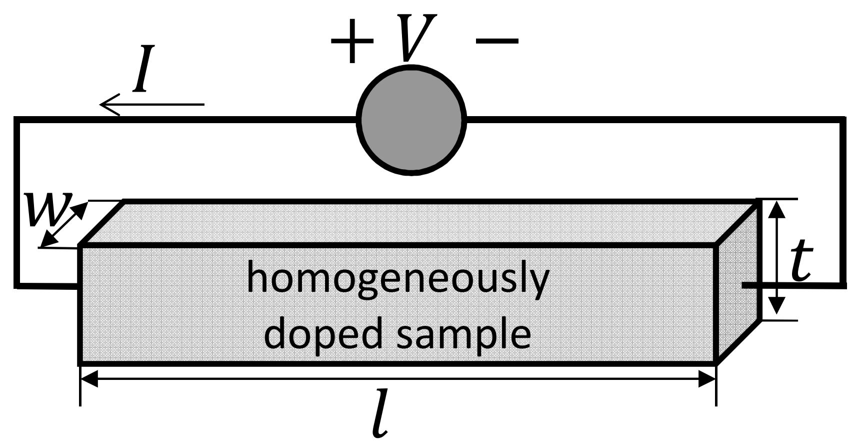
Note: This plot does not apply to the compensated material!



Electrical Resistance

Resistance (Ohms)

$$\sigma = \frac{1}{\rho} = \frac{J}{\mathcal{E}} = \frac{I/wt}{V/l} \quad \rightarrow \quad R \equiv \frac{V}{I} = \rho \frac{l}{wt}$$



Q: Consider a Si sample doped with $10^{16}/cm^3$ Boron. What is its resistivity?

A: $N_A = 10^{16}/cm^3$, $N_D = 0$ ($N_A \gg N_D$ hence p-type) $p \approx 10^{16}/cm^3$ and $n \approx 10^4/cm^3$

$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \sim \frac{1}{qp\mu_p}$$
$$= [1.6 \times 10^{-19} \times 10^{16} \times 450]^{-1}$$
$$= 1.4 \ \ \Omega cm$$

Q: Consider the same Si sample doped with $10^{17}/cm^3$ Arsenic. What is its resistivity?

A: $N_A = 10^{16}/cm^3$, $N_D = 10^{17}/cm^3$ ($N_D \gg N_A$ hence n-type) $n \approx 9 \times 10^{16}/cm^3$ and $p \approx 1.1 \times 10^3/cm^3$

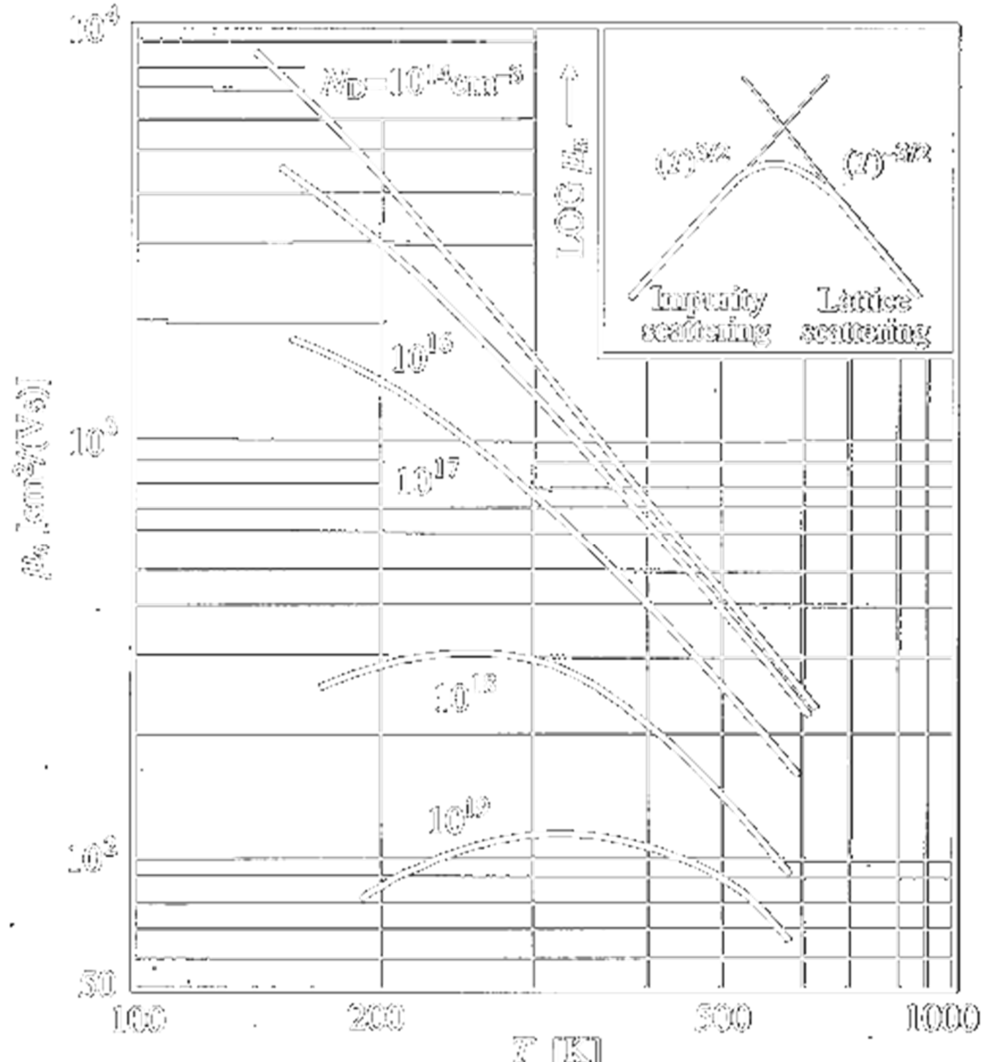
$$\rho = \frac{1}{qn\mu_n + qp\mu_p} \sim \frac{1}{qn\mu_n}$$
$$= [1.6 \times 10^{-19} \times 9 \times 10^{16} \times 650]^{-1}$$
$$= 0.12 \ \ \Omega cm$$

Potential vs. Kinetic Energy

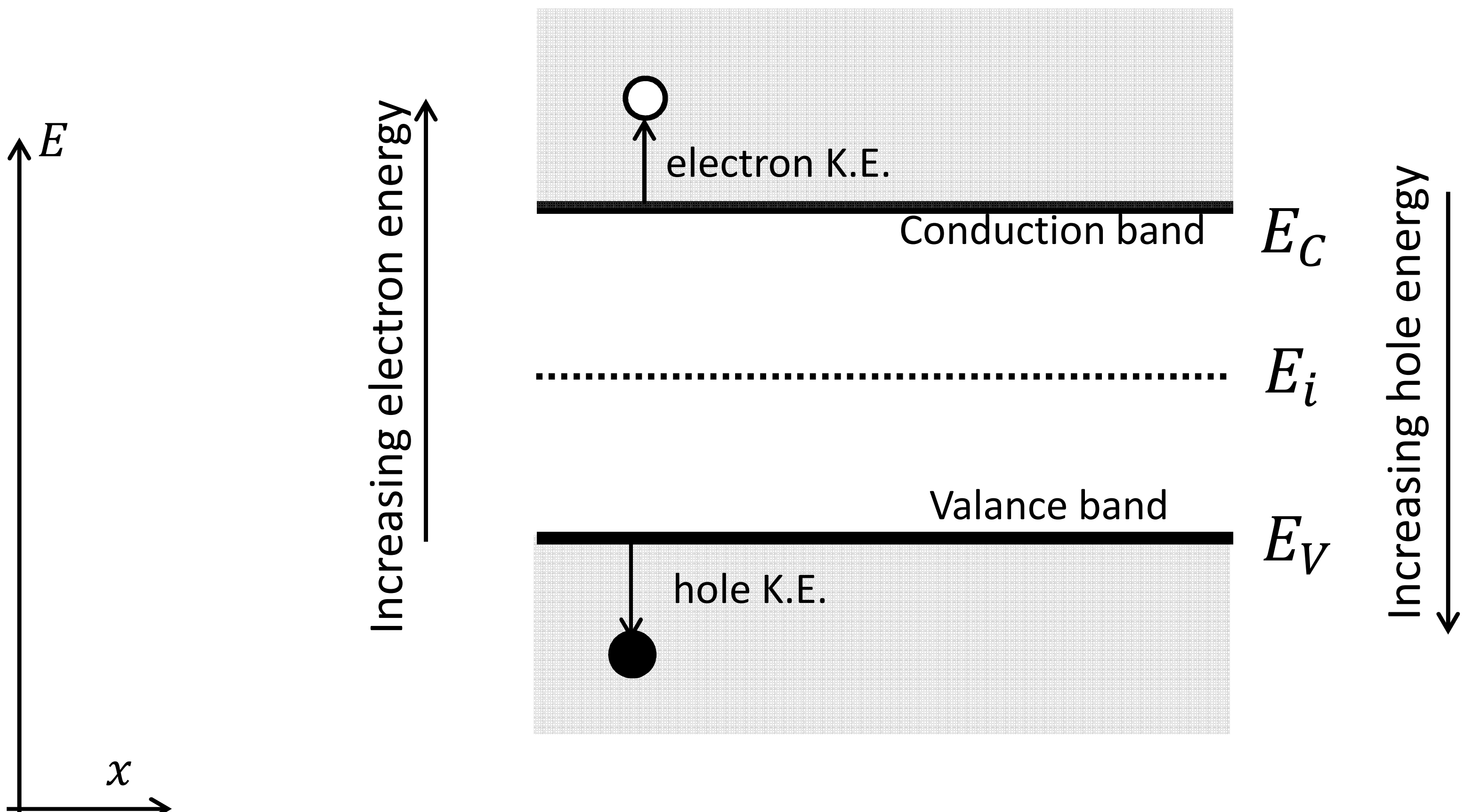
Q: Consider a Si sample doped with $10^{17} cm^{-3}$ As. How will its resistivity change when the temperature is increased from T=300K to T=400K?

A: The temperature dependent factor in ρ (and therefore ρ) is μ_n. From the mobility *vs.* temperature curve for $10^{17} cm^{-3}$, we find that μ_n decreases from $770\ cm^2/Vs$ at 300K to $400\ cm^2/Vs$ at 400K. As a result, ρ **increases** by

$$\frac{770}{440} = 1.93$$



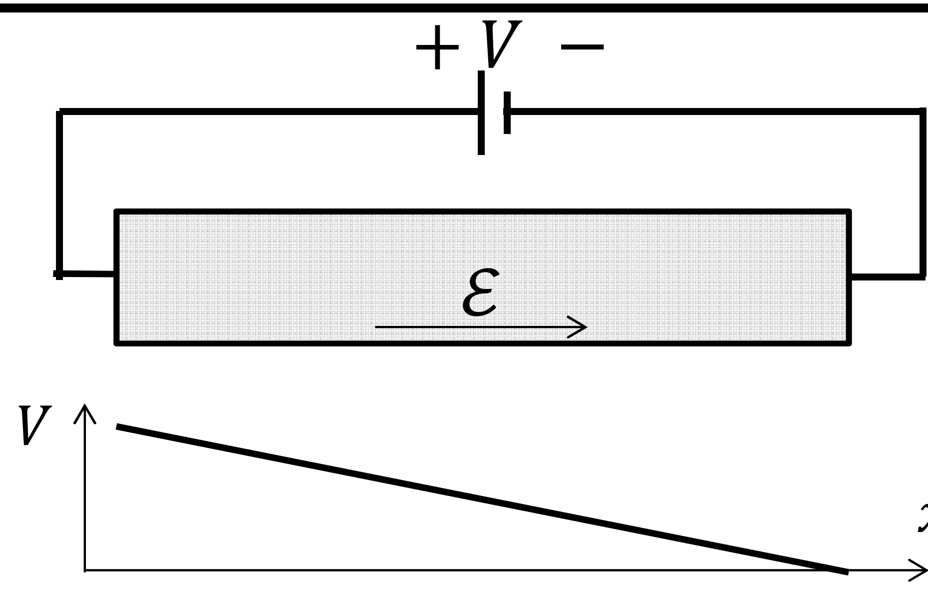
Potential vs. Kinetic Energy



E_C represents the electron potential energy:

$$P.E. = E_C - E_{\text{reference}}$$

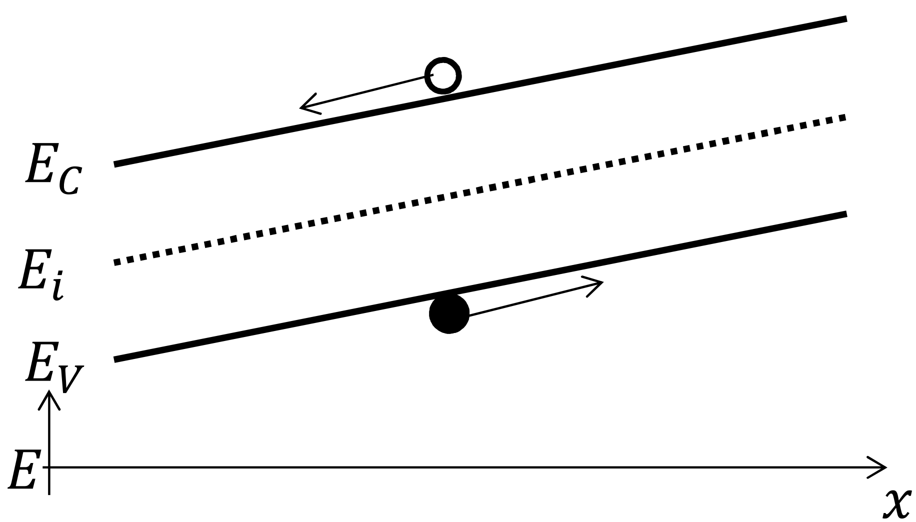
Band Bending



The potential energy of a particle with charge $-q$ is related to the electrostatic potential $V(x)$:

$$P.E. = -qV$$

$$V = \frac{1}{q}(E_{\text{ref.}} - E_C)$$



$$\mathcal{E} = -\frac{dV}{dx} = \frac{1}{q}\frac{dE_C}{dx}$$

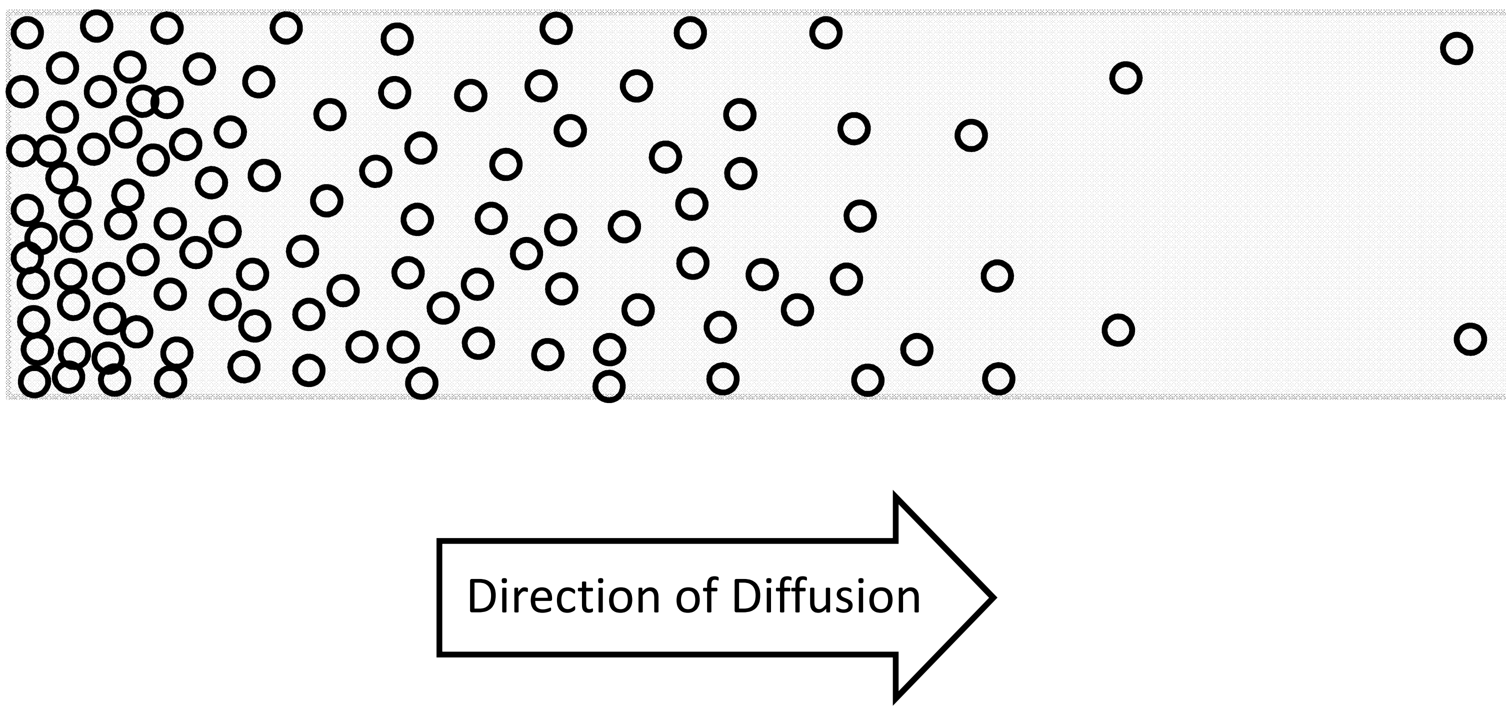
Variation of Ec with position is called "band bending."

Diffusion

Particles diffuse from regions of higher concentration to regions of lower concentration region, due to random thermal motion

Higher particle concentration

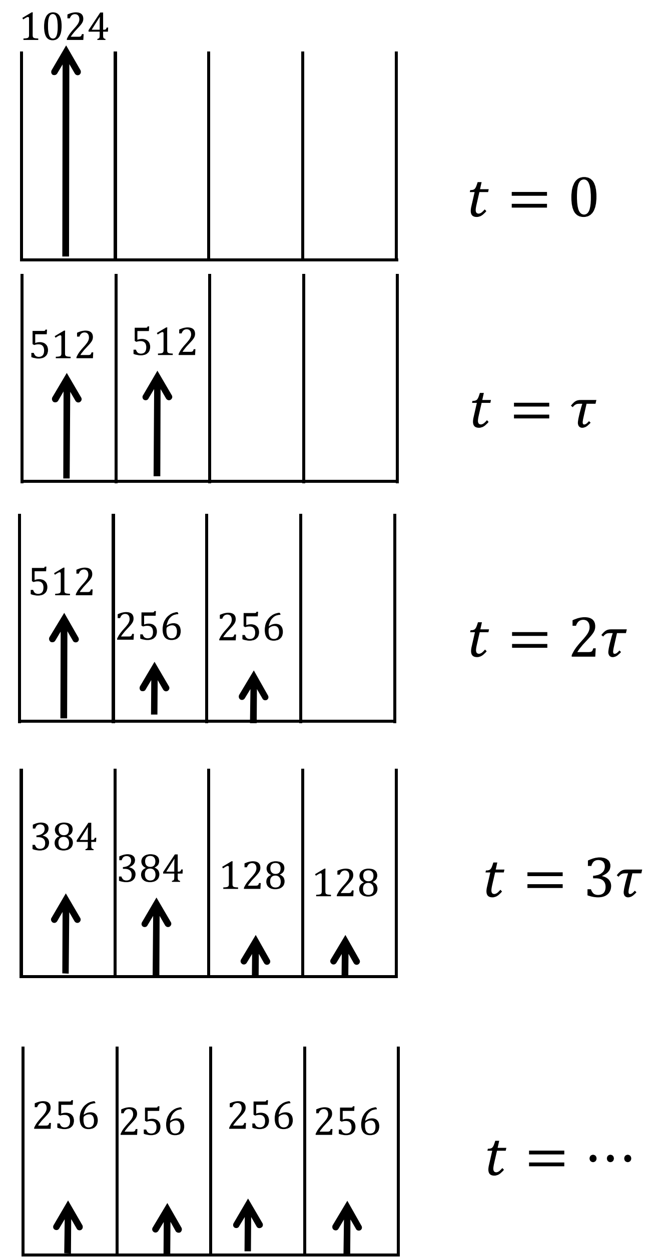
Lower particle concentration



1-D Diffusion Example

Thermal motion causes particles to move into an adjacent compartment every t seconds.

Each particle has an equal probability of jumping to the left and to the right.

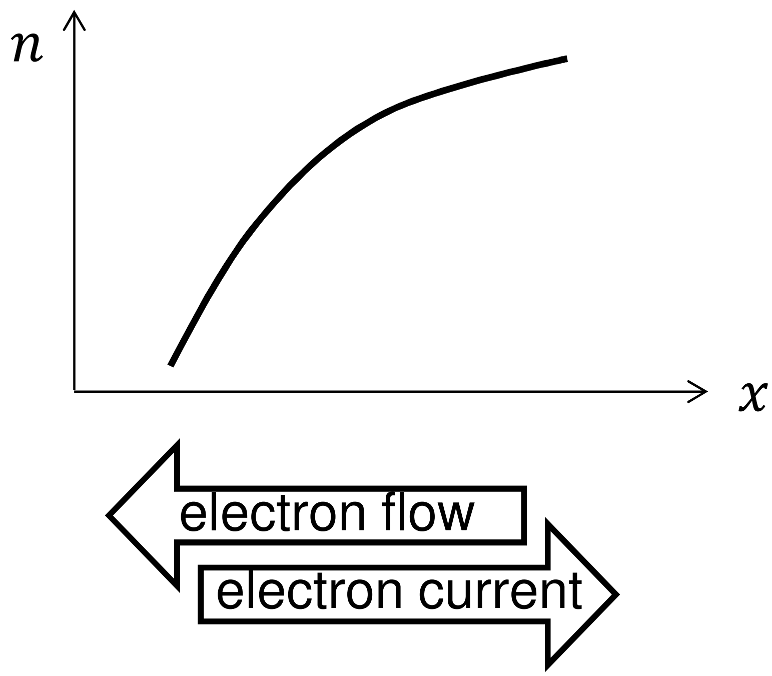


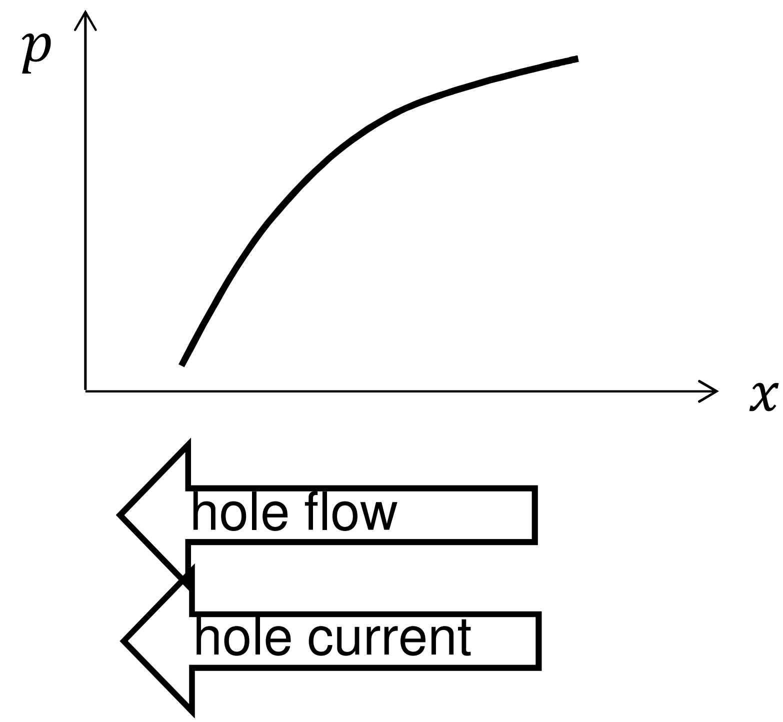
Diffusion Current

$$J_{n\_diff} = qD_n \frac{dn}{dx}$$

D is the diffusion constant, or diffusivity.

$$J_{p\_diff} = -qD_p \frac{dp}{dx}$$





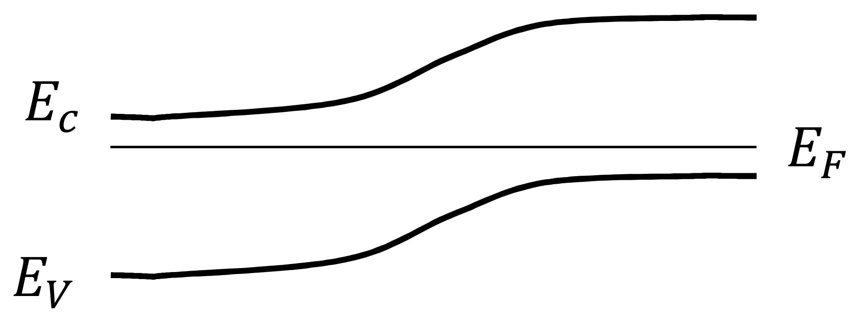
$$J = J_n + J_p$$

$$J_n = J_{n_{diff}} + J_{n_{drift}} = qn\mu_n\mathcal{E} + qD_n \frac{dn}{dx}$$

$$J_p = J_{p_{diff}} + J_{p_{drift}} = qp\mu_p\mathcal{E} - qD_p \frac{dp}{dx}$$

Einstein Relationship

Consider a piece of a non-uniformly doped semiconductor:



In equilibrium, E_F is constant; therefore, the band energies vary with position:

In equilibrium, there is no net flow of electrons or holes

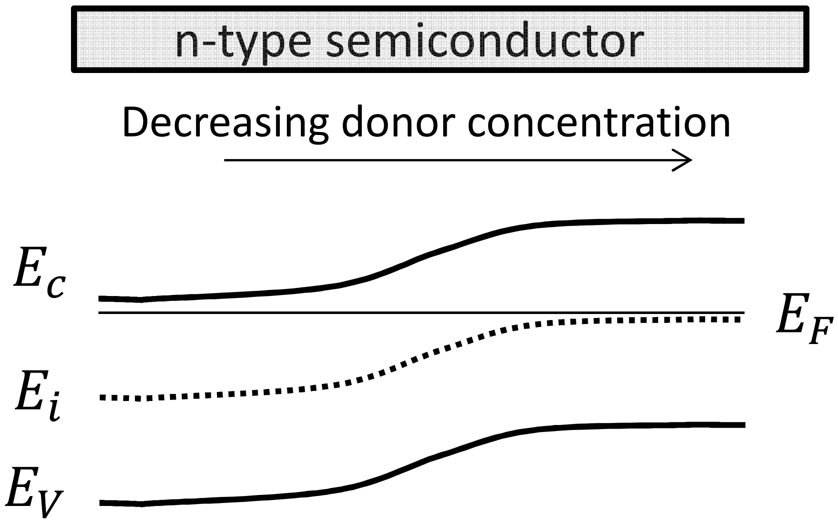
$$J_n = 0 \ and \ J_p = 0$$

The drift and diffusion current components must balance each other exactly. (A built-in electric field exists, such that the drift current exactly cancels out the diffusion current due to the concentration gradient.)

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

Non Uniformly Doped Semiconductor

The position of EF relative to the band edges is determined by the carrier concentrations, which is determined by the dopant concentrations.



$$n = N_C\, e^{-(E_C - E_F)/kT}$$

$$\frac{dn}{dx} = -\frac{N_C}{kT}\, e^{-(E_C - E_F)/kT}\, \frac{dE_C}{dx}$$

$$= -\frac{n}{kT}\,\frac{dE_C}{dx} = -\frac{n}{kT}\, q\mathcal{E}$$

Under equilibrium conditions: $J_n \ = \ 0 \ and \ J_p \ = \ 0$

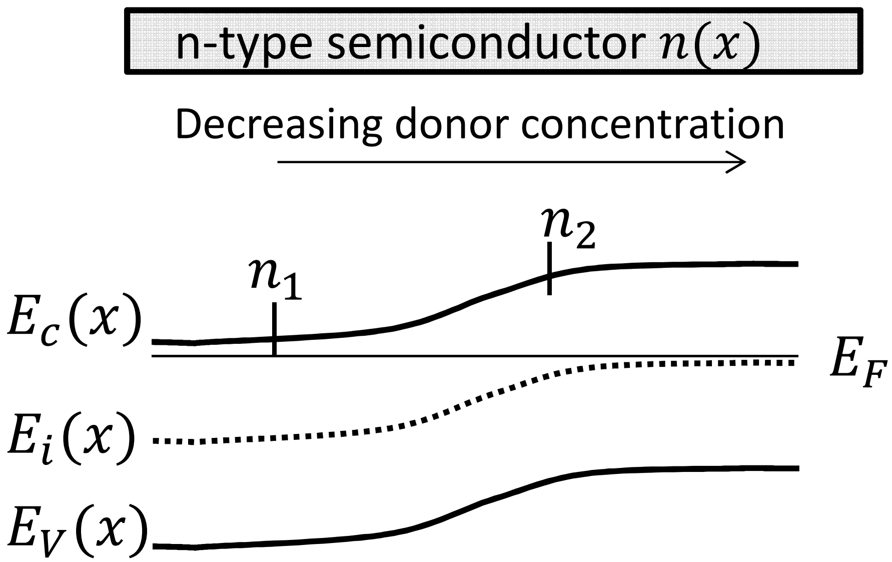
$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} = 0 = qn\mu_n\mathcal{E} - qD_n\frac{n}{kT}\, q\mathcal{E} \qquad \rightarrow \quad D_n = \frac{kT}{q}\mu_n$$

Similarly: $\rightarrow \quad D_p = \dfrac{kT}{q}\mu_p$

Note: The Einstein relationship is valid for a non-degenerate semiconductor, even under non-equilibrium conditions

Potential Difference due to n(x) , p(x)

Consider a piece of a non-uniformly doped semiconductor:



$$n(x) = n_i \, e^{(E_F - E_i(x))/kT}$$

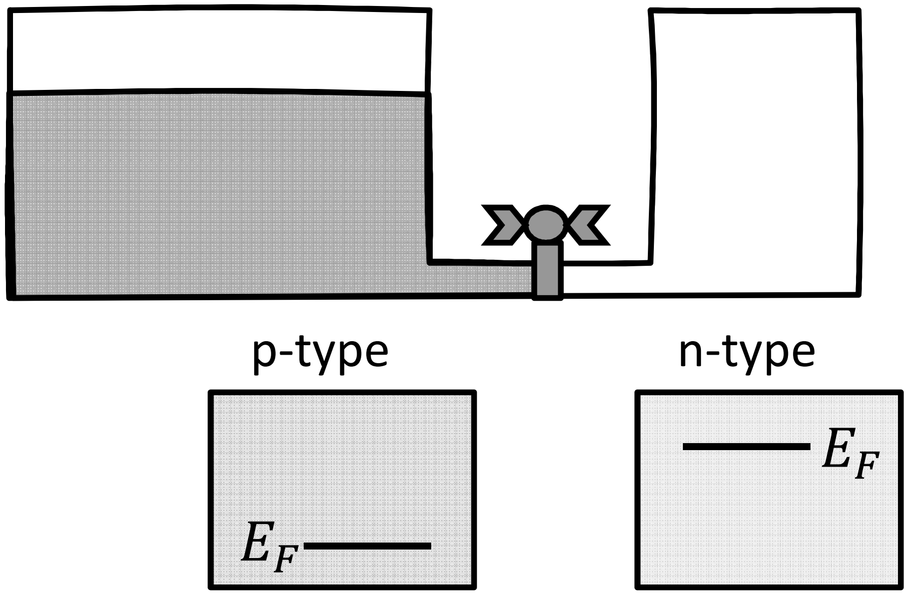
$$\rightarrow \; E_{i1} = E_F - kT \, \ln(\frac{n_1}{n_i})$$

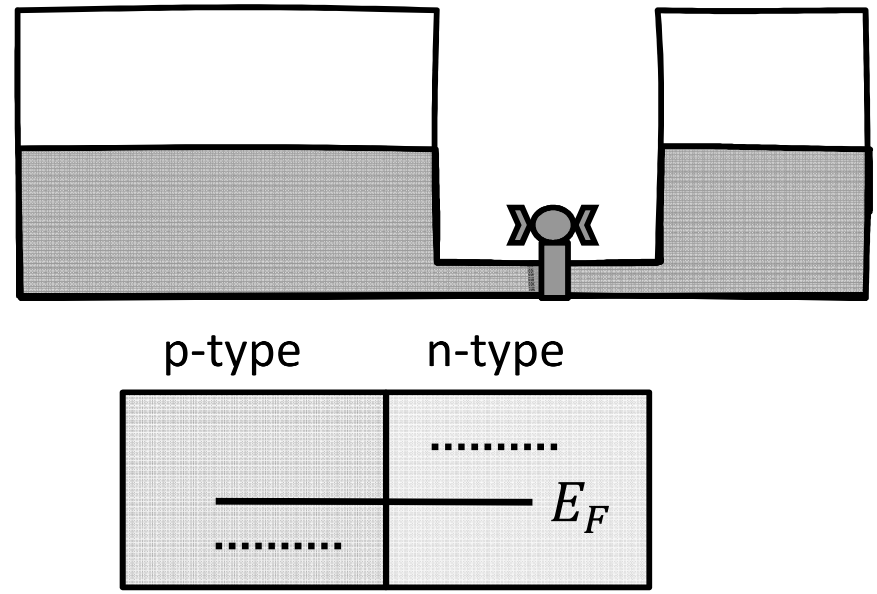
Similarly: $$E_{i2} = E_F - kT \, \ln(\frac{n_2}{n_i})$$

Therefore: $$E_{i1} - E_{i2} = kT \, \ln(\frac{n_2}{n_1})$$

$$V_2 - V_1 = \frac{1}{q}(E_{i1} - E_{i2}) = kT \, \ln(\frac{n_2}{n_1})$$

Fermi Energy





Density of state: $D_1(E)$ $D_2(E)$

Fermi dist.: $f_1(E)$ $f_2(E)$

$D_1(E)f_1(E)[D_2(E)(1-f_2(E)]$

$D_2(E)f_2(E)[D_1(E)(1-f_1(E)]$

then

$$D_1(E)f_1(E)[D_2(E)(1-f_2(E)] = D_2(E)f_2(E)[D_1(E)(1-f_1(E)]$$

$$\rightarrow \quad f_1(E) = f_2(E) \qquad \rightarrow \quad E_{F1} = E_{F2}$$

Non-Equilibrium Process

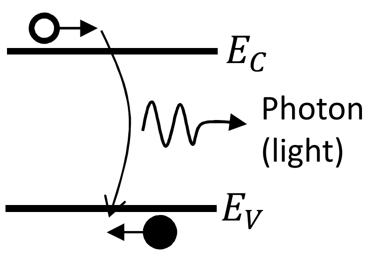
Whenever the thermal-equilibrium condition of a semiconductor system is disturbed $pn \neq n_i^2$ processes exist to restore the system to equilibrium

Generation and recombination processes act to change the carrier concentrations, and thereby indirectly affect current flow

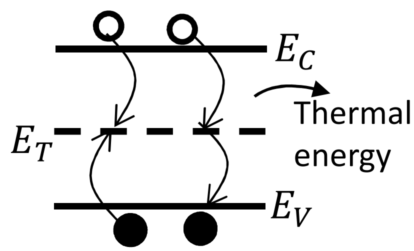
Recombination mechanisms: $pn > n_i^2$

Generation mechanisms: $pn < n_i^2$

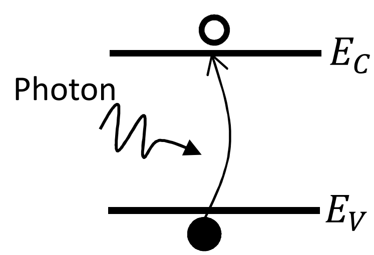
Direct



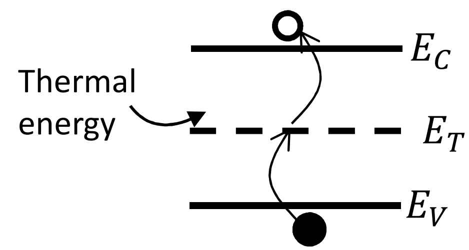
R-G Center



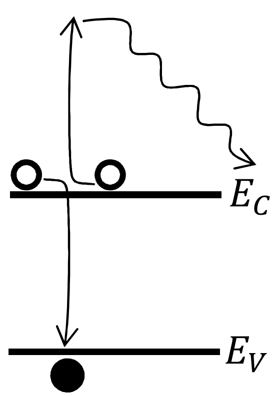
Band-to-band



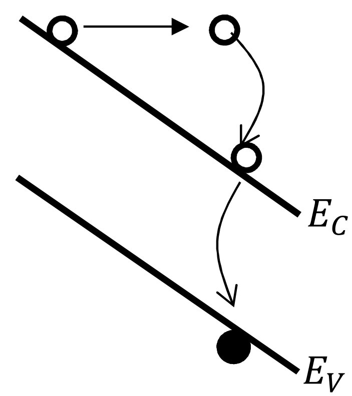
R-G Center



Auger

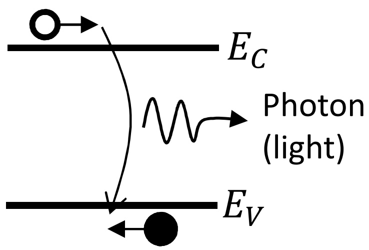


Impact Ionization

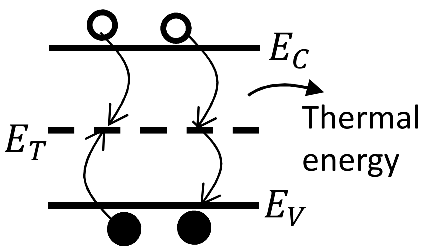


Recombination in Si is primarily via R-G centers

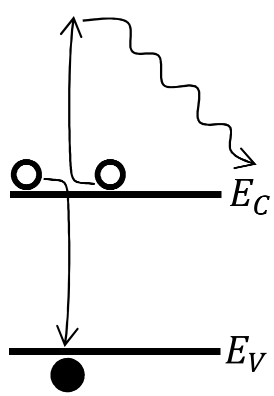
Recombination Mechanisms



Direct or Band to Band:
Basis for light emission devices
Photon (single particle of light) or multiple phonons (single quantum of lattice vibration – equivalent to saying thermal energy)

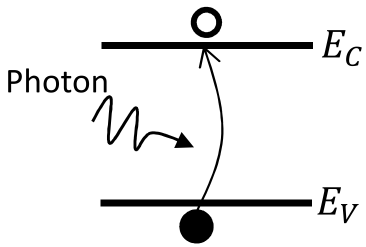


R-G Center:
Also known as Schockley-Read-Hall (SRH) recombination
Photon (single particle of light) or multiple phonons (single quantum of lattice vibration – equivalent to saying thermal energy)
Note: Trap level, Two steps: 1st Carrier is trapped at a defect/impurity, 2nd Carrier (opposite type) is attracted to the RG center and annihilates the 1st carrier

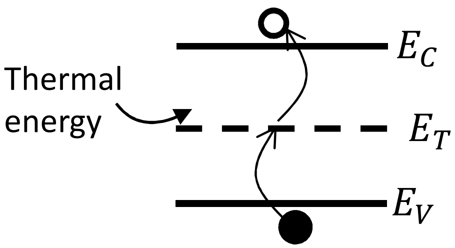


Auger:
Requires 3 particles, Two steps:
1st carrier and 2nd carrier of the same type collide instantly annihilating the electron hole pair (1st and 3rd carrier).
The energy lost in he annihilation process is given o the 2nd carrier.
2nd carrier gives off a series of phonons until it’s energy returns to equilibrium energy (E~Ec)

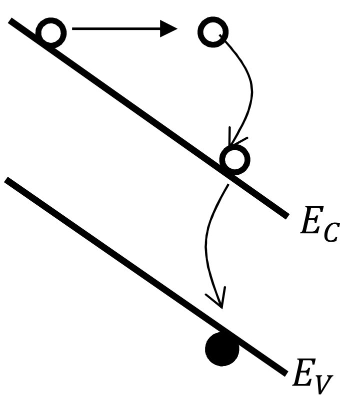
Generation Mechanisms



Direct or Band to Band:
Does not have to be a direct bandgap material
Mechanism that results in n_i
Basis for light absorption devices such as semiconductor photodetectors, solar cells, etc.



R-G Center:
Two steps:
A bonding electron is trapped at an unintentional defect/impurity generating a hole in the valence band
This trapped electron is then promoted to the conduction band resulting ina new electron-hole pair
Almost always detrimental to electronic devices



Impact Ionization:
Requires 3 particles and typically high electric fields
1st carrier is accelerated by high electric fields
Collides with a lattice atom
Knocks out a bonding electron
Creates an electron hole pair
What is it called when this process repeats and what device is it useful for?

Low-Level Injection

Excess Carrier Concentrations: $\Delta n \equiv n - n_0$ equilibrium values

$\Delta p \equiv p - p_0$

Charge neutrality condition: $\Delta n = \Delta p$

Low-Level Injection: Often the disturbance from equilibrium is small, such that the majority-carrier concentration is not affected significantly:

For an n-type material $|\Delta n| = |\Delta p| \ll n_0$ so $n \cong n_0$

For a p-type material $|\Delta n| = |\Delta p| \ll p_0$ so $p \cong p_0$

However, the minority carrier concentration can be significantly affected

Indirect Recombination Rate

Suppose excess carriers are introduced into an n-type Si sample (e.g. by temporarily shining light onto it) at time t = 0. How does p vary with time t > 0?

1. Consider the rate of hole recombination via traps:

$$\left.\frac{\partial p}{\partial t}\right|_R = -c_p N_T p$$

$c_p =$ Capture coefficient

$N_T =$ # or traps

2. Under low-level injection conditions, the hole generation rate is not significantly affected:

$$\left.\frac{\partial p}{\partial t}\right|_G \cong \left.\frac{\partial p}{\partial t}\right|_{G_{equil.}} = -\left.\frac{\partial p}{\partial t}\right|_{R_{equil.}} = -c_p N_T p_0$$

3. The net rate of change in p is therefore

$$\left.\frac{\partial p}{\partial t}\right|_{R-G} = \left.\frac{\partial p}{\partial t}\right|_R + \left.\frac{\partial p}{\partial t}\right|_G = -c_p N_T p + c_p N_T p_0 = -c_p N_T (p - p_0)$$

$$\left.\frac{\partial p}{\partial t}\right|_{R-G} = \frac{\Delta p}{\tau_p} \quad \text{where} \quad \tau_p \equiv \frac{1}{c_p N_T}$$

Relaxation to Equilibrium State

Consider a semiconductor with no current flow in which thermal equilibrium is disturbed by the sudden creation of excess holes and electrons. The system will relax back to the equilibrium state via the R-G mechanism:

for electrons in p-type material: $$\frac{\partial n}{\partial t} = -\frac{\Delta n}{\tau_n} \qquad \tau_n \equiv (c_n N_T)^{-1}$$

for holes in n-type material: $$\frac{\partial p}{\partial t} = -\frac{\Delta p}{\tau_p} \qquad \tau_p \equiv \left(c_p N_T\right)^{-1}$$

The minority carrier lifetime τ is the average time an excess minority carrier "survives" in a sea of majority carriers.
τ ranges from 1ns to 1ms in Si and depends on the density of metallic impurities (contaminants) such as Au and Pt, and the density of crystalline defects. These deep traps capture electrons or holes to facilitate recombination and are called recombination-generation centers.

Example: Photoconductor

Consider a sample of Si doped with $10^{16}\ cm^{-3}$ boron, with recombination lifetime 1µs. It is exposed continuously to light, such that electron-hole pairs are generated throughout the sample at the rate of 10^{20} per cm^3 per second, i.e. the generation rate $G_L = 10^{20}/cm^3/s$

1. What are p_0 and n_0? $\quad p_0 = 10^{16}\ \mathrm{cm}^{-3} \qquad n_0 = 10^4\ \mathrm{cm}^{-3}$

2. What are Δn and Δp?

$$G_L = \Delta n/\tau_n = 10^{20}$$

$$\Delta n = \Delta p = G_L\tau = 10^{20} \times 10^{-6} = 10^{-14} cm^{-3}$$

3. What are n and p?

$$p = p_0 + \Delta p = 10^{16} + 10^{14} \approx 10^{16} cm^{-3}$$

$$n = n_0 + \Delta n = 10^4 + 10^{14} \approx 10^{14} cm^{-3}$$

3. What is the np product? $\quad np = 10^{30}\ cm^{-3} \gg n_i^2$

Net Recombination Rate (General Case)

For arbitrary injection levels and both carrier types in a non-degenerate semiconductor, the net rate of carrier recombination is:

$$\frac{\partial \Delta n}{\partial t} = \frac{\partial \Delta p}{\partial t} = -\frac{pn - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

where $n_1 \equiv n_i e^{(E_T - E_i)/kT}$ and $p_1 \equiv n_i\, e^{(E_i - E_T)/kT}$

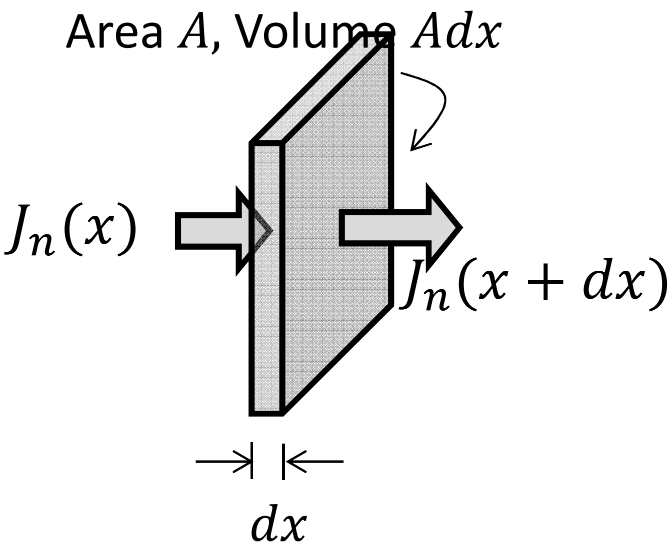
For low level injection:

for electrons in p-type material: $\frac{\partial n}{\partial t} = -\frac{\Delta n}{\tau_n}$

for holes in n-type material: $\frac{\partial p}{\partial t} = -\frac{\Delta p}{\tau_p}$

Derivation of Continuity Equation

Consider carrier-flux into/out-of an infinitesimal volume:



$$Adx\left(\frac{\partial n}{\partial t}\right) = -\frac{1}{q}\left[J_n(x)A - J_n(x+dx)A\right] - \frac{\Delta n}{\tau_n}Adx$$

$$J_n(x+dx) = J_n(x) + \frac{\partial J_n(x)}{\partial x}dx$$

$$\rightarrow \quad \frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n}$$

Continuity Equation:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p(x)}{\partial x} - \frac{\Delta p}{\tau_p} + G_L$$

Minority Carrier Diffusion Equation

The minority carrier diffusion equations are derived from the general continuity equations, and are applicable only for minority carriers.

Simplifying assumptions

1. The electric field is small, such that

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx} \sim qD_n\frac{dn}{dx}$$ in p-type material

$$J_p = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx} \sim qD_p\frac{dp}{dx}$$ in n-type material

2. n_0 and p_0 are independent of x (uniform doping)

3. low-level injection conditions prevail

Starting with the continuity equation for electrons

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n(x)}{\partial x} - \frac{\Delta n}{\tau_n} + G_L \;\rightarrow\; \frac{\partial(n_0 + \Delta n)}{\partial t} = \frac{1}{q}\frac{\partial}{\partial x}\left[qD_n\frac{\partial(n_0+\Delta n)}{\partial x}\right] - \frac{\Delta n}{\tau_n} + G_L$$

$$\frac{\partial \Delta n}{\partial t} = D_n\frac{\partial^2 \Delta n}{\partial x^2} - \frac{\Delta n}{\tau_n} + G_L$$

Minority Carrier Diffusion Equation

The subscript "n" or "p" is used to explicitly denote n-type or p-type material, e.g.

p_n is the hole (minority-carrier) concentration in n-type material

n_p is the electron (minority-carrier) concentration in p-type material

Thus the minority carrier diffusion equations are

$$\frac{\partial \Delta n_p}{\partial t} = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} + G_L \qquad \text{in p-type material}$$

$$\frac{\partial \Delta p_n}{\partial t} = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p} + G_L \qquad \text{in n-type material}$$

Simplifications (Special Cases):

Steady state:

$$\frac{\partial \Delta n_p}{\partial t} = 0 \ , \qquad \frac{\partial \Delta p_n}{\partial t} = 0$$

No diffusion current:

$$D_n \frac{\partial^2 \Delta n_p}{\partial x^2} = 0 \ , D_p \frac{\partial^2 \Delta p_n}{\partial x^2} = 0$$

No R-G:

$$\frac{\Delta n_p}{\tau_n} = 0 \ , \frac{\Delta p_n}{\tau_p} = 0$$

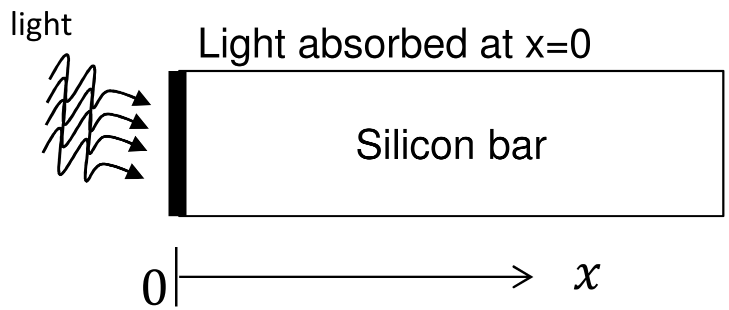
No light:

$$G_L = 0$$

Example

Consider the special case:

1. constant minority-carrier (hole) injection at $x = 0$
2. steady state; no light absorption for $x > 0$: $\Delta p_n(0) = \Delta p_n 0$



$$0 = D_p \frac{\partial^2 \Delta p_n}{\partial x^2} - \frac{\Delta p_n}{\tau_p}$$

$$\frac{\partial^2 \Delta p_n}{\partial x^2} = \frac{\Delta p_n}{D_p \tau_p} = \frac{\Delta p_n}{L_p^2}$$

L_p is the hole diffusion length: $L_p \equiv \sqrt{D_p \tau_p}$

The general solution to the equation is

$$\Delta p_n(x) = Ae^{-x/L_p} + Be^{x/L_p}$$

where A, B are constants determined by boundary conditions:

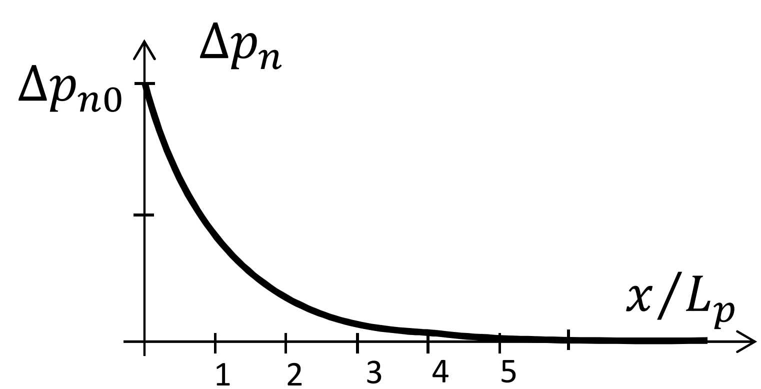
B.C.

$$\Delta p_n(\infty) = 0 \quad \rightarrow \quad B = 0$$

$$\Delta p_n(0) = \Delta p_{n0} \quad \rightarrow \quad A = \Delta p_{n0}$$

Hence solution is:

$$\Delta p_n(x) = \Delta p_{n0} e^{-x/L_p}$$



Example – Minority Carrier Diffusion length

Physically, L_p and L_n represent the average distance that minority carriers can diffuse into a sea of majority carriers before being annihilated.

Q: Find L_p if $N_D = 10^{16} cm^{-3}$; $\tau_p = 10^{-6}\ sec$

$$L_p = \sqrt{D_p \tau_p}$$

$$D_p = \frac{kT}{q} \mu_p$$

$$\mu_p = 400\ cm^2/Vs$$

$$D_p = 10\ cm^2/s$$

$$L_p = 30\ \mu m$$

Quasi-Fermi Levels

Whenever $\Delta n = \Delta p \neq 0, np \neq n_i^2$. However, we would like to preserve and use the relations:

$$n = n_i e^{(E_F - E_i)/kT} \qquad p = n_i e^{(E_i - E_F)/kT}$$

These equations imply $np \neq n_i^2$, however. The solution is to introduce two quasi-Fermi levels F_N and F_P such that

$$n = n_i e^{(F_N - E_i)/kT} \qquad p = n_i e^{(E_i - F_P)/kT}$$

$$F_N = E_i + kT \ln\left(\frac{n}{n_i}\right) \qquad F_P = E_i - kT \ln\left(\frac{p}{n_i}\right)$$

Example: Quasi-Fermi Levels

Consider a Si sample with $N_D = 10^{17} cm^{-3}$ and $\Delta n = \Delta p = 10^{14}\ cm^{-3}$.

What are p and n ?

$$n_0 = N_D = 10^{17} cm^{-3}, p_0 = n_i^2/n_0 = 10^3\ cm^{-3}$$

$$n = n_0 + \Delta n = 10^{17} + 10^{14} \approx 10^{17} cm^{-3}$$

$$p = p_0 + \Delta p = 10^3 + 10^{14} \approx 10^{14} cm^{-3}$$

What is the np product ?

$$np = 10^{31} cm^{-3}$$

Find F_N and F_P :

$$F_N = E_i + kT \ln\left(\frac{n}{n_i}\right) \rightarrow F_N - E_i = kT \ln 10^7 = 0.42\ eV$$

$$F_P = E_i + kT \ln\left(\frac{p}{n_i}\right) \rightarrow E_i - F_P = kT \ln 10^4 = 0.24\ eV$$