Time Domain Modeling of Lossy Interconnects

Christer Svensson, *Member, IEEE,* and Gregory E. Dermer

***Abstract*—A new model for dielectric loss, suitable for time domain modeling of printed circuit boards, is proposed. The model is based on a physical relaxation model. Complete time domain modeling of skin effect and dielectric losses in FR-4 boards are demonstrated and experimentally verified. Finally, the developed model is used to predict that FR-4 boards are useful for data rates up to 10 Gb/s.**

***Index Terms*—Dielectric loss, interconnect, maximum data-rates, printed circuit boards, time domain model, transmission line.**

I. Introduction

When utilizing printed circuit boards for multi-gigabit wide-band signals, both skin-effect loss and dielectric loss become important. The loss not only introduces attenuation of the signals but, far more important, distortion [1]. The distortion will in turn introduce inter-symbol interference, which seriously limits the data rate or calls for equalization. It therefore becomes very important to accurately characterize the time domain behavior of transmission lines on PC boards. However, existing modeling methods fail to correctly describe the time-domain effects of these losses [2]–[4]. The general problem is that the models have a noncausal behavior. Arabi *et al.* solve this problem in the case of skin effect and demonstrate causal time-domain behavior of transmission lines with skin effect. However, they fail to formulate a corresponding model for dielectric loss. Alonso *et al.* present a modeling method based on general matching of measured transmission line characteristics to circuit models, but do not explicit treat dielectric loss. Hjellen discuss dielectric loss in circuit boards, but only in the frequency domain. One reason for the lack of time-domain models is that gigahertz frequencies have been used primarily in narrow-band systems (microwave systems) in the past, so all methods and tools has been developed for the frequency domain. Also, our knowledge of material parameters, such as dielectric loss, is based on frequency domain measurements.

One method dealing with this problem is to use a frequency domain description and then use Fourier transformation to convert the calculated signals to the time domain. This needs some care though, as a "nonphysical" loss model leads to a nonphysical result [2]. Particularly, an overly simplified model of skin effect or dielectric loss gives rise to noncausal behavior of the transmission line. Arabi introduced a physically correct model of skin effect loss, and demonstrated that it behaved well [2]. So far, a corresponding model for dielectric loss has not been demonstrated. Some recent tools support this method, by allowing a general s-parameter model in frequency domain as input to a system, which then calculates time response through Fourier transformation [5].

Another method to deal with the problem is to develop an equivalent circuit, which approximates the correct frequency behavior of the transmission line, and then uses this circuit in a circuit simulator (like SPICE) [3], [4]. However, it is not very easy to describe the complex frequency dependence of skin effect (square root of frequency) or of dielectric loss (which may vary between different materials). These methods tend to be accurate only within a certain frequency range (or around a certain center frequency), which makes them less useful for broadband digital signals.

We have developed a physically correct model of dielectric loss in the frequency domain, and demonstrated that it is well-behaved after transformation to the time domain. In Section II, we describe the modeling principles and in Section III, the method of calculation. In Section IV, we describe the specific model for FR-4 boards, which is verified by experiments in Section V. We present some analytical results in Section VI and discuss applications of our model in Section VII. In Section VIII, we use our model to predict the upper limit to the data rate in FR-4 boards in section. Finally we present some conclusions.

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II. Modeling of Skin Effect and Dielectric Loss in a Transmission Line

The transfer function of a transmission line can be written as

$$H(\omega) = e^{-\gamma x} \tag{1}$$

$$\gamma = \sqrt{(j\omega l + r)(j\omega c)} \tag{2}$$

where we have introduced inductance, capacitance and resistance per unit length, l, c and r and the wire length x. Furthermore, we assume that the dielectric loss is included in the capacitance as a complex dielectric constant, $c(\varepsilon)$, where ε is the complex dielectric constant. In order to be physically feasible (giving rise to real and causal time domain results), H must fulfill the requirements

$$H(-\omega) = H^*(\omega) \tag{3}$$

$$h(t) = 0, \qquad \text{for } t < 0. \tag{4}$$

This puts certain constraints to the frequency dependence of the complex values of $j\omega l + r$ and $j\omega c$ respectively. Arabi [2] showed that by describing both the resistive and the inductive part of skin effect properly, it was possible to fulfill the constraints above. This can be accomplished by letting r be complex

$$r = r_s (1 + j)\sqrt{\omega}. \tag{5}$$

For dielectric loss, we introduce a model based on physics. In the polymers used as dielectric in PCB's, dielectric loss is dominated by relaxation loss [6]. The complex dielectric constant in presence of relaxation loss can be described by

$$\varepsilon = \frac{a_r}{1 + j\omega\tau_r} \tag{6}$$

where

- ε contribution to the dielectric constant of the particular loss mechanism;
- a_r strength;
- τ_r relaxation time.

The value of τ_r varies in a very wide range between different materials [6]. Furthermore, many materials have multiple relaxation processes with different relaxation times. For simple polymers, a relatively narrow range of relaxation times (for each material) is observed as a broad peak in the frequency dependence of the imaginary part of ε. For more complex materials, as for example PCB dielectrics, no peaks are observed. Instead the imaginary part of ε shows a slow variation with frequency over a very large frequency range. To describe this case, we have chosen to use multiple relaxation processes with a broad distribution of relaxation times

$$\varepsilon = \sum_i \frac{a_i}{1 + j\omega\tau_i}. \tag{7}$$

By adjusting the a_i, τ_i pairs of the different processes, it is possible to describe any form of slow frequency dependence of ε. This complex dielectric constant is then used in the expression for c, and thus in (1) and (2).

III. Calculation of Step and Pulse Responses

The step and pulse responses of a transmission line with losses were calculated as follows. A well-behaved step function was defined through

$$g = \tfrac{1}{2}\left(1 + erf\left(b_1 (t - t_1)\right)\right) \tag{8}$$

where t_1 and b_1 are parameters giving the starting time and the steepness of the step, respectively. A pulse was defined as the difference between two step functions with different starting times. This input function was then Fourier transformed to G and combined with the transmission line response function, H, defined above, to form the output response, HG. Finally the time domain output response is found from the inverse Fourier transform

$$o(t) = ifft(HG). \tag{9}$$

All calculations were performed using Matlab.

The usefulness of this approach is demonstrated in Fig. 1, where we compare the step response calculated from a traditional dielectric constant approach (assuming $\varepsilon = \varepsilon_r(1 - j\tan(\delta))$, independent of frequency), Fig. 1(a), with the new approach (using the formulation for ε used for FR-4 below) [Fig. 1(b)]. In this example we have set $r_{\mathrm{DC}} = 0$. From this figure we can clearly see that the new formulation gives causal results. While the previous formulation do not.

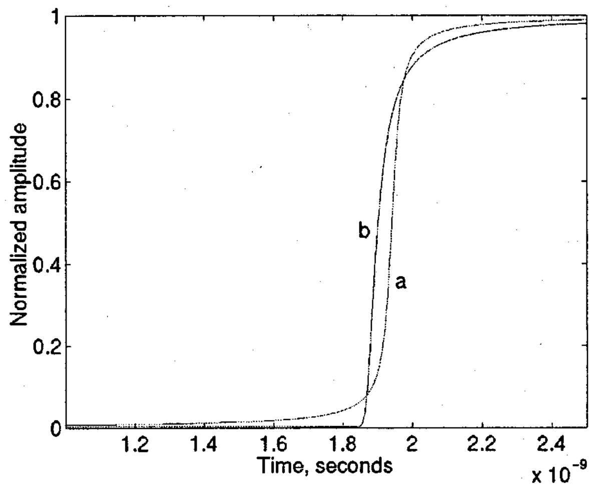

Fig. 1. (a) Step response calculated using traditional approach. (b) Step response calculated using new approach.

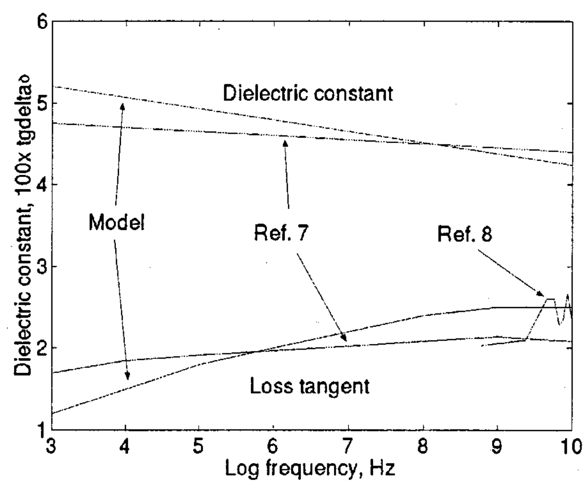

Fig. 2. Dielectric constant and loss tangent versus frequency, from experiments, and the present model.

IV. Modeling of a Case—FR-4

The most common dielectric material in printed circuit boards today is FR-4. In Fig. 2, we show the dielectric constant and dielectric loss of FR-4 from two sources [7], [8]. We see that the loss varies very slowly with frequency over a very large frequency range. Instead of using a large number of discrete relaxation times to describe this behavior, we have chosen a continuous distribution of relaxation times. Thus, we describe the dielectric constant with

$$\varepsilon = \varepsilon_1 + \int_{\tau_1}^{\tau_2} \frac{a/\tau}{1 + j\omega\tau}\, d\tau \tag{10}$$

where the sum in (7) has been replaced by an integral and we have added ε_1 being the nonrelaxation contribution to the dielectric constant. The individual strengths a_i have been replaced by a strength distribution function, chosen to a/τ. τ_1 and τ_2 are the limits to the relaxation time distribution. This integral can be solved and yields

$$\varepsilon = \varepsilon_1 + a\left(\frac{1}{2}\ln\left(\frac{\tau_2^2\left(1+\omega^2\tau_1^2\right)}{\tau_1^2\left(1+\omega^2\tau_2^2\right)}\right) - j\,\mathrm{arctg}\left(\omega\tau_2\right) + j\,\mathrm{arctg}\left(\omega\tau_1\right)\right). \quad (11)$$

By choosing appropriate values of the parameters in (10) and (11), we can make (11) fit the measured loss relatively well, see Fig. 2. The parameters used are $\varepsilon_1 = 4.1$, $\tau_1 = 1.6$ ps, $\tau_2 = 1.6$ ms, and $a = 0.06$. Note, that also the real part of the dielectric constant follows the experimental data reasonably well.

Concerning the skin effect loss, we model that in the conventional way [9]. For a microstrip, the dc-resistance per unit length (of a single trace) is given by

$$r_{\mathrm{DC}} = \frac{\rho}{wh} \quad (12)$$

where
ρ resistivity;
w wire width;
h height.
The onset of skin effect (occurring when the skin depth is equal to the height) occurs at f_s

$$f_s = \frac{\rho}{h^2\pi\mu_0} \quad (13)$$

where μ_0 is the magnetic permittivity (for vacuum). Using r_{DC} and f_s, we may express the resistance per unit length at frequencies above f_s according to (5), with r_s given by (where we have added a factor of two to approximate the resistance in the return path [8])

$$r_s = \frac{2r_{\mathrm{DC}}}{\sqrt{2\pi f_s}}. \quad (14)$$

Finally, we assume the transmission line to have a characteristic impedance of 50 Ω, which together with the average dielectric constant give values of l and c.

Actual parameters used in our calculation are for the dielectric constant the parameters shown above, and for inductance and nominal capacitance, $l = 0.355$ μH/m, and $c_0 = 0.137$ nF/m, which gives $Z_0 = 50\ \Omega$ and $v =$ (velocity of light)/sqrt(ε_r) with $\varepsilon_r = 4.2$. The complex dielectric constant is then incorporated by setting the real capacitance to $c = c_0(\varepsilon/\varepsilon_r)$. r_{DC} was chosen to 4.7 Ω and f_s to 54.4 MHz, calculated from wire width and height of 203 μm and 17.8 μm, respectively [9]. [In this reference, a stripline was used, so h is replaced by $h/2$ in (13)].

In Fig. 3 we show calculated step responses for a 40 cm trace on FR-4, with skin effect only, with dielectric loss only and with both mechanisms active. We can see that both mechanisms are important in this case. We can also note that skin effect has a larger effect on the tail of the step response, whereas dielectric loss has a larger effect on the rise time. In Fig. 4 we show the corresponding attenuation values versus frequency.

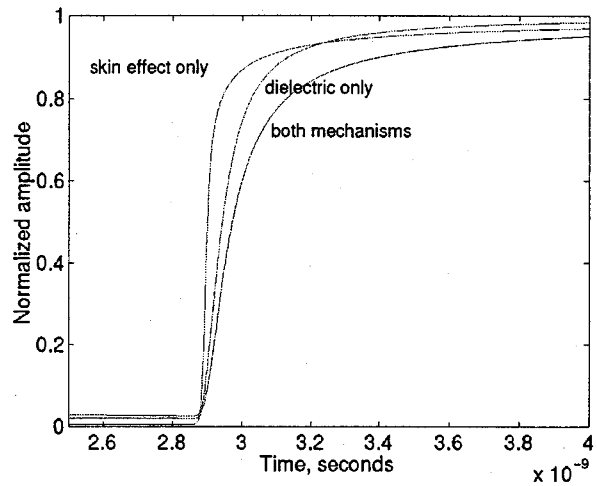

Fig. 3. Step response from skin effect or dielectric loss only, and with both active.

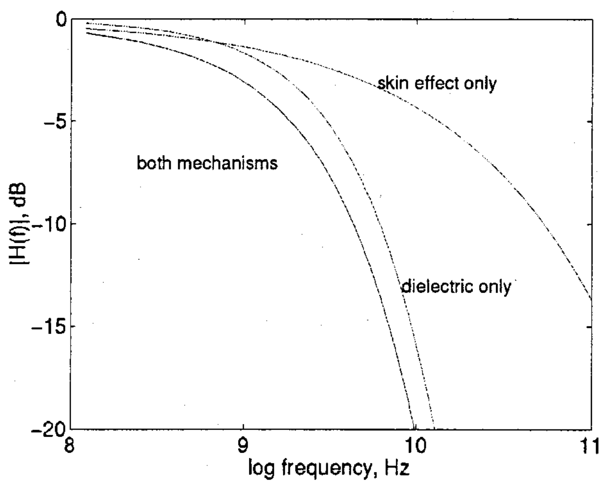

Fig. 4. Absolute value of transfer function versus frequency with skin effect or dielectric loss only, and with both active.

V. Verification Experiments

We have performed measurements on a 10 in long embedded microstrip trace on an FR-4 board using a Tektronix 11 801B sampling oscilloscope equipped with an SD-24 step generator/sampling head. This instrument is capable of generating and observing a step with a risetime of less than 40 ps. The trace was terminated in both ends. When coupled to the trace under study via edge launch connectors to minimize transmission line discontinuities, the composite risetime of the instrumentation and trace launch was approximately 40 ps The propagation delay of the 10 in trace, computed from TDR analysis, was 1.64 ns for the 10 in length. The characteristic impedance was found to be $Z_0 = 58.6\ \Omega$. From these measurements, the dielectric constant was computed to be $\varepsilon_1 = 3.75$, with line constants $l = 0.378$ μH/m and $c_0 = 0.110$ nF/m. The dc resistance was in this case measured as 0.871 Ω, giving $r_{\mathrm{DC}} = 3.43$ Ω/m. From the width of the trace, 0.01″ and r_{DC} we calculated $h = 19.5$ μm. This corresponds to standard (1/2) ounce copper thickness. Using this value of h we find $f_s = 11.3$ MHz. In Fig. 5 we show the measured far end step response, compared

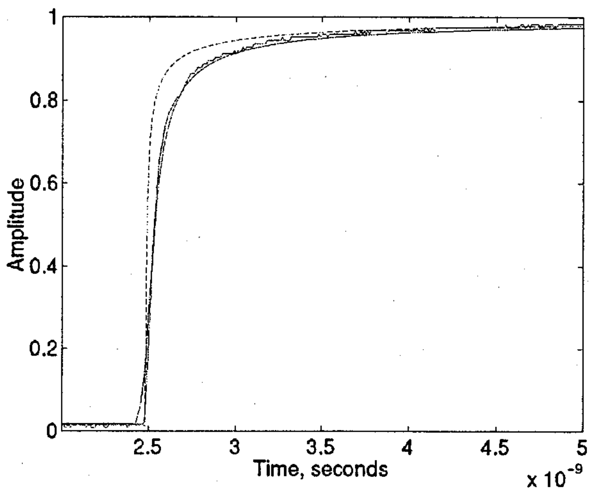

Fig. 5. Measured and calculated (smooth, solid line) response from a 10 in trace on an FR-4 board. The dotted curve is calculated without dielectric loss.

to the calculated step response using the parameters calculated above. We have also indicated the calculated step response without dielectric loss in the figure. We can conclude that the calculated response agrees very well with the measured one, and that the dielectric loss has a large impact in this case. This also indicates that other loss mechanisms, as for example surface roughness loss, are less important, in agreement with the findings in [10].

VI. Some Analytical Results

By a series expansion of γ from (2) with skin effect only, the resistive loss can be described as $\exp(-rx/2Z_0)$ [9]. Inserting r from (5), noticing that $1+j = \mathrm{sqrt}(2j)$ yields

$$\begin{aligned} H &\propto e^{-j\omega\sqrt{lc}x} e^{-(rx/2Z_0)} \\ &= e^{-j\omega\sqrt{lc}x} e^{-(r_s x/\sqrt{2}Z_0)\sqrt{j\omega}} \\ &= e^{-s\sqrt{lc}x} e^{-(r_s x/\sqrt{2}Z_0)\sqrt{s}} \end{aligned} \tag{15}$$

where we have changed to Laplace notation in the third part of the expression. After removing the first part of the Laplace expression (corresponding to a time delay) the remainder corresponds to the known step response [1]

$$o(t) = \mathrm{erfc}\left(\frac{r_s x}{2\sqrt{2}Z_0\sqrt{t}}\right) \tag{16}$$

where erfc is the complementary error function. This equation can be used for the prediction of the maximum data-rate that can be carried on a wire. Assuming simple NRZ data (non return to zero, i.e., a simple stream of binary symbols) with the symbol length t a fixed argument in (16) corresponds to a fixed eye opening in an eye diagram. With a bit-rate B equal to $1/t$ we have

$$B \propto \frac{Z_0^2}{r_s^2}\frac{1}{x^2} \tag{17}$$

or, by combining (17) with (12), (13) and (14)

$$B \propto \frac{1}{\rho}\frac{A}{x^2} \tag{18}$$

where we have assumed constant Z_0 and replaced w^2 with the wire cross section A (as the wire aspect ratio is given by Z_0 in practice). This is the result in [1].

If we instead consider (2) with dielectric loss only, we find after reordering, expansion and change to Laplace notation

$$H \propto \exp\left(-ax\sqrt{lc_0}s\ln\left(\frac{\tau_1(1+s\tau_2)}{\tau_2(1+s\tau_1)}\right)\right). \tag{19}$$

Unfortunately, we have not found an analytical step response corresponding to this function. We can conclude, though, that the step response is invariant under constant ax, which means that a reduction of the loss (expressed as a) by half, leads to that we can accept double wire length for the same step response. Including also $\sqrt{lc_0}$ in the discussion, and noting that $\sqrt{lc_0}$ is equal to the inverse of the signal velocity on the wire, v, we conclude that the step response is invariant under constant ax/v.

By analyzing the step response calculated numerically with the method above, we find the following empirical relation:

$$o(t) \approx 1 - e^{-\sqrt{(t/t_1)}} \tag{20}$$

$$t_1 = 0.27\frac{ax}{v} = 0.27ax\sqrt{lc_0}. \tag{21}$$

Using this expression we find that the maximum data rate can be expressed as

$$B \propto \frac{1}{a\sqrt{lc_0}}\frac{1}{x}. \tag{22}$$

We can thus conclude, that pure skin effect loss leads to a maximum data rate proportional to x^{-2} and pure dielectric loss leads to a data rate proportional to x^{-1}. This means that skin effect dominates in very long wires, whereas dielectric loss dominates at short distances. We can also conclude that the maximum data-rates are proportional to the inverse of metal resistivity and dielectric loss in the two cases, respectively.

VII. Applications

We demonstrated how our proposed model for dielectric loss can be used for correctly calculating the step response of a transmission line. The model is therefore reliable for time-domain calculations. The model can be used for general nonlinear time-domain circuit simulations by utilizing a transient simulator using convolution simulation. One example of such a simulator is Agilent ADS [5]. The transmission line is implemented in the convolution simulator through a table of its frequency-dependent s-parameters [which are calculated from (1) and (2), with the model of ε included].

As our model is a general model of the dielectric constant, we also expect that it can be implemented in a general field-solver (which must allow complex and frequency-dependent dielectric constant) to obtain the frequency-dependent s-parameters of any PCB structure (capacitor, resonator, multiconductor etc.). An example of a field-solver with the desired properties is HFSS from Ansoft [11].

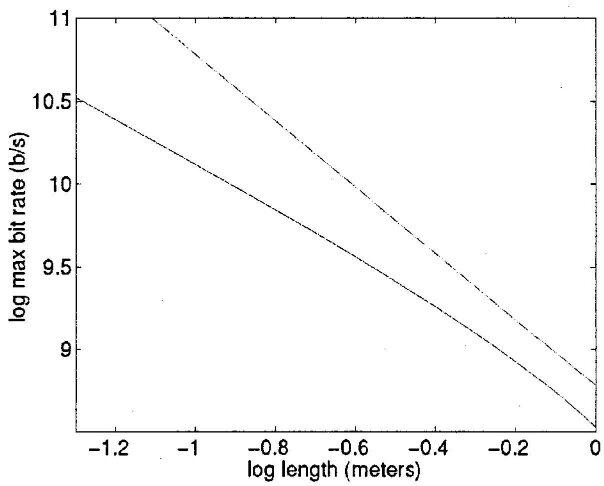

Fig. 6. Max bit-rate versus wire length, assuming an eye opening of 0.68. The upper curve is for skin effect only and the lower one includes both skin effect and dielectric loss.

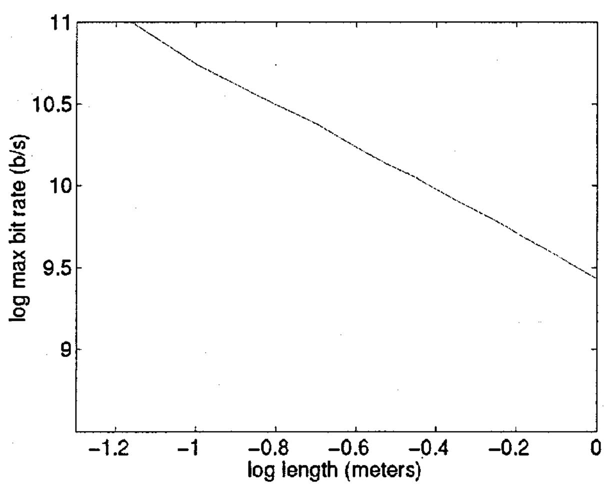

Fig. 7. Max bit-rate versus wire length assuming an eye opening just approaching zero.

VIII. Maximum Usable Data Rates

Signal distortion introduced by the frequency dependent attenuation in the wires will limit the data rate on the wire [1]. The degree to which the data rate is limited depends on the degree of sophistication of the transmitter and receiver. For the simplest case, NRZ signals between a simple logic driver and a simple comparator receiver, we need to have a clear data eye. The worst case occurs when comparing a single one following a large number of zeros with a single zero following a large number of ones. Miller and Özatkas proposed that we need a minimum relative pulse amplitude (at the end of the data) of 0.84, corresponding to a window size of 0.68, for reliable data detection. For the pure skin effect case this corresponds to $t_D = 50 * (RC/4)$, where t_D is the bit length of the data and R and C are the dc values of transmission line resistance and capacitance [1]. More advanced receivers may contain an equalizer, partly compensating for the inter-symbol interference introduced by the distortion. If we assume that the receiver still should be relatively simple (as we consider very high data rates), we may demand that data recovery should be possible with a reasonably simple equalizer, compensating for short range inter-symbol interference.

We have calculated the maximum data rates obtainable on a circuit board versus the wire length, assuming a "clean" eye of size 0.68 as described above. This was done by calculating the step response of the wire using our model and note the time at which the step has reached 0.84 of its final value. The maximum data rate is then taken as the inverted value of this time. Parameters used are the same as for the experimental FR-4 board above. Some results are depicted in Fig. 6 (lower curve). We note that the maximum data rate goes from about 300 Mb/s for a 1 m long wire to about 30 Gb/s for a wire 6 cm in length. For comparison we show the corresponding data rate if the dielectric loss is omitted (skin effect only). We note that for wires of about 10 cm in length, dielectric loss reduces the data rate about 4×, whereas the reduction is smaller for longer wires. If a smaller step response is accepted the bit-rate can be increased. In Fig. 7 we show an example where the step response has reached 0.5 of its final value (eye opening = 0).

From these simulations we find that data rates of up to 10 Gb/s are easily attainable for short board traces (<10 cm). For longer traces, up to 0.5 m, the data rate limit without equalization is of the order of 1 Gb/s. We also note, that the dielectric loss has a very strong impact on the maximum data rate, especially for shorter traces (a 10 cm trace may carry nearly 50 Gb/s without dielectric loss). If we assume an eye opening just approaching zero (which can be handled by a quite simple equalizer), considerably higher data rates are possible. 10 Gb/s can then be run on traces up to about 0.4 m.

IX. Conclusion

We introduced a new model of dielectric loss in transmission lines, based on a physical model of relaxation loss. The model was applied to a Fourier transform method for the calculation of step responses in printed circuit board traces. The model was verified by the demonstration of its causality and by comparison to experimental measurements. Simple analytical expressions for maximum data rates on wires were discussed. We explained how the new model can be applied to fully nonlinear transient simulations including lossy transmission lines and to general field-solvers for more complex structures, using commercially available tools. Finally we used the model to predict data rates of ordinary FR-4 boards, demonstrating the importance of dielectric loss for the maximum data rate and concluding that rates of 10 Gb/s are feasible on reasonably sized FR-4 boards.

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