Session 4: Analog Circuits

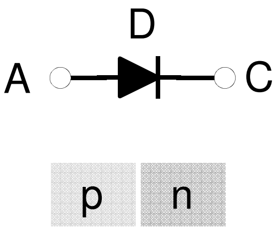
BJT
Biasing
Single stage amplifier

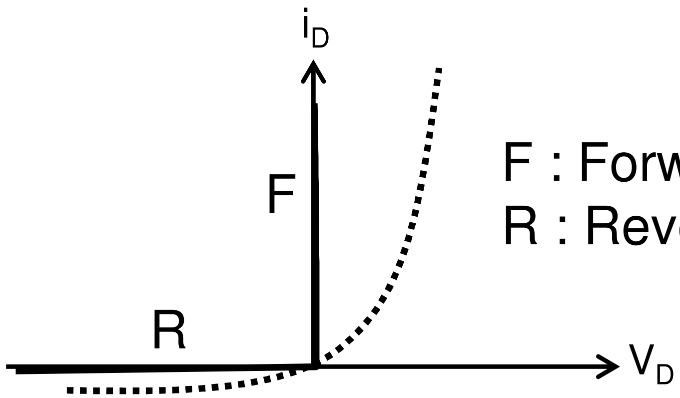
Outline

- BJT Amplifier

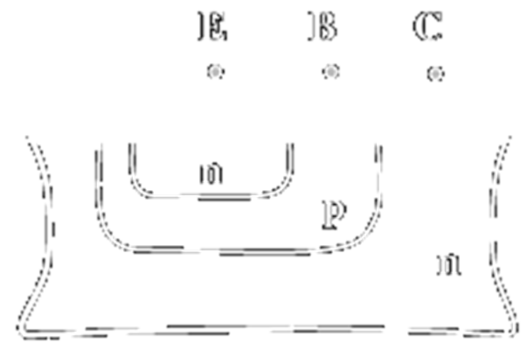
BJT: Bipolar Junction Transistor

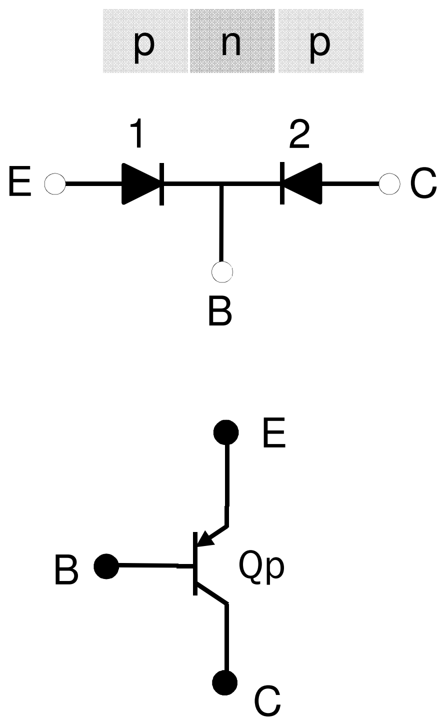
1. Lab
2. Power
3. Ser/Parl
4. Small sig.
5. Applic

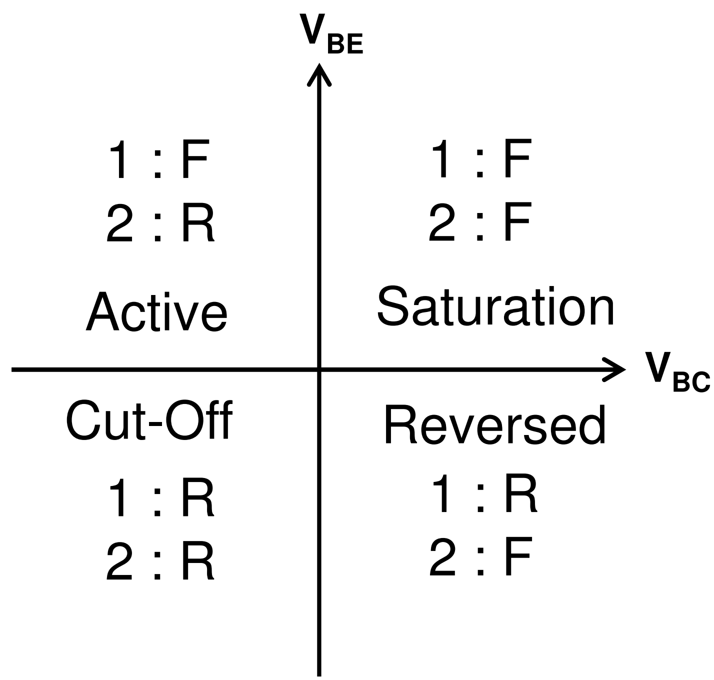


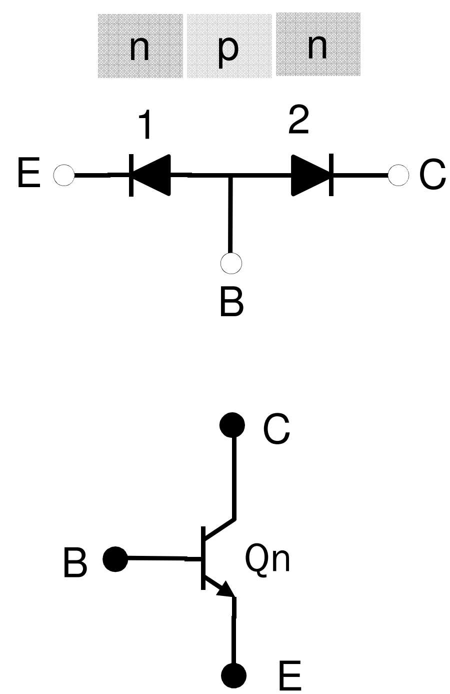


F : Forward
R : Reverse

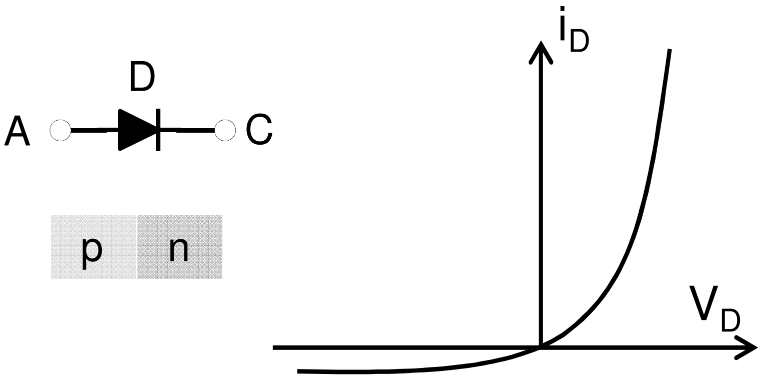








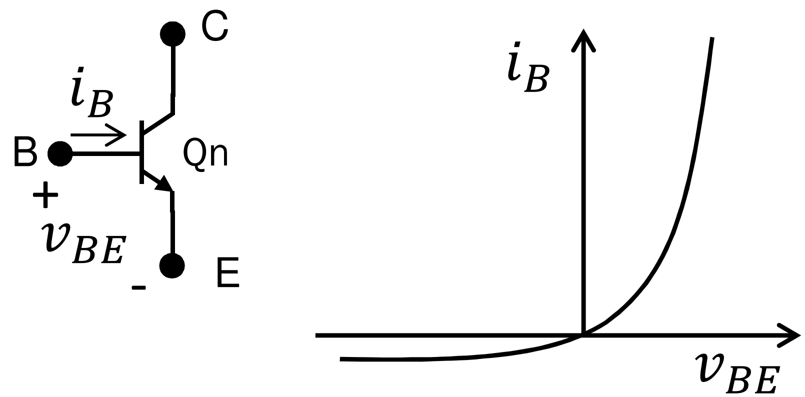
BJT: Bipolar Junction Transistor



$$i_D \propto e^{v_D/nV_T}$$

$$\frac{\Delta v_D}{\Delta T} \approx -2\frac{mV}{K}\bigg|_{i_D=cte}$$

$$\frac{i_D(T_2)}{i_D(T_1)} \approx 2^{(T_2-T_1)/10K}$$



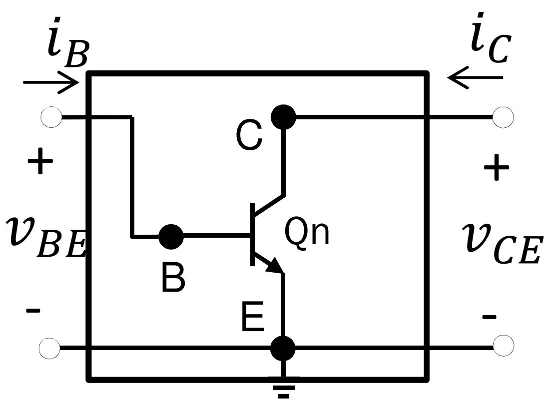
$$i_B \propto e^{v_{BE}/nV_T}$$

$$\frac{\Delta v_{BE}}{\Delta T} \approx -2\frac{mV}{K}\bigg|_{i_E=cte}$$

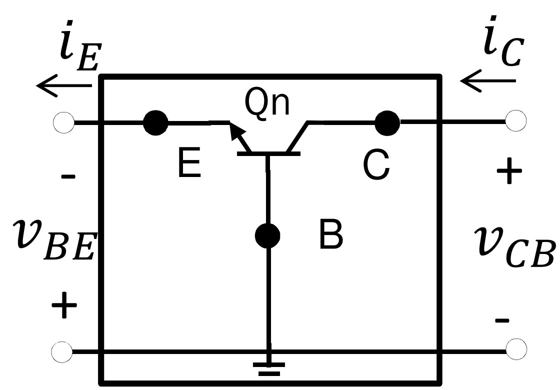
$$\frac{i_C(T_2)}{i_C(T_1)} \approx 2^{(T_2-T_1)/10K}$$

BJT Configurations

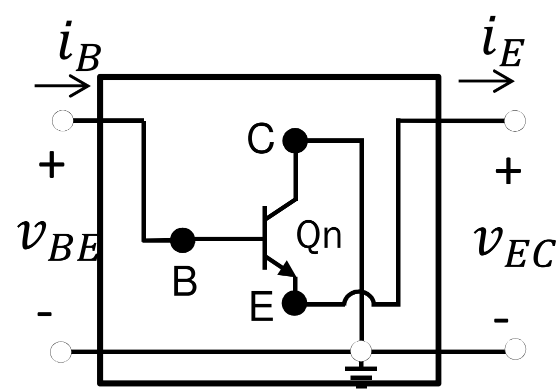
CE: Common Emitter



CB: Common Base



CC: Common Collector



Input Characteristic

i_B vs. v_{BE} for different V_{CE}

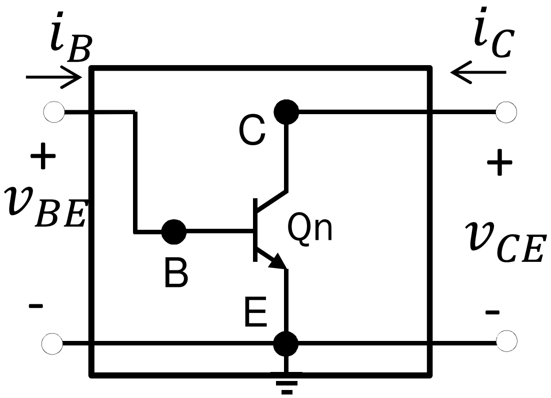
Output Characteristic

i_C vs. v_{CE} for different V_{BE} or i_B

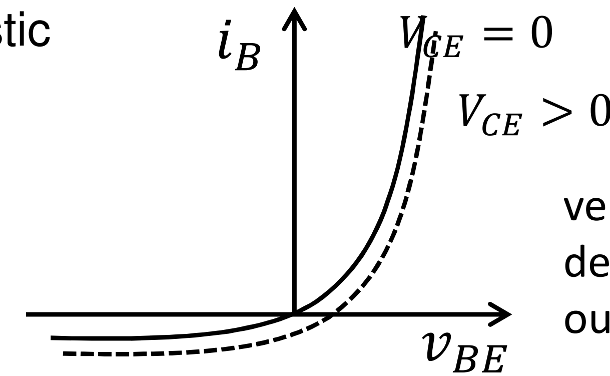
Transfer Characteristic

i_C vs. v_{BE} or i_B for different V_{CE}

CE: Common Emitter

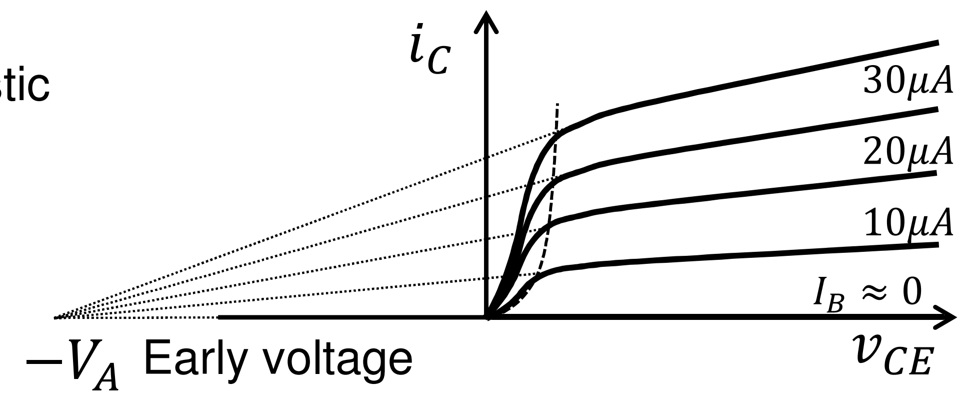


Input Characteristic



very little dependence to output voltage

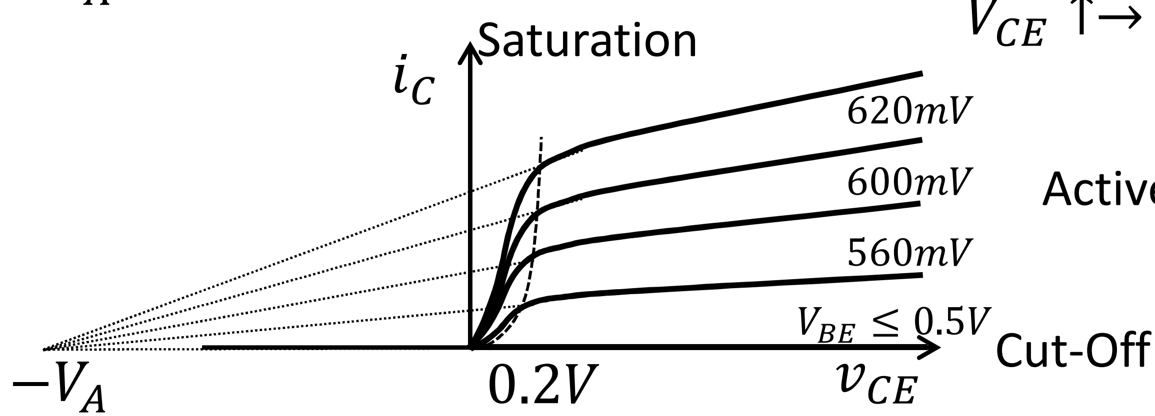
Output Characteristic



Active:

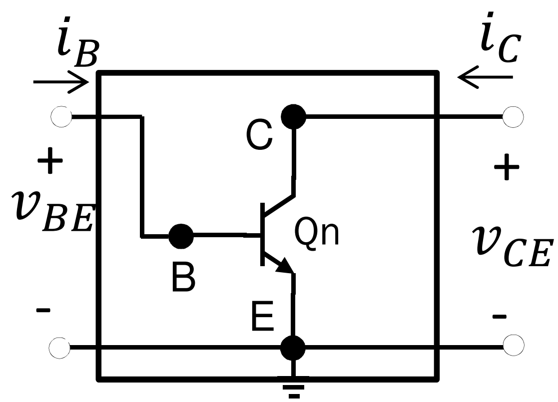
$$i_C = I_S e^{\frac{v_{BE}}{nV_T}}(1 + \frac{v_{CE}}{V_A})$$

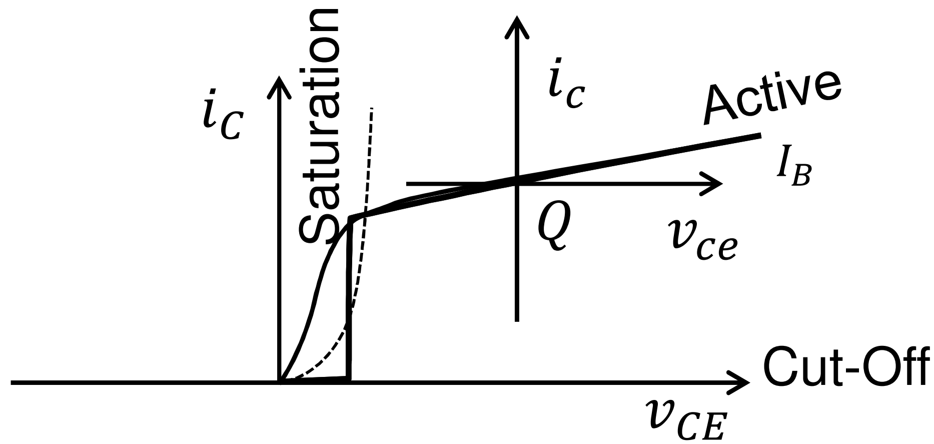
$$V_{CE} \uparrow \to W \uparrow \to \beta \uparrow \to I_C \uparrow$$



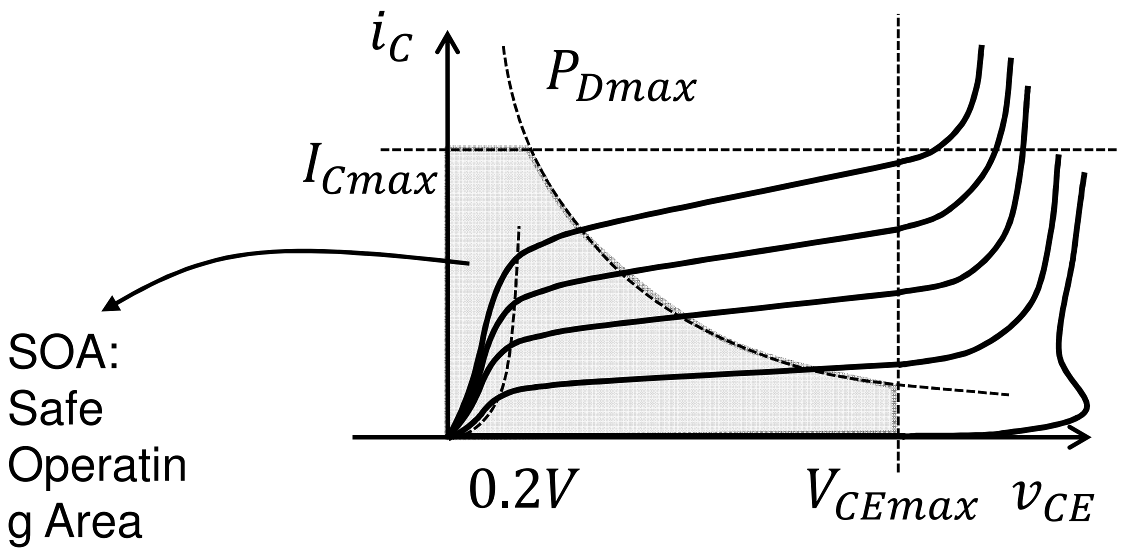
CE: Transistor Model

1. Lab
2. Power
3. Ser/Parl
4. Small sig.
5. Applic

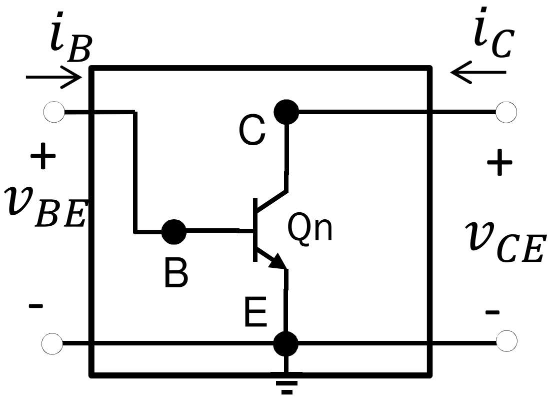


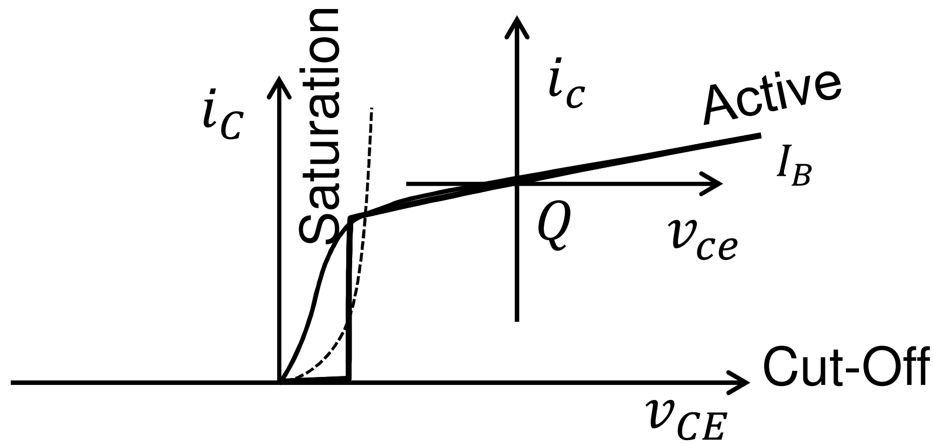


Output Characteristic



CE: Output Characteristic





$$i_B = I'_S e^{\frac{v_{BE}}{nV_T}}$$

$$i_C \approx I_S e^{\frac{v_{BE}}{nV_T}}(1 + \frac{v_{CE}}{V_A})$$

$$i_B = \frac{I_S}{\beta} e^{\frac{v_{BE}}{nV_T}}$$

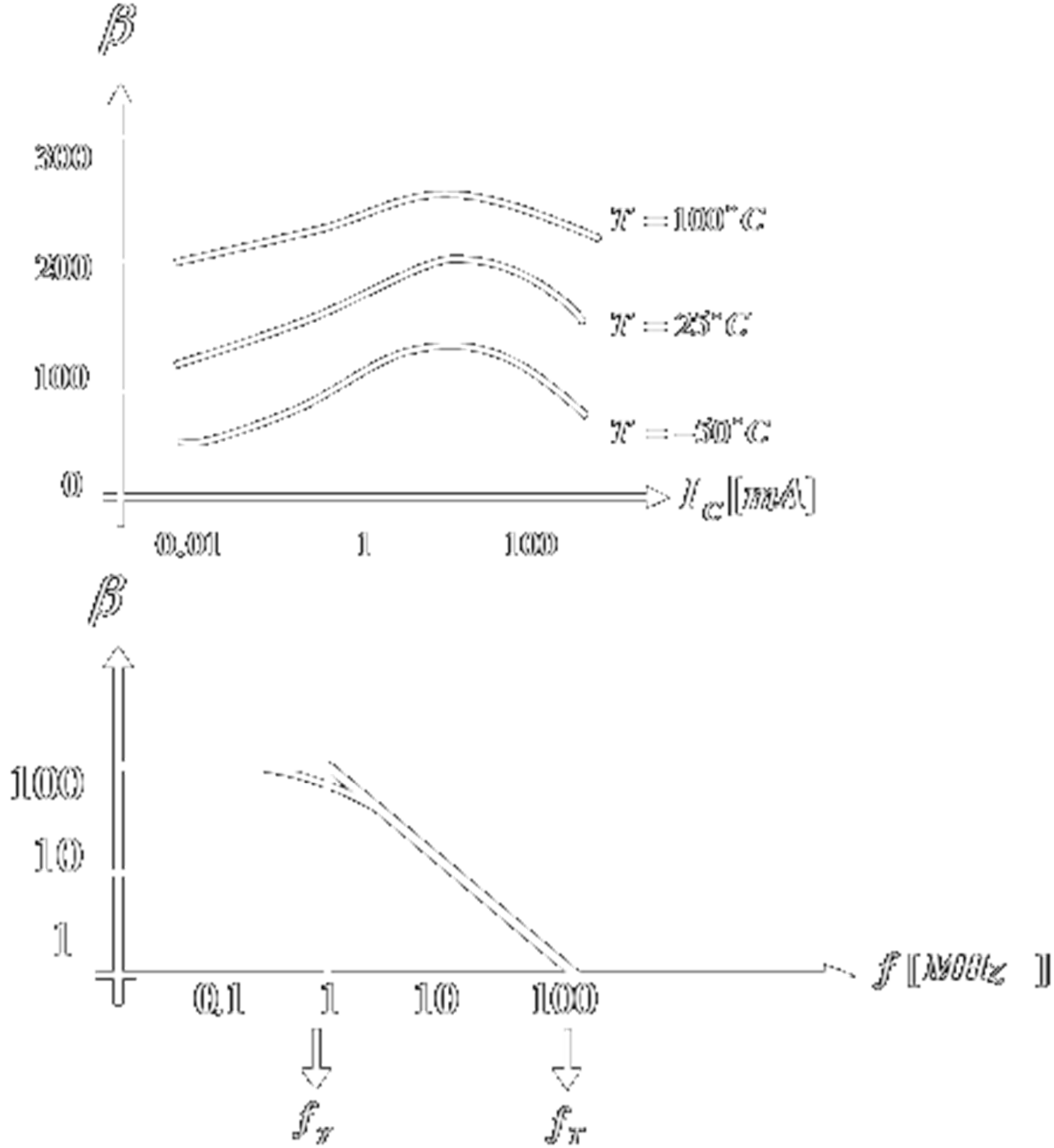
Ideally linear: $\beta = \frac{i_C}{i_B} = \frac{I_C}{I_B} = \frac{i_c}{i_b}$

$$BDC = \beta_{DC} = \left.\frac{i_C}{i_B}\right|_{I_C=I_Q} = \beta_F(1 + \frac{v_{CE}}{V_A})$$

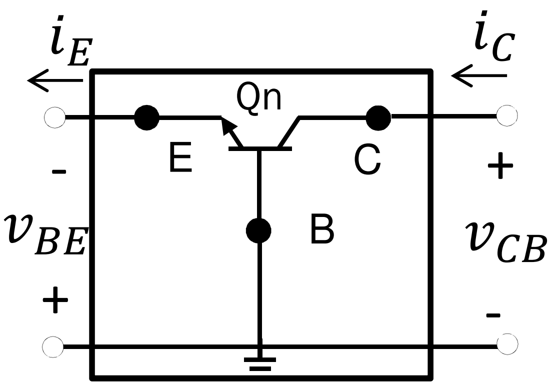
SPICE $BF = \beta_F = \left.\frac{i_C}{i_B}\right|_{V_{CB}=0}$

$$BAC = \left.\frac{\partial i_C}{\partial i_B}\right|_{I_C=I_Q} = \frac{i_c}{i_b}$$

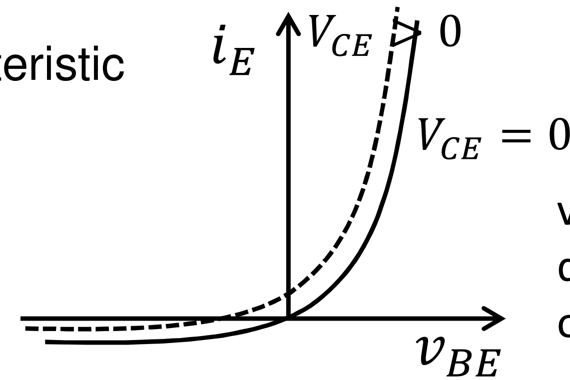
Beta : Current Gain



CB: Common Base

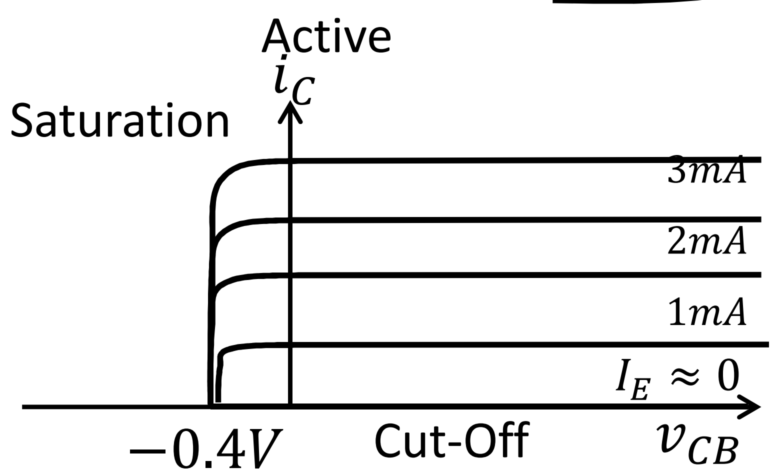


Input Characteristic



very little dependence to output voltage

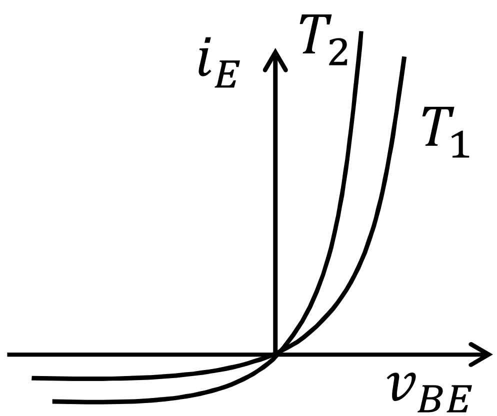
Output Characteristic



Active:

$$i_C = \alpha i_E$$

$$\alpha = \frac{\beta}{\beta + 1}$$

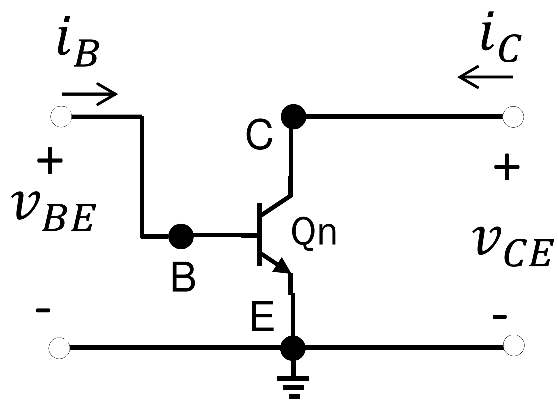


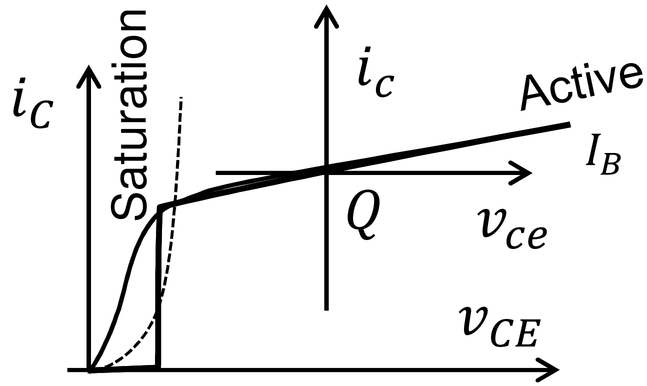
$$T_2 > T_1$$

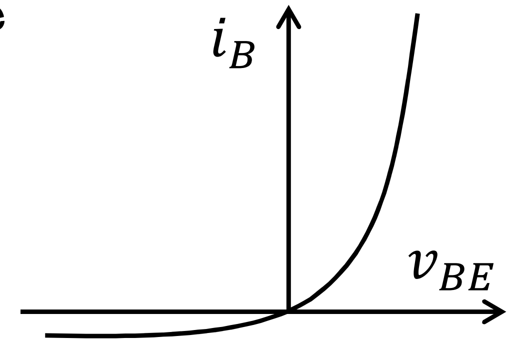
$$\frac{\Delta v_{BE}}{\Delta T} \approx -2\frac{mV}{K}\bigg|_{i_E = cte}$$

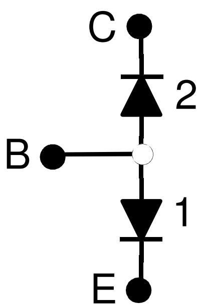
Large Signal Model

1. Lab
2. Power
3. Ser/Parl
4. Small sig.
5. Applic

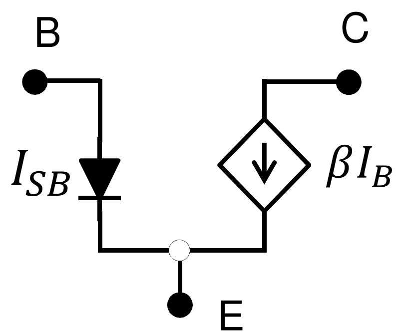




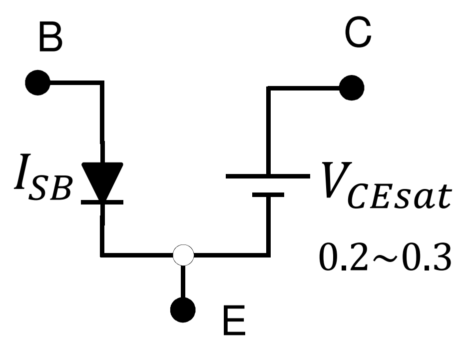


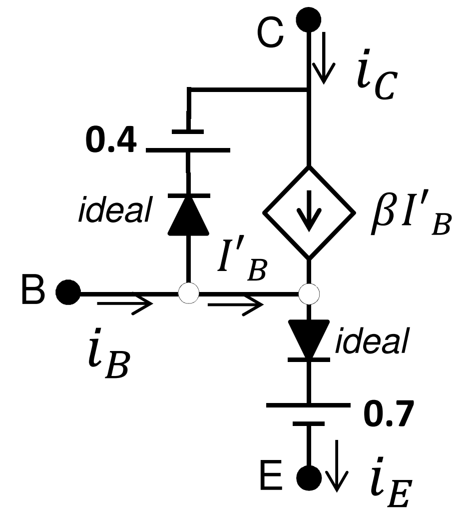


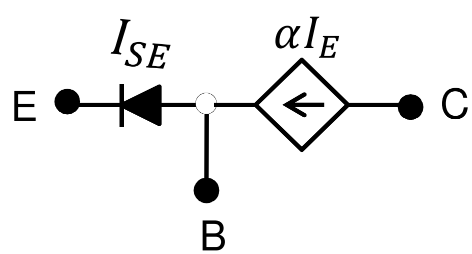
Active



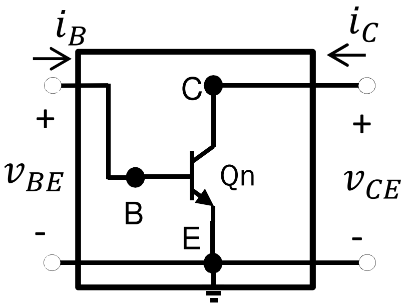
Saturation

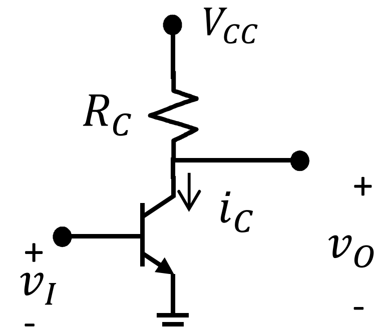






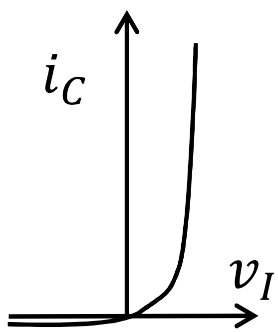
Voltage Amplifier

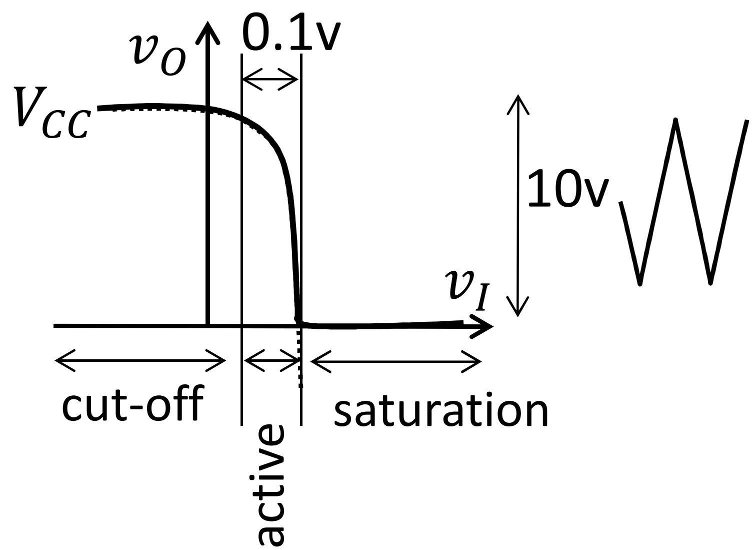


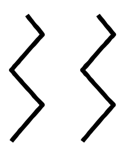


$$v_O = V_{CC} - R_C i_C$$

$$i_C = \beta i_B = I_s(e^{\frac{v_I}{V_T}} - 1)$$







Biasing: V_{BE} = cte

1. Lab
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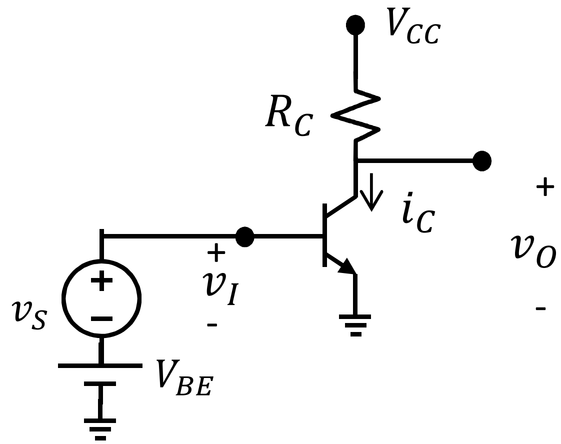
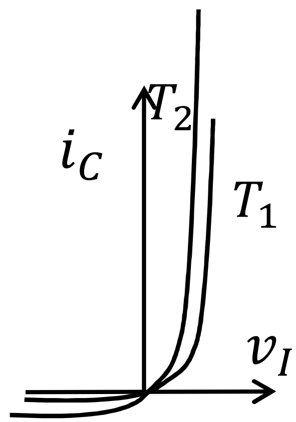
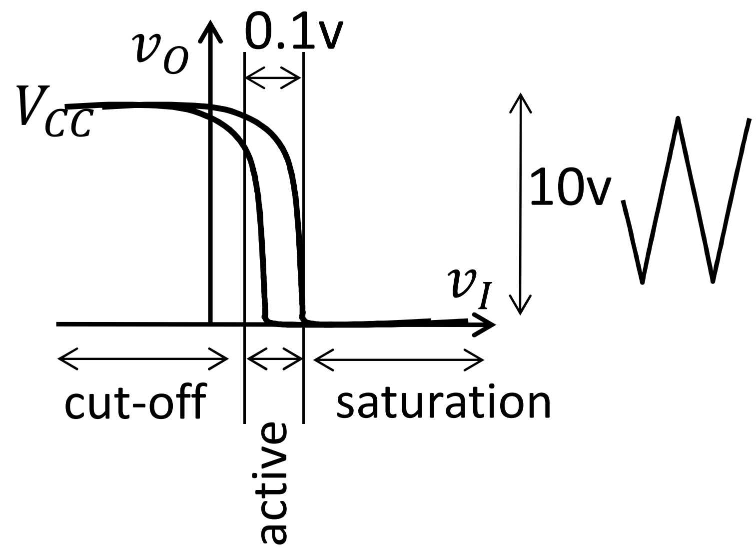
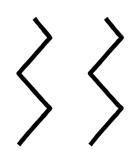
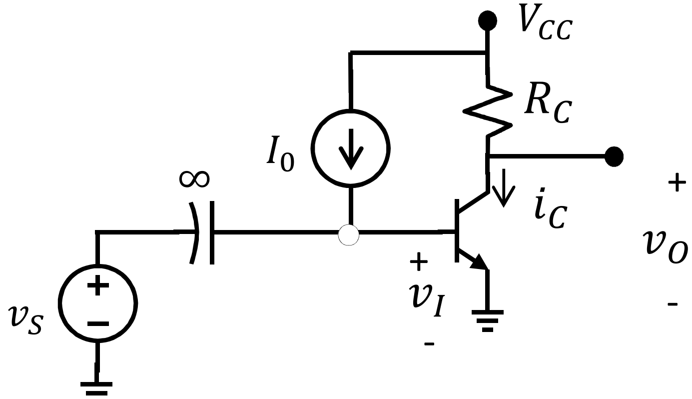








Biasing: I_B = cte



$$I_B = I_0$$
$$I_C = \beta I_B$$

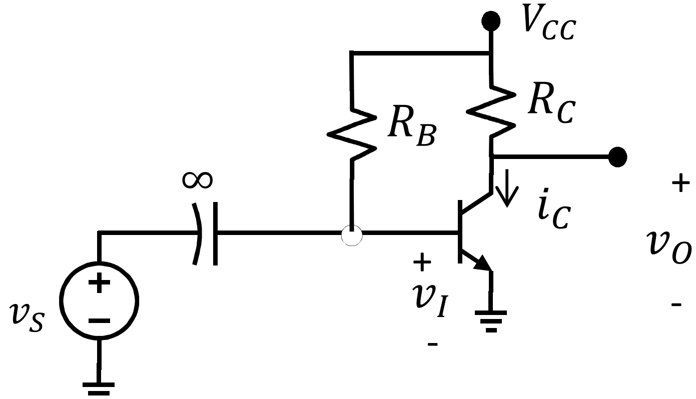
$V_{CC} = 10V$

$I_C = 1mA$ $\quad \beta = 100$

$I_0 = 10\mu A$

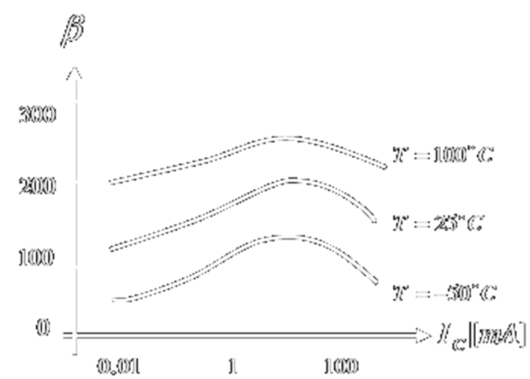
For max swing: $V_C \sim 5V \quad \rightarrow R_C \sim 5k\Omega$

For max gain: $V_C \sim 0.3V \quad \rightarrow R_C \sim 9.7k\Omega$



$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

What is the problem?



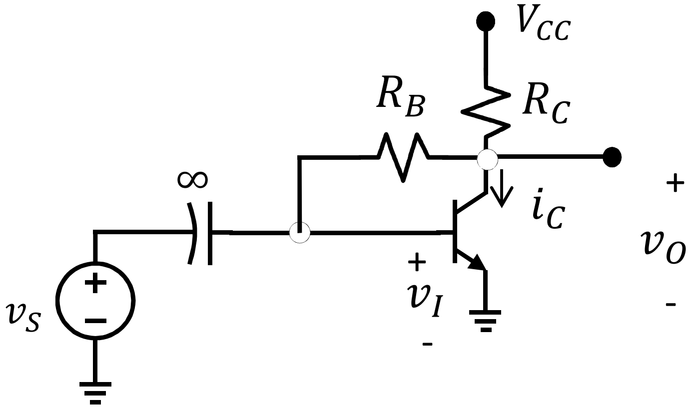
$V_{CC} = 10V$

$I_C = 1mA$ $\quad \rightarrow R_B = 930k\Omega$

Replace it with transistor with $\beta = 250$

$I_C = 2.5mA \qquad V_{CE} = 10 - 5 \times 2.5 = -2.5 < V_{CEsat}$

Biasing: I_B = cte



$V_{CC} = 10V$ $\quad I_C = 1mA$ $\quad \beta = 100$

For max swing: $V_C \sim 5V$

$$R_B = \frac{V_C - V_{BE}}{I_B} = \beta \frac{V_C - V_{BE}}{I_C}$$

$\rightarrow R_B = 430k\Omega$

Negative feedback!

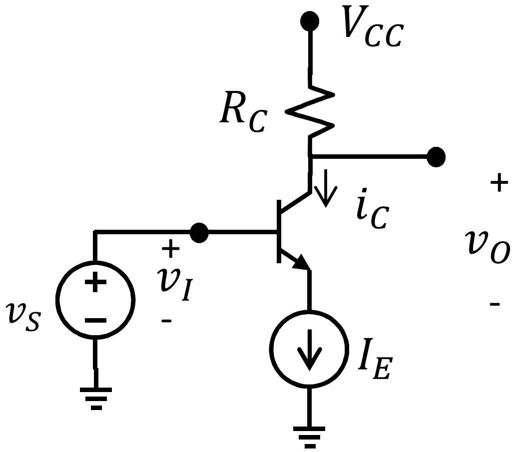
$\beta \uparrow \quad \rightarrow I_C \uparrow$

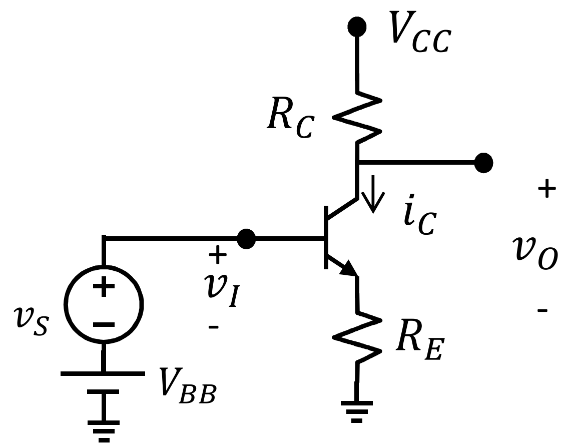
$\rightarrow V_C \downarrow$

$\rightarrow I_B \downarrow$

$\rightarrow I_C \downarrow$

Biasing: I_E = cte

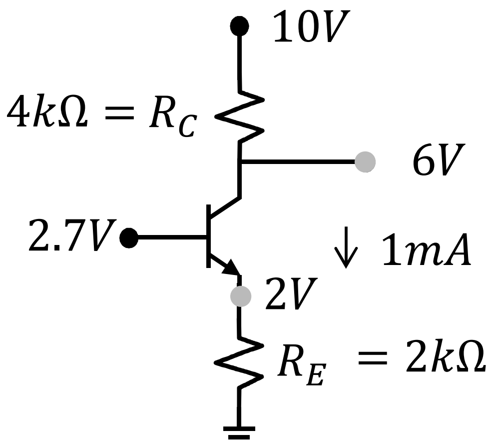


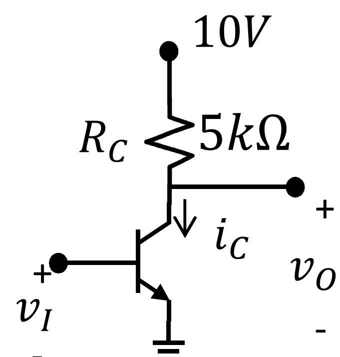


$$I_E = \frac{V_{BB} - V_{BE}}{R_E}$$

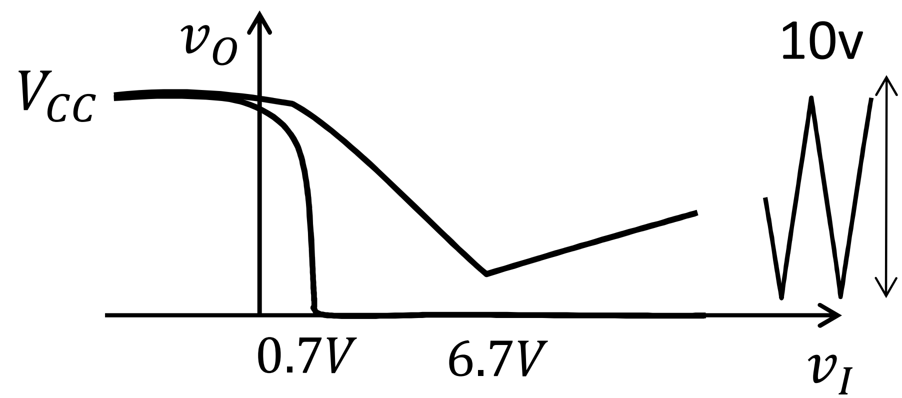
I_E independent of V_{BE}

$V_{BB} \gg V_{BE}$ $\qquad$ $V_{BB} \sim 2.7V$

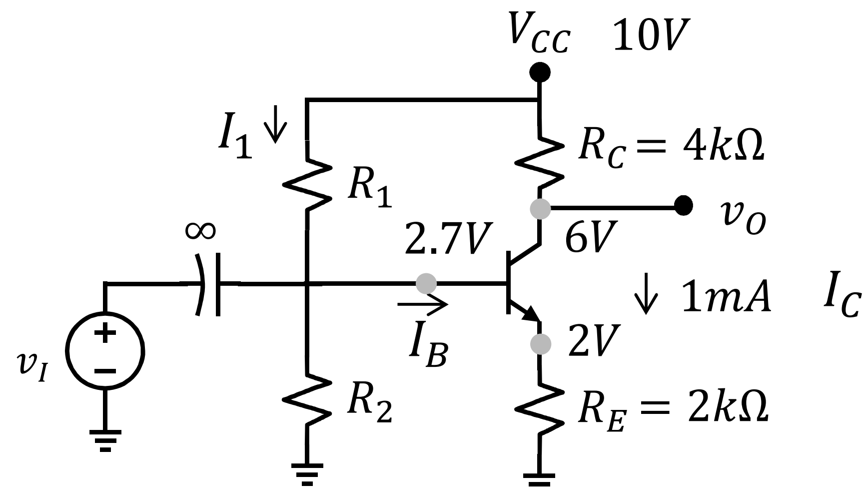




Where is the trade-off?



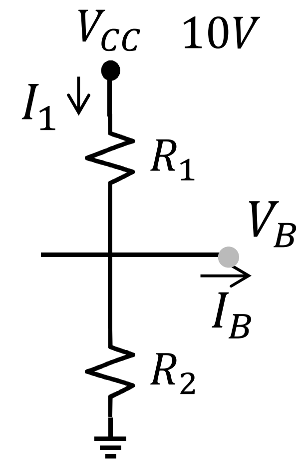
Biasing: I_E = cte

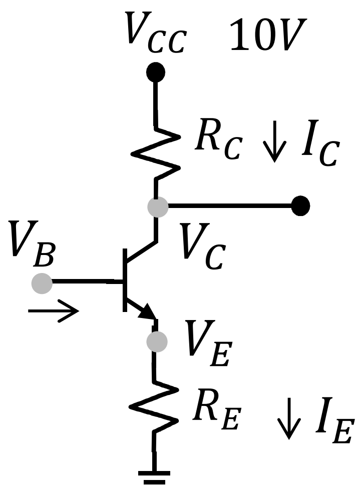


$$I_B = \frac{I_C}{\beta}$$

Assume: $I_1 \gg I_B$

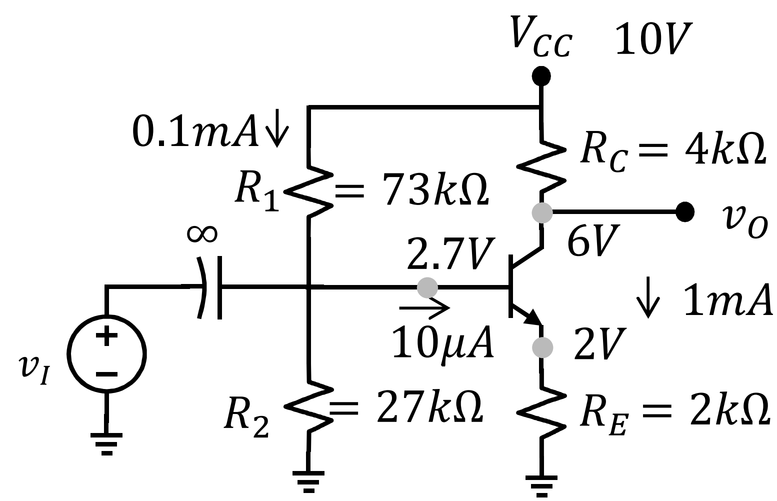
$$V_B = \frac{R_2}{R_2 + R_1} V_{CC}$$





$$V_E = V_B - V_{BEOn}$$

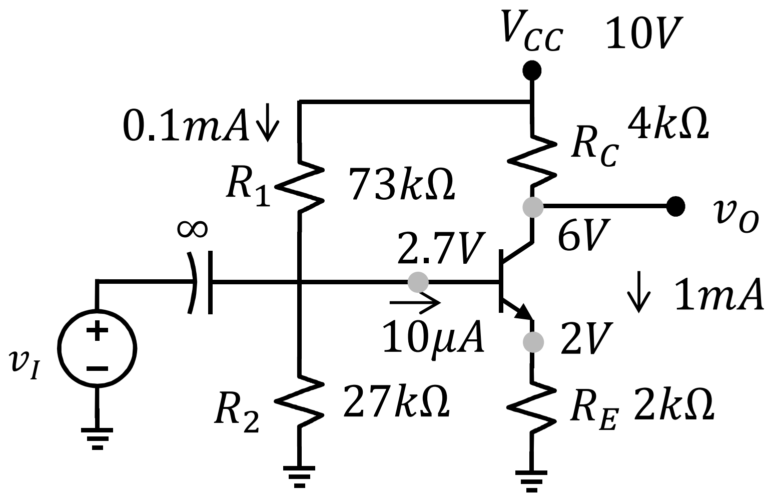
$$I_E = \frac{V_E}{R_E} \qquad I_C = \alpha I_E \approx I_E \qquad V_C = V_{CC} - I_C R_C$$



?

This is for design
how about analysis

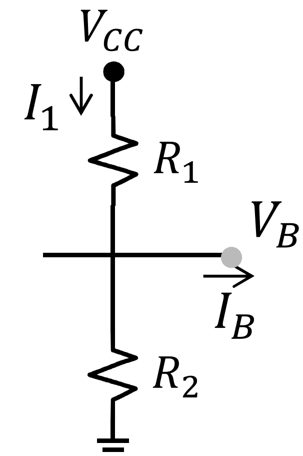
Biasing: I_E = cte



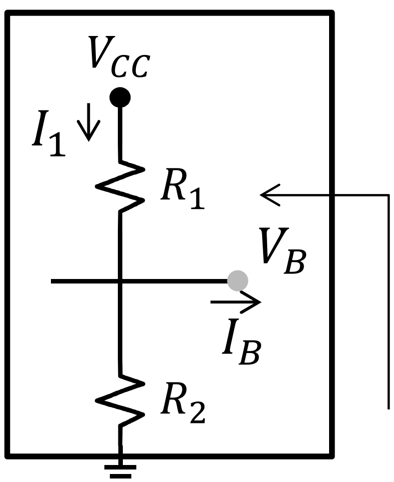
Assume: $I_1 \gg I_B$ $\quad I_1 = 0.1mA$

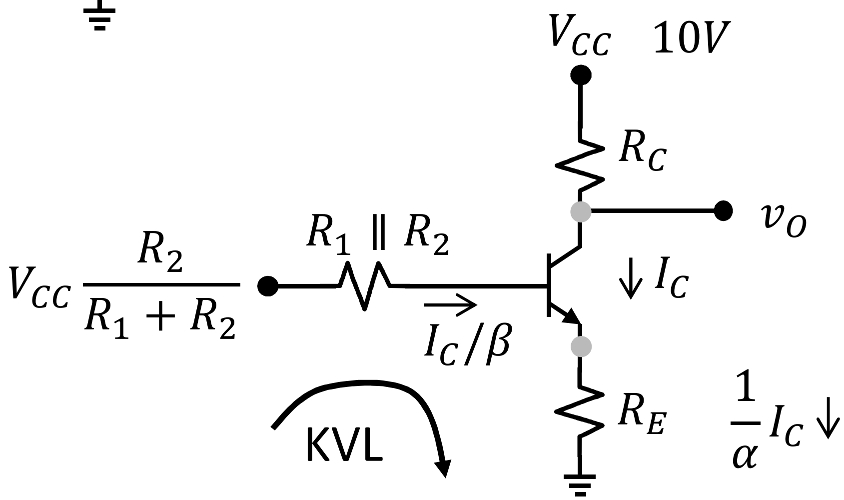
$V_B = 2.7V \quad V_E = 2V$

$I_E = 1mA \quad I_B = 0.01mA$



What if β was 10!

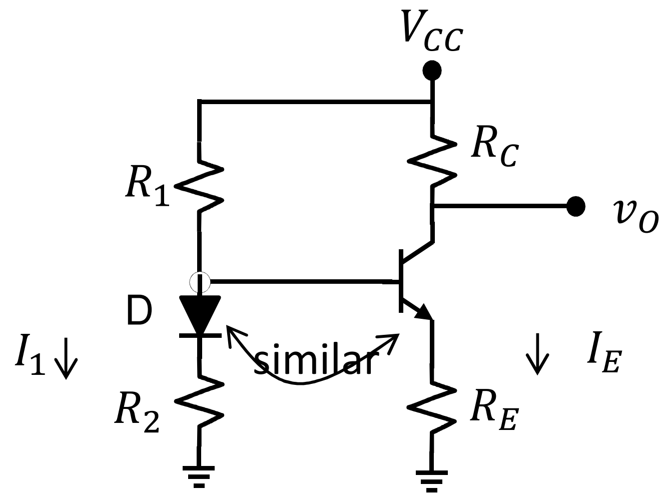




$$\frac{R_2}{R_1+R_2}V_{CC} = I_C\frac{R_1 \parallel R_2}{\beta} + V_{BEon} + \frac{1}{\alpha}R_E I_C \qquad \rightarrow I_C = \cdots$$

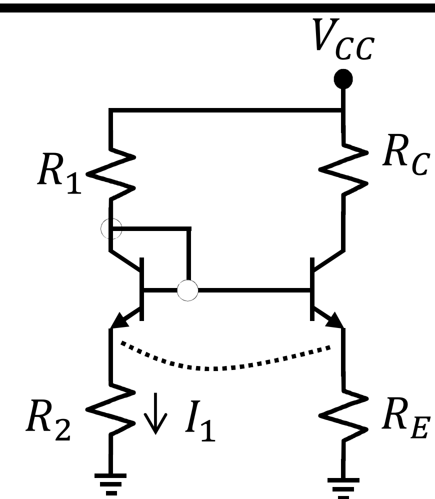
For the above numbers: $I_C = \dfrac{2}{1.01 \times 2k + 0.01 \times 19.7k} = 0.92mA$

Biasing: I_E = cte



$$I_1 R_2 + V_D = V_{BEon} + I_E R_E$$

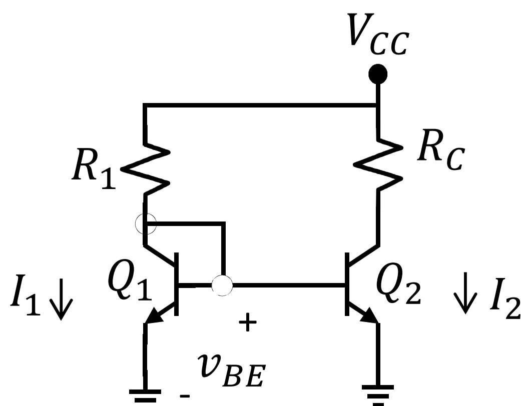
$$I_1 \approx \frac{V_{CC}}{R_1 + R_2} \qquad I_E = \frac{R_E}{R_2} I_1$$



$$I_1 R_2 = I_E R_E$$

$$I_1 = \frac{V_{CC} - V_{BEon}}{R_1 + R_2}$$

Only in Integrated Circuits!



$$I_1 = \frac{V_{CC} - V_{BEon}}{R_1}$$

$$I_1 = I_{S1}\left(e^{v_{BE}/V_T} - 1\right)$$

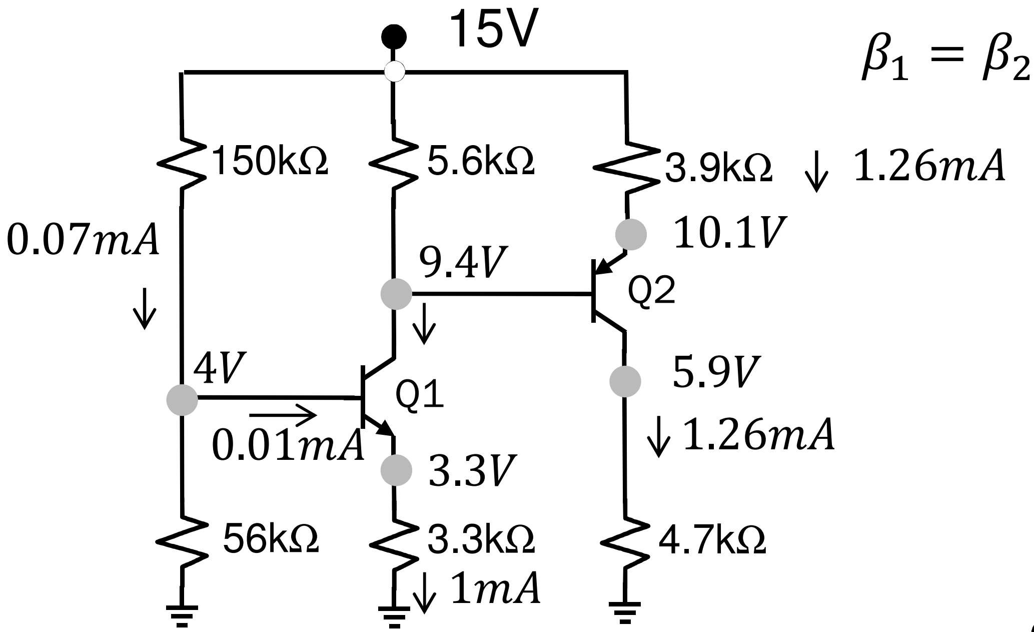
$$I_2 = I_{S2}\left(e^{v_{BE}/V_T} - 1\right)$$

$$\frac{I_2}{I_1} = \frac{I_{S2}}{I_{S1}} = \frac{A_{Q2}}{A_{Q1}}$$

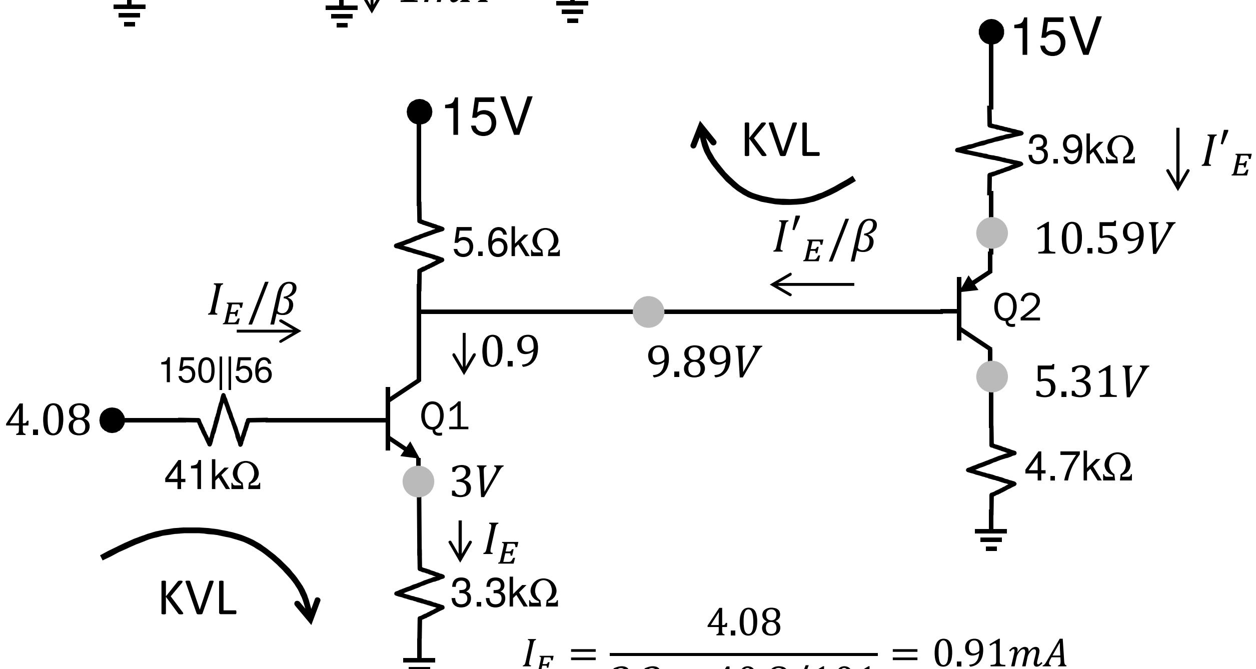
area

Biasing: Example 01

$\beta_1 = \beta_2 = 100 \quad V_{BE_{on}} = 0.7$



	Q_1	Q_2
I_C[mA]	1	1.26
V_{CE}[V]	6.1	4.2



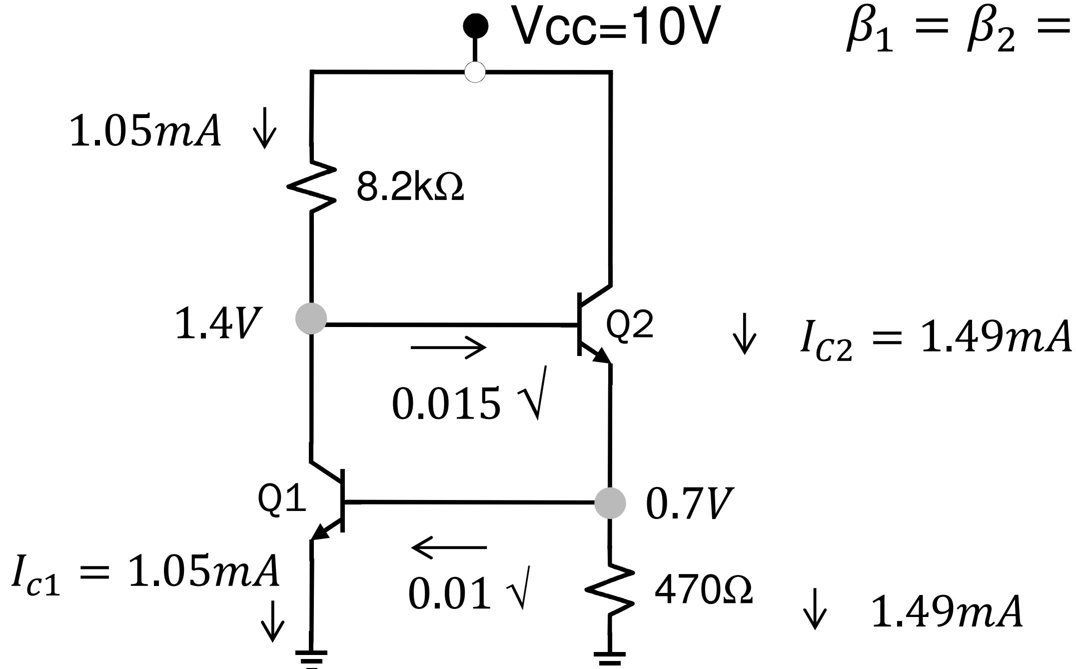
$$3.9I'_E + 0.7 = 5.6(0.9 - \frac{I'_E}{\beta})$$

$$I'_E = 1.13mA$$

$$I_E = \frac{4.08}{3.3 + 40.8/101} = 0.91mA$$

Biasing: Example 02

Vcc=10V $\beta_1 = \beta_2 = 100$ $V_{BE_{on}} = 0.7$

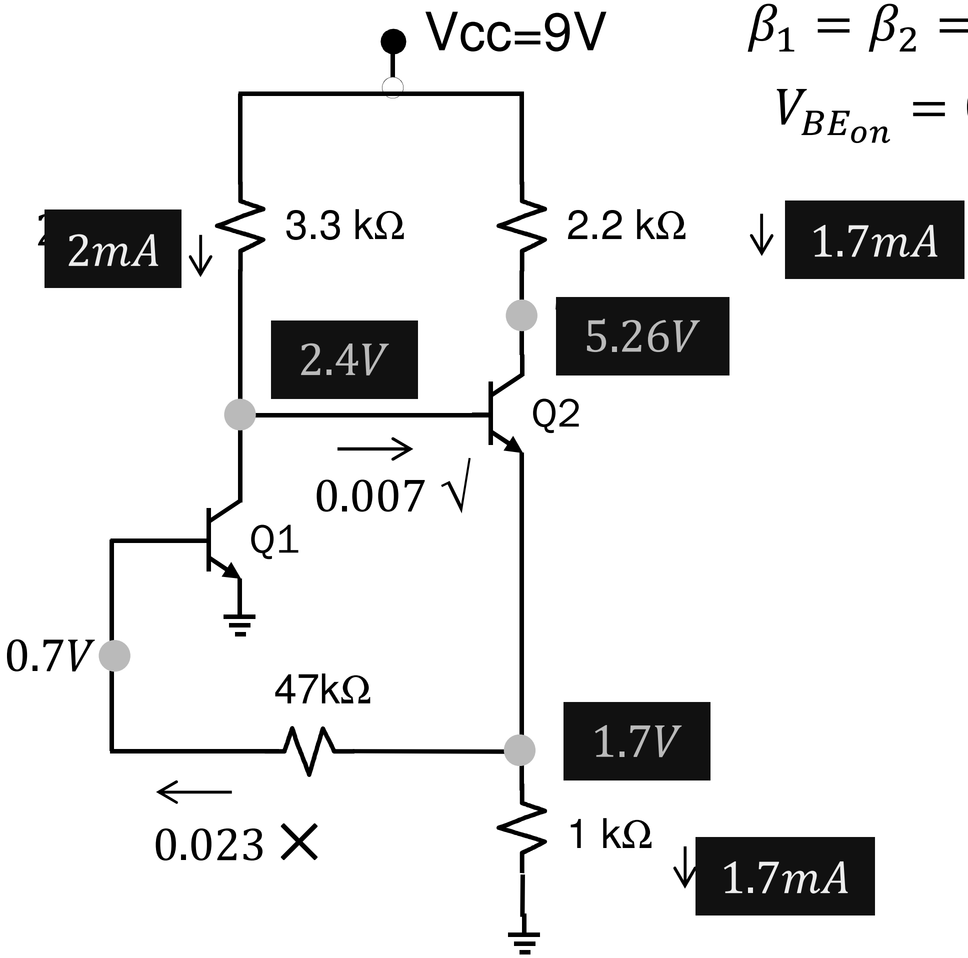


	Q_1	Q_2
I_C[mA]	1.05	1.49
V_{CE}[V]	1.4	9.3

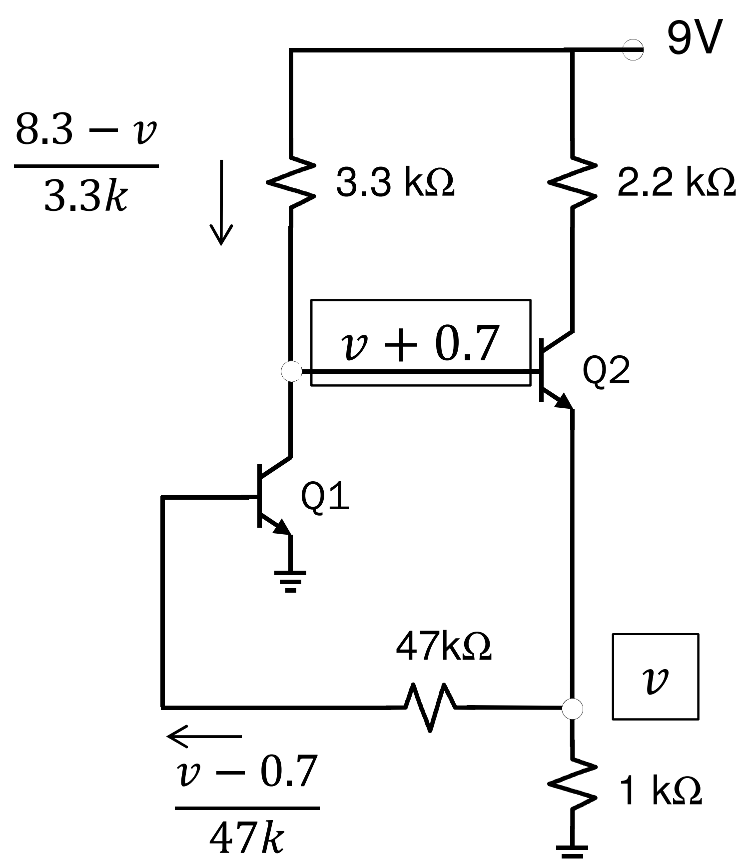
Biasing: Example 03

$$\beta_1 = \beta_2 = 100$$

$$V_{BE_{on}} = 0.7$$

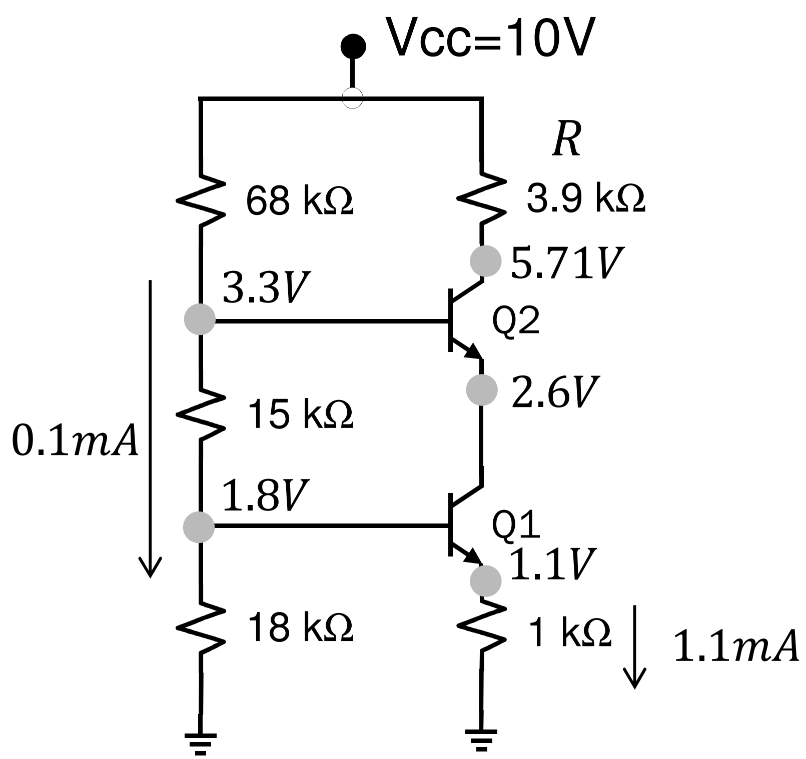


	Q_1	Q_2
I_C[mA]	2	1.7
V_{CE}[V]	2.4	3.56



$$\frac{8.3 - v}{330} = \frac{v - 0.7}{47} \quad \Rightarrow \quad v = 1.65$$

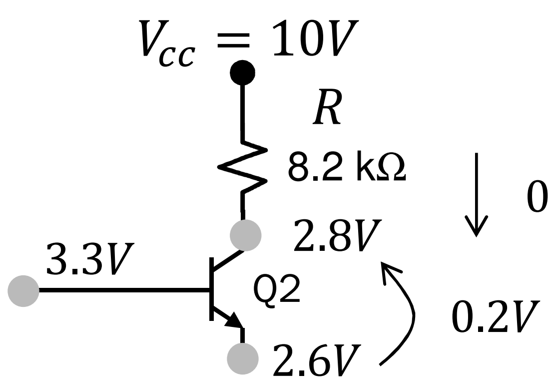
Biasing: Example 04



$\beta_1 = \beta_2 = 100 \quad V_{BE_{on}} = 0.7$

	Q_1	Q_2
I_C[mA]	1.1	1.1
V_{CE}[V]	1.5	3.11

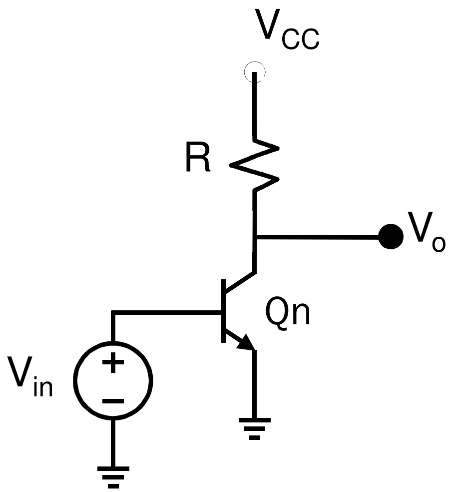
Find bias points if
$R = 8.2\ k\Omega$



$$V_{CE2} = (10 - 1.1 \times 8.2) - 2.6$$
$$= -1.62$$

	Q_1	Q_2
I_C[mA]	1.1	0.88
V_{CE}[V]	1.5	0.2

Linear BJT Amplifier



$$V_T = 26mV \qquad V_A = 200V$$

$$i_C = I_S e^{\frac{v_{BE}}{nV_T}}\left(1 + \frac{v_{CE}}{V_A}\right) \approx I_S e^{\frac{v_{BE}}{nV_T}}$$

$$v_{BE} = V_B + \hat{v}_i \sin \omega t$$

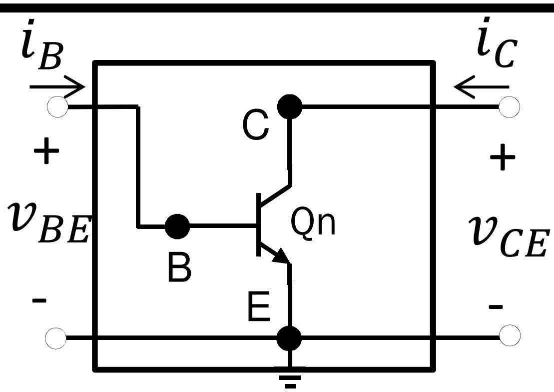
$$v_O = V_{CC} - Ri_C = V_{CC} - R\underbrace{I_S e^{\frac{V_B}{V_T}}}_{I_C} e^{\frac{\hat{v}_i \sin \omega t}{V_T}}$$

$$= V_{CC} - RI_C\left(1 + \frac{\hat{v}_i}{V_T}\sin \omega t + \frac{\hat{v}_i^2}{2V_T^2}\sin^2 \omega t + \cdots\right)$$

$$\approx \underbrace{V_{CC} - RI_C}_{V_O} \underbrace{- RI_C \frac{\hat{v}_i}{V_T}\sin \omega t}_{v_o}$$

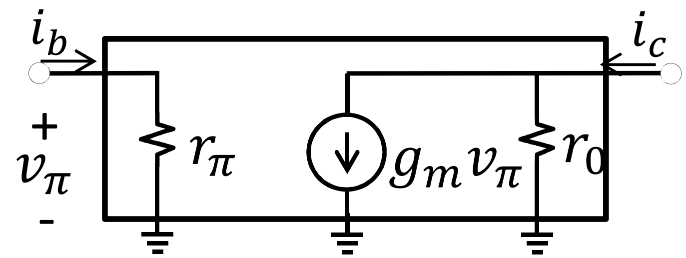
$$A_V = \frac{v_o}{v_{in}} = \frac{-R_C I_C}{V_T} = -g_m R_C \qquad g_m = \frac{I_C}{V_T}$$

BJT Small Signal Model (h-π)



$$i_C = I_S\left(e^{\frac{v_{EB}}{V_T}} - 1\right)\left(1 + \frac{v_{CE}}{V_A}\right)$$

$$\cong \underbrace{I_S e^{\frac{v_{EB}}{V_T}}}_{I_C}\left(1 + \frac{v_{CE}}{V_A}\right)$$



Input resistance:

$$r_\pi \equiv \frac{\partial v_{BE}}{\partial i_B} = \left(\frac{\partial i_B}{\partial v_{BE}}\right)^{-1} = \beta\left(\frac{\partial i_C}{\partial v_{BE}}\right)^{-1} = \beta\left(\frac{I_S}{V_T} e^{\frac{v_{EB}}{V_T}}\right)^{-1} = \beta\frac{V_T}{I_C} = \frac{\beta}{g_m} = \beta r_m$$

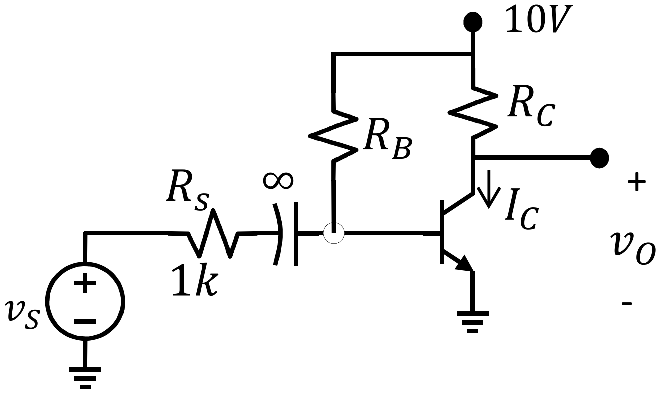
Output resistance:

$$r_o \equiv \frac{\partial v_{CE}}{\partial i_C} = \left(\frac{\partial i_C}{\partial v_{CE}}\right)^{-1} = \left(\frac{I_C}{V_A}\right)^{-1} = \frac{V_A}{I_C}$$

Trance-Conductance:

$$g_m \equiv \frac{\partial i_C}{\partial v_{BE}} = \frac{I_S}{V_T} e^{\frac{v_{EB}}{V_T}} = \frac{I_C}{V_T} = \frac{1}{r_m}$$

Example 01 - CE

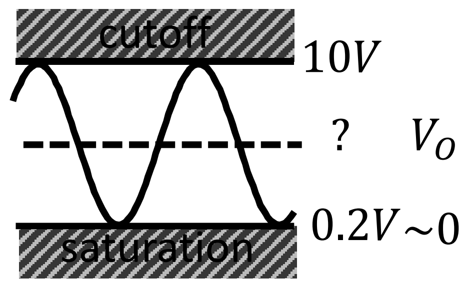


Assume $\beta = 100 \quad V_A \sim \infty$

Design for $I_C = 1mA$ and maximum swing

Find A_v, R_{in}, R_{out}

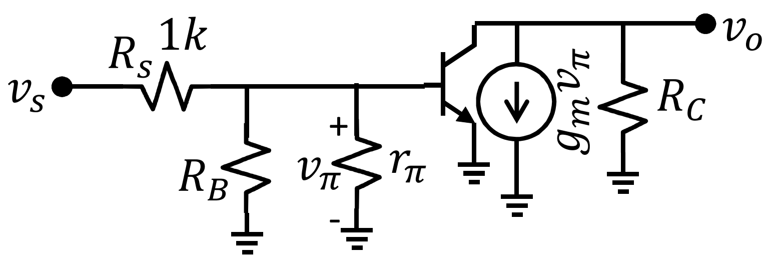
window for v_O



$V_O = 5 \qquad R_C = 5k\Omega$

DC: $R_B = \dfrac{10 - 0.7}{0.01mA} = 930k\Omega$

AC:



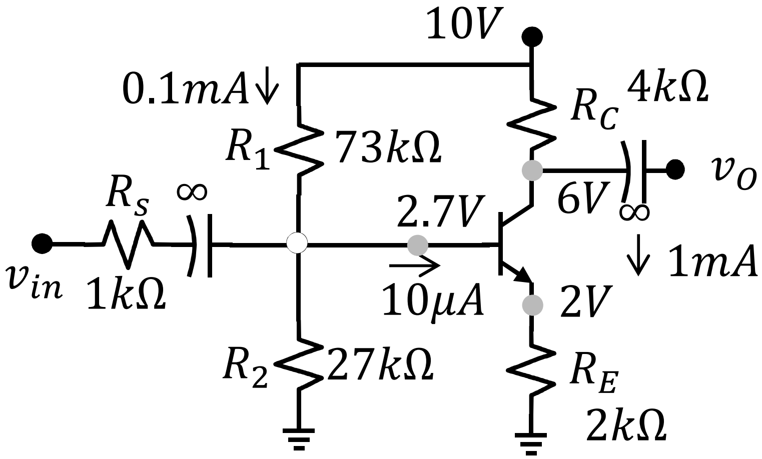
$$v_\pi = v_s \frac{r_\pi \parallel R_B}{r_\pi \parallel R_B + R_s} \sim v_s \frac{r_\pi}{r_\pi + R_s}$$

$$v_0 = -g_m v_\pi R_C$$

$$A_v = \frac{v_0}{v_s} = -g_m R_C \frac{r_\pi}{r_\pi + R_s} = \frac{-\beta R_C}{r_\pi + R_s} = \frac{-R_C}{r_m + R_s/\beta} = -\frac{\text{Collector resistance}}{\text{Emitter's circuit resistance}}$$

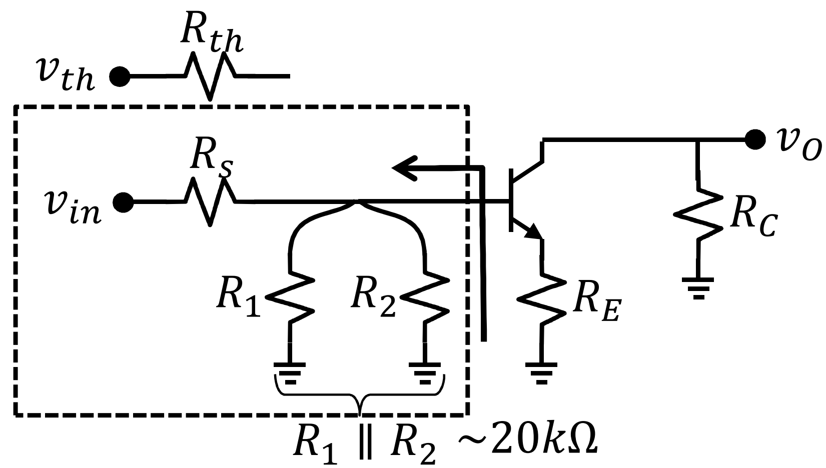
if $R_s \to 0$: $\quad A_v = -g_m R_C$

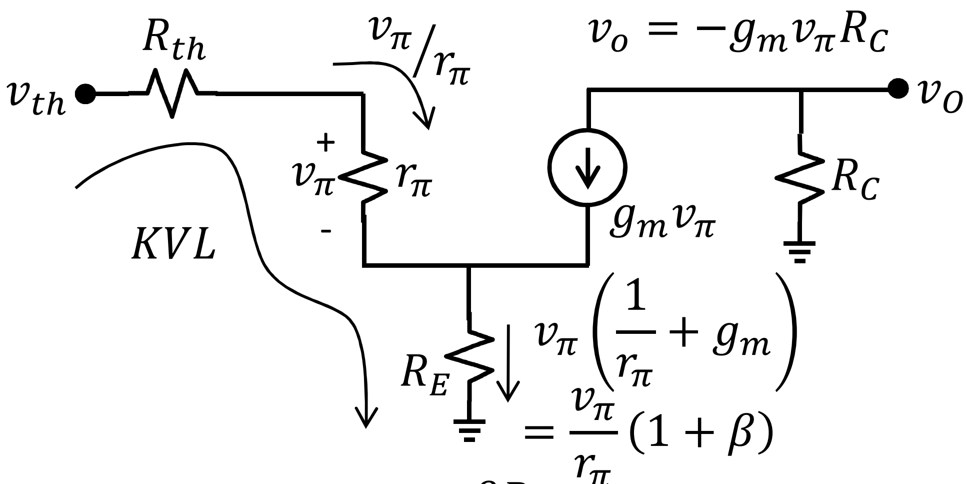
Example 02 - CE



Assume $\beta = 100$ $V_A \sim \infty$ Find A_v, R_{in}, R_{out}

AC circuit





$$v_{th} = \frac{R_1 \parallel R_2}{R_1 \parallel R_2 + R_s} v_{in} \qquad R_{th} = R_1 \parallel R_2 \parallel R_s$$

$$KVL: \quad -v_{th} + R_{th}\frac{v_\pi}{r_\pi} + v_\pi + R_E \frac{v_\pi}{r_\pi}(1+\beta) = 0$$

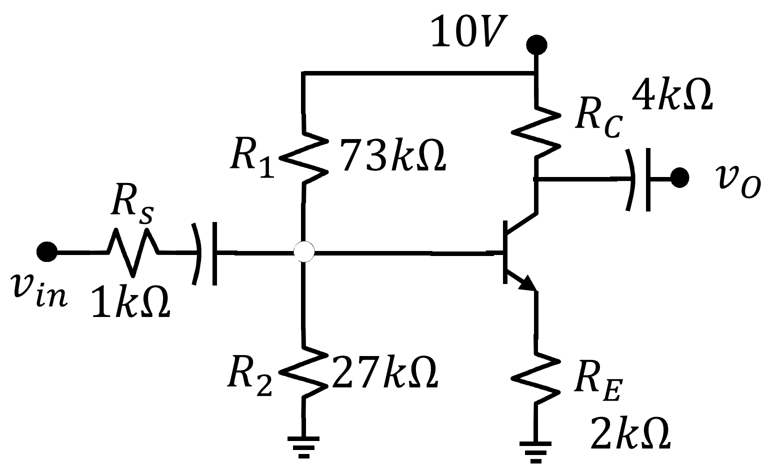
$$v_\pi = v_{th} \frac{r_\pi}{R_{th} + r_\pi + R_E(1+\beta)}$$

$$A'_v = \frac{v_o}{v_{th}} = \frac{-\beta R_C}{R_{th} + r_\pi + R_E(1+\beta)}$$

$$= \frac{-R_C}{\frac{R_{th} + r_\pi}{\beta} + R_E\left(\frac{1+\beta}{\beta}\right)}$$

$$A_v = \frac{v_o}{v_s} = \frac{v_{th}}{v_s} \cdot \frac{v_o}{v_{th}} = -\frac{20}{21} \cdot \frac{4}{\frac{3.5}{100} + 2 \times \frac{101}{100}} = -1.8$$

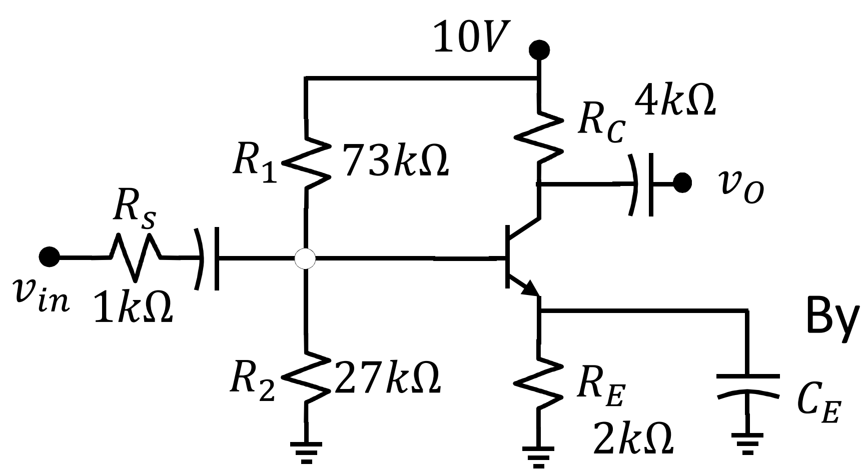
Example 02 - CE



$A_v = -1.8$

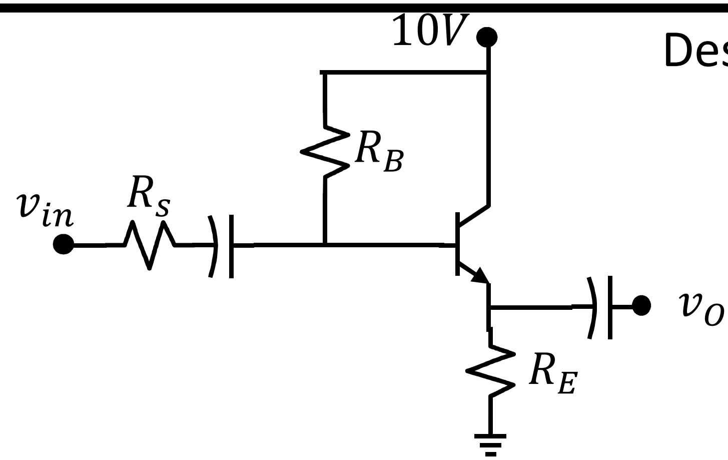
[redacted]

How we can increase gain?



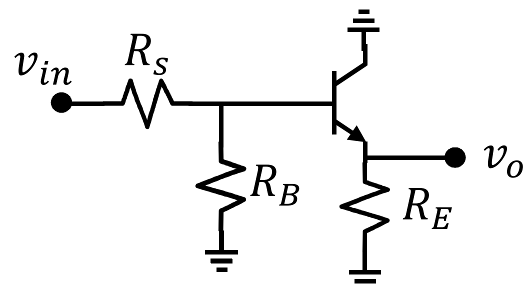
Bypass capacitance

Example 03 - CC

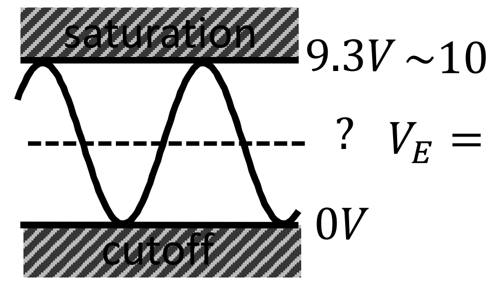


Design a buffer $I_C = 1mA$

AC circuit

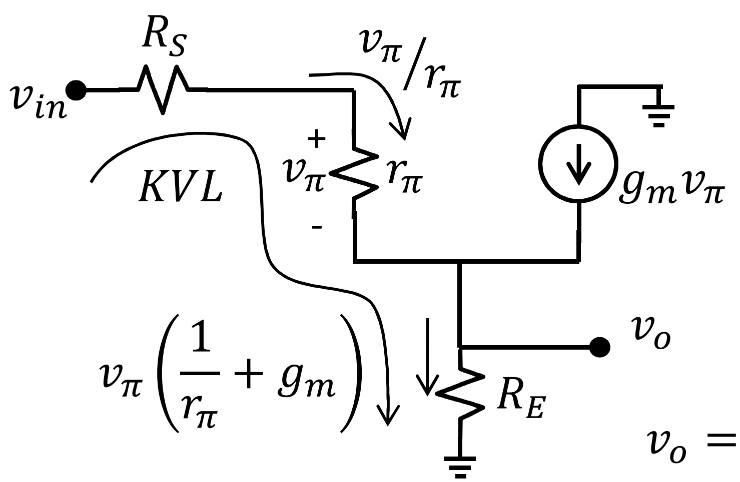


window for v_E



$$R_E = 5k\Omega$$

$$R_B = \frac{10 - 5.7}{0.01m} = 430k\Omega$$



KVL:

$$-v_{in} + R_S \frac{v_\pi}{r_\pi} + v_\pi + R_E \left(\frac{v_\pi}{r_\pi} + g_m v_\pi \right) = 0$$

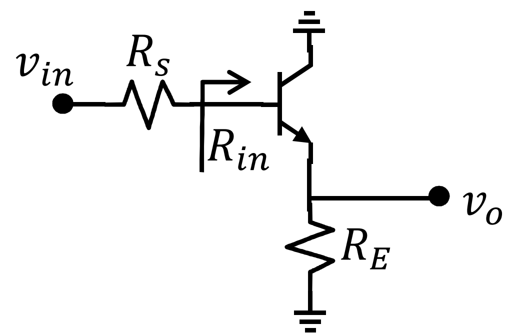
$$v_\pi = \frac{v_{in}}{\frac{R_S}{r_\pi} + 1 + R_E \left(g_m + \frac{1}{r_\pi} \right)}$$

$$v_o = v_\pi \left(\frac{1}{r_\pi} + g_m \right) R_E$$

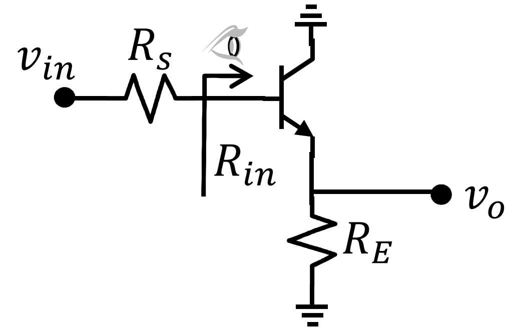
$$A_v = \frac{v_o}{v_{in}} = \frac{\left(\frac{1}{r_\pi} + g_m \right) R_E}{\frac{R_S}{r_\pi} + 1 + R_E \left(g_m + \frac{1}{r_\pi} \right)} = \frac{R_E}{\frac{R_S + r_\pi}{1 + \beta} + R_E} \sim 1$$

Example 03 - CC

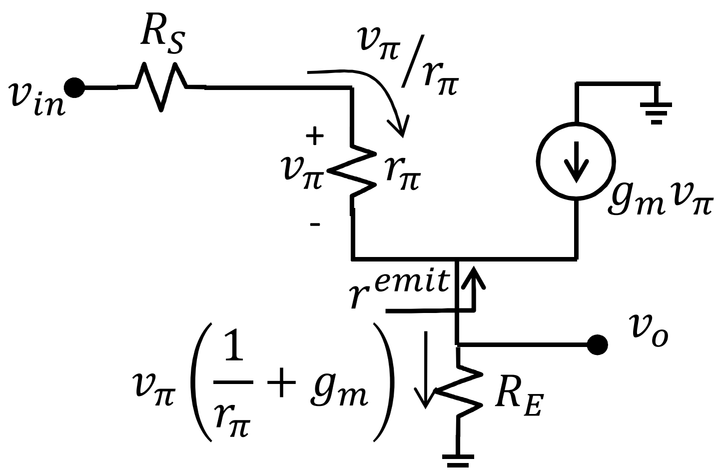
1. Lab
2. Power
3. Ser/Parl
4. Small sig.
5. Applic



$$\frac{v_o}{v_{in}} = \frac{R_E}{\frac{R_S + r_\pi}{1+\beta} + R_E}$$

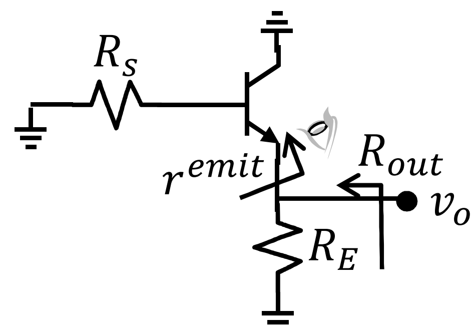


$$R_{in} = r_\pi + R_E(1+\beta)$$



$$R_{in} = \frac{v_\pi + R_E\left(\frac{v_\pi}{r_\pi} + g_m v_\pi\right)}{\frac{v_\pi}{r_\pi}}$$

$$= r_\pi + R_E(1+\beta)$$

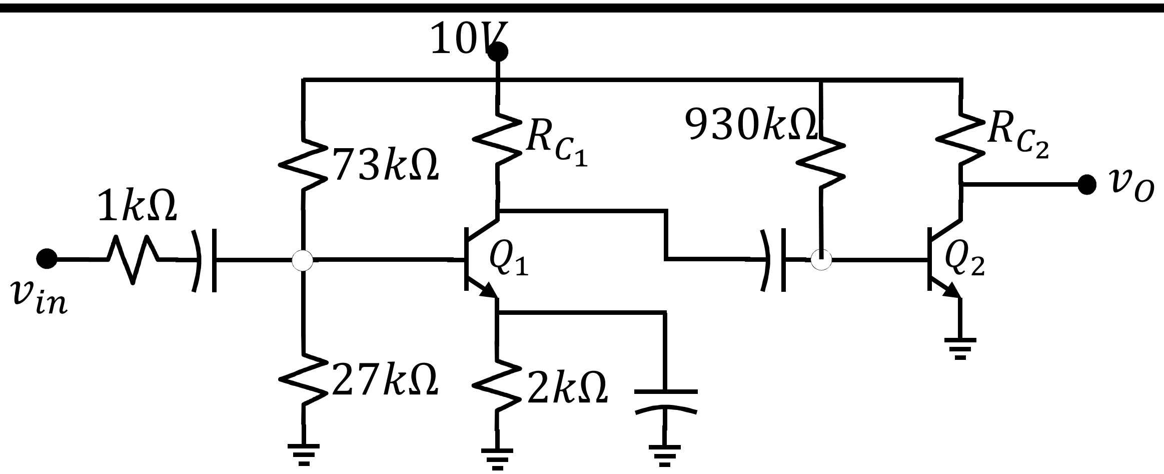


$$R_{out} = R_E \parallel r^{emit}$$

$$= R_E \parallel \frac{R_S + r_\pi}{1+\beta}$$

Example 04 – Multi-stage Amplifier

1. Lab
2. Power
3. Ser/Parl
4. Small sig.
5. Applic



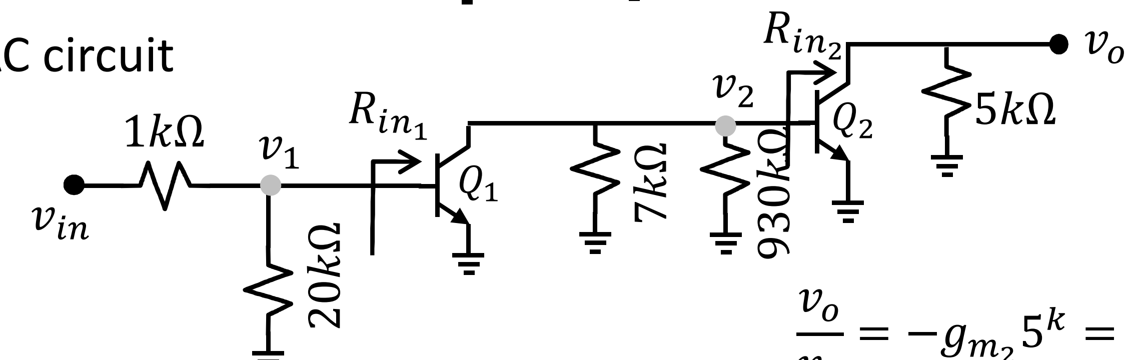
Design an amplifier:

$I_C = 1mA$
$\beta = 100$
$A_v \geq 1000$
$V_{cc} = 10V$
$R_S = 1k\Omega$

$$R_{C_1} = \frac{10-3}{1m} = 7k\Omega$$

$$R_{C_2} = \frac{10-5}{1m} = 5k\Omega$$

AC circuit



$27 \parallel 73$

$$A_v = \frac{v_o}{v_i} = \frac{v_1}{v_i} \cdot \frac{v_2}{v_1} \cdot \frac{v_o}{v_2}$$

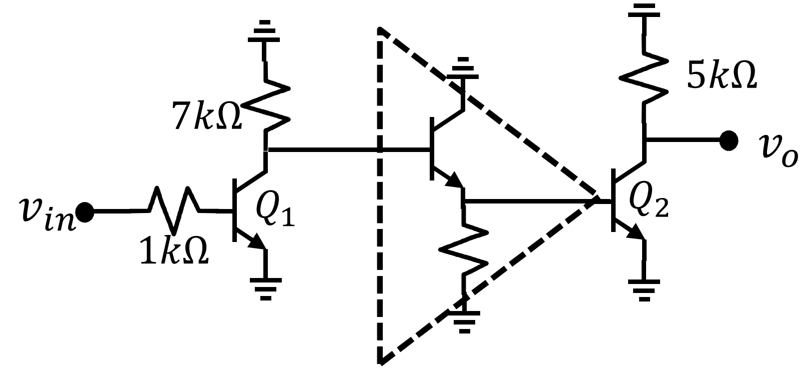
$$A_v = 10143$$

$$\frac{v_o}{v_2} = -g_{m_2} 5^k = -200$$

$$\frac{v_2}{v_1} = -g_{m_1}(7 \parallel 930 \parallel r_{\pi_2}) = -73$$

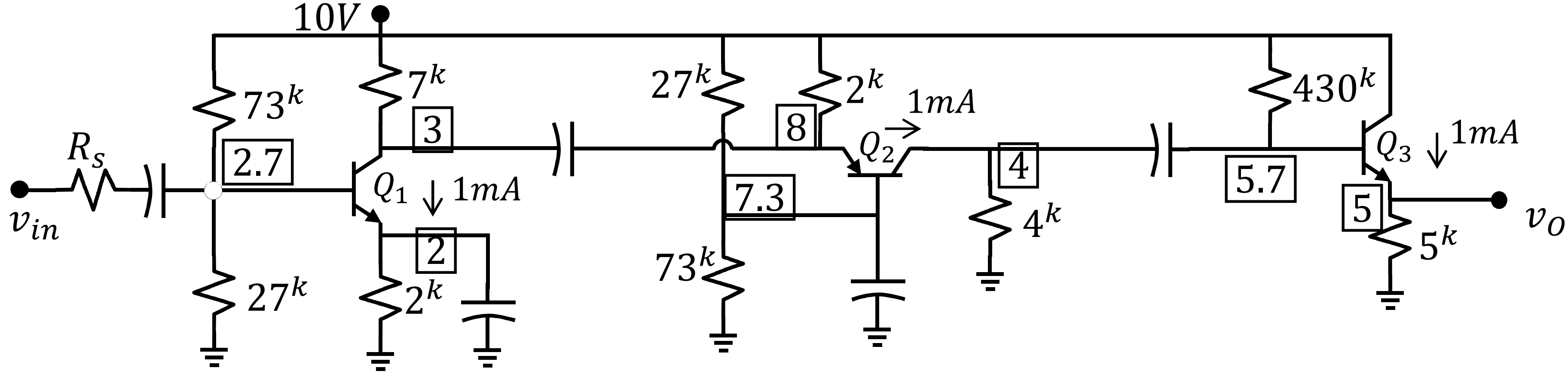
$$\frac{v_1}{v_i} = \frac{20^k \parallel r_{\pi_1}}{20^k \parallel r_{\pi_1} + 1^k} = 0.69$$

buffer

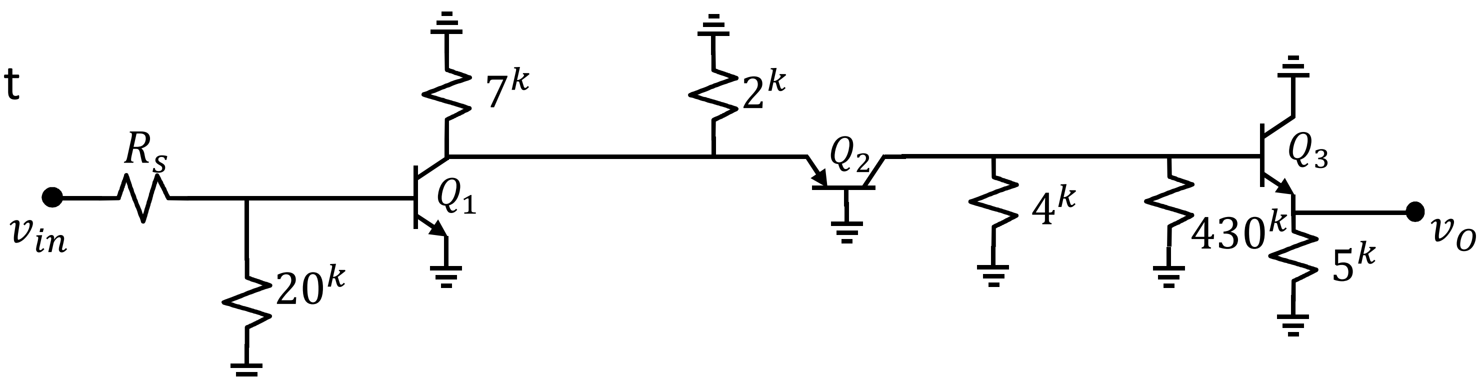


Example 05 – CE , CB, CC

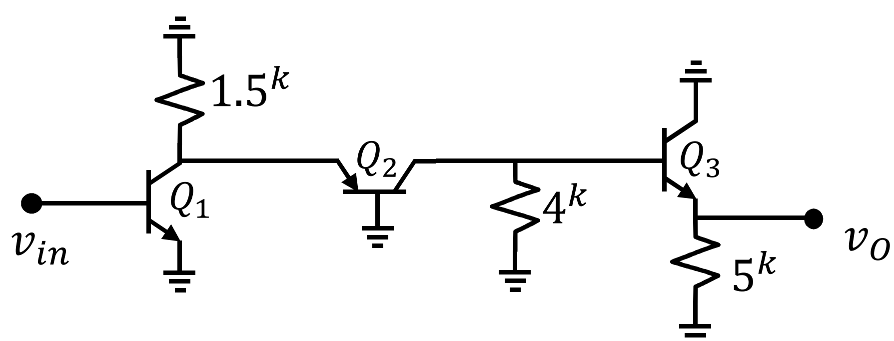
1. Lab
2. Power
3. Ser/Parl
4. Small sig.
5. Applic



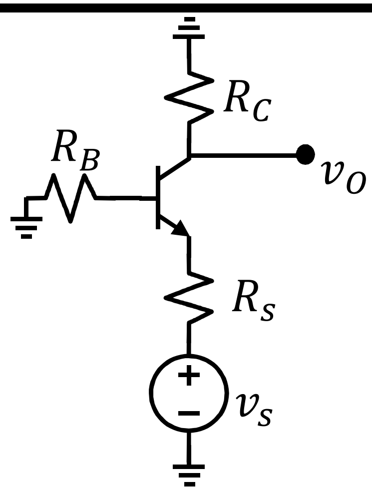
AC circuit

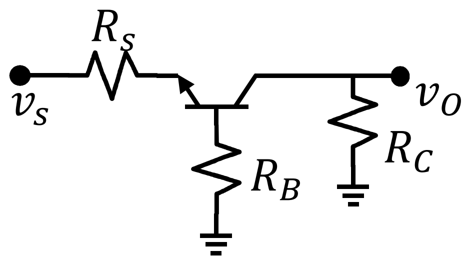


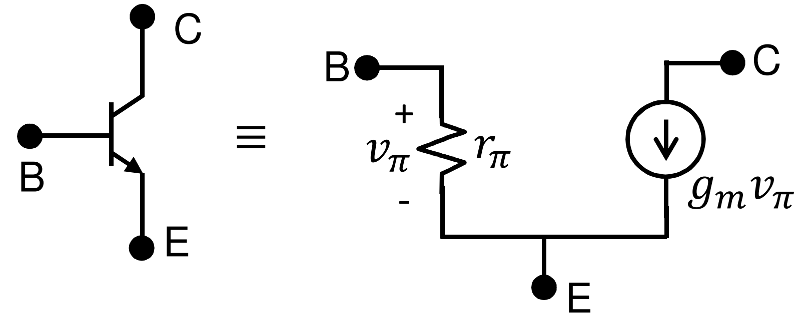
$R_s \ll ?$

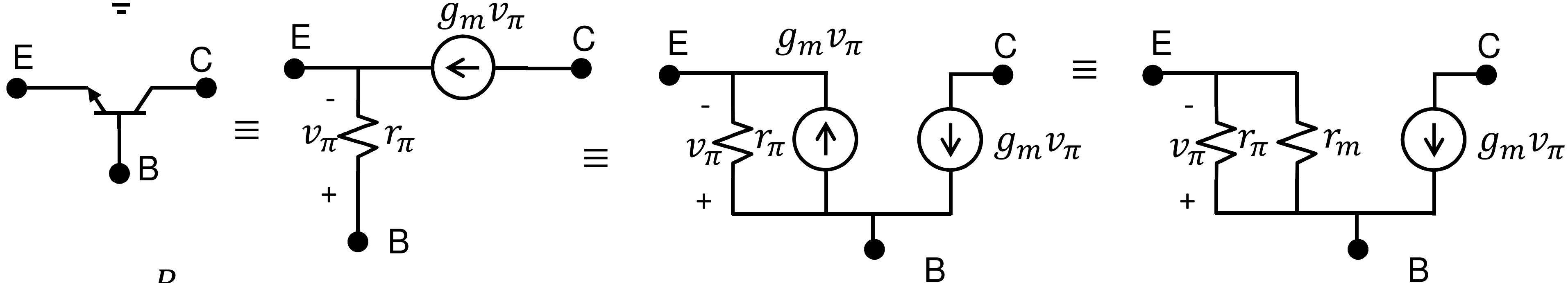


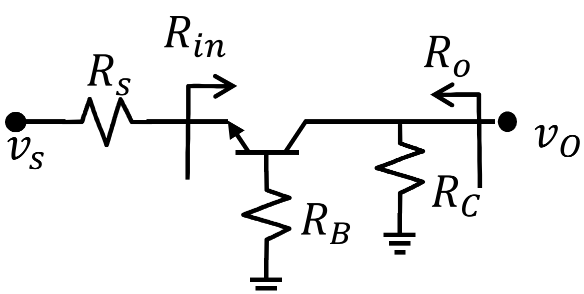
Common Base





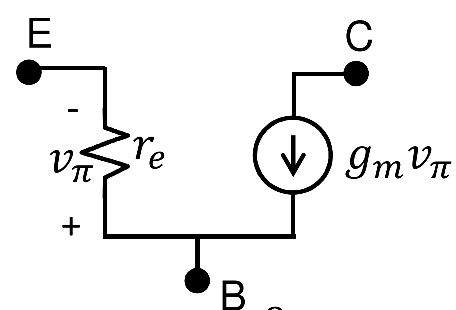






$$R_{in} = \frac{R_B + r_\pi}{\beta + 1}$$

$$R_o = R_c$$

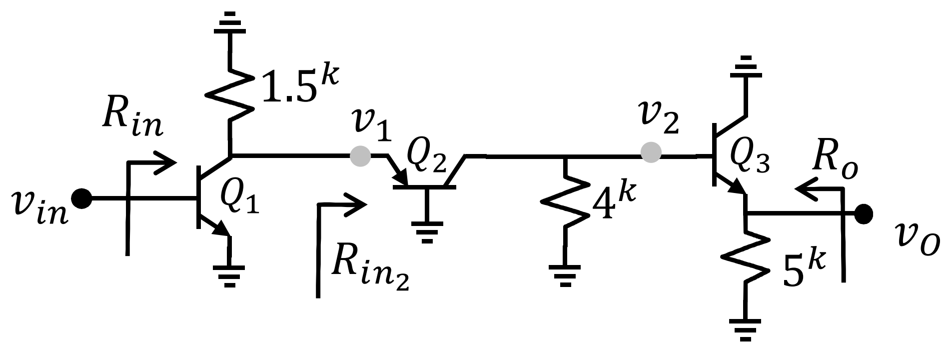


$$A_v = \frac{v_o}{v_s} = \frac{R_C}{\frac{R_B + r_\pi}{\beta + 1} + R_s}$$

$$r_e = r_m \parallel r_\pi = r_m \frac{\beta}{\beta + 1} \cong r_m$$

Example 05 – CE , CB, CC

1. Lab
2. Power
3. Ser/Parl
4. Small sig.
5. Applic



$\approx r_m$

$$\frac{v_1}{v_s} = -g_m\left(1.5^k \parallel R_{in_2}\right) = -1$$

$$\frac{v_2}{v_1} = +g_m\left(4^k \parallel (r_{\pi_2} + \beta 5^k)\right) = 160$$

$$\frac{v_o}{v_2} = \frac{5^k}{5^k + \frac{r_{\pi_2}}{\beta}} \cong 1$$

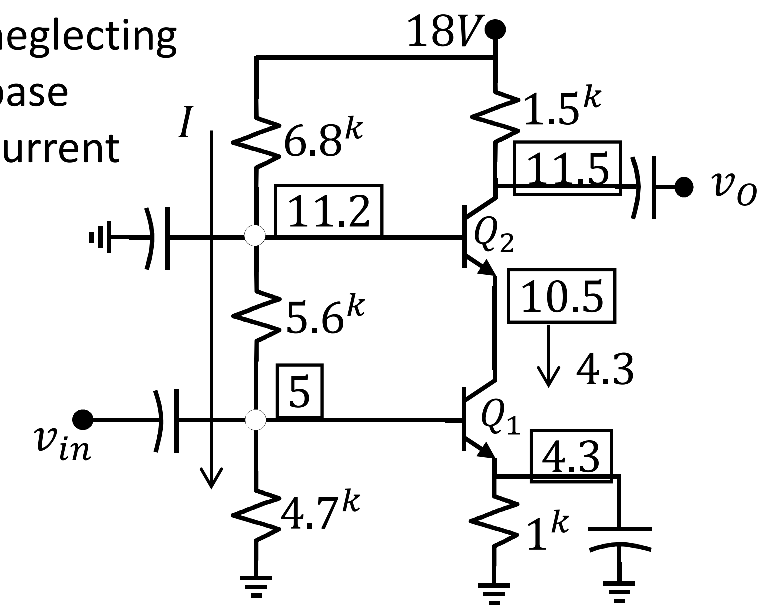
$$A_v = \frac{v_o}{v_s} = \frac{v_1}{v_s} \cdot \frac{v_2}{v_1} \cdot \frac{v_o}{v_2} = -160$$

$$R_{in} = r_{\pi_1} = 2.5^k$$

$$R_o = 4^k \parallel \frac{4^k + r_{\pi_2}}{\beta + 1} = 63\Omega$$

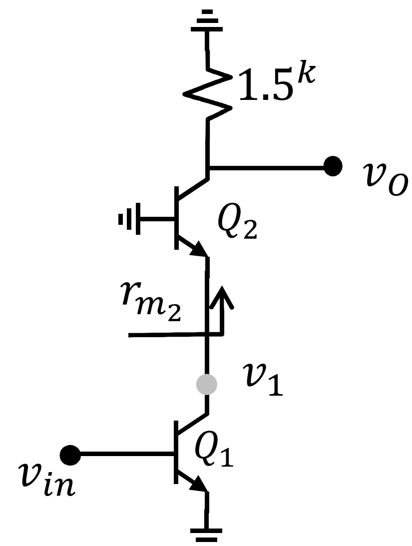
Cascode Amplifier , CE-CB

neglecting base current



$$I = \frac{18}{6.8 + 5.6 + 4.7} = 1.1^{mA}$$

AC circuit:

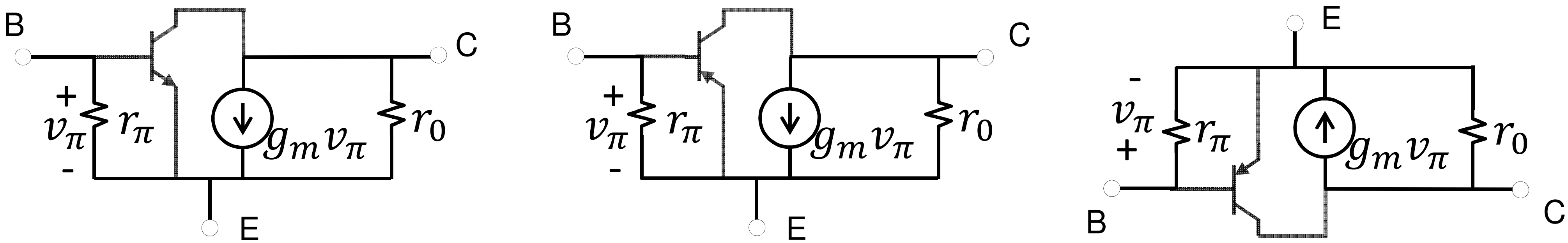


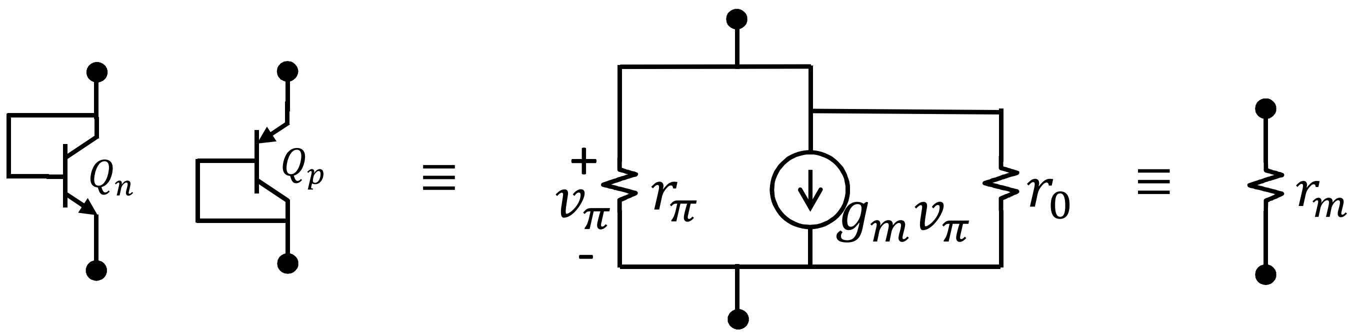
$$\frac{v_1}{v_{in}} = -g_{m_1}(r_{m_2}) = -1$$

$$\frac{v_o}{v_1} = \frac{1.5^k}{\frac{r_{\pi_2}}{\beta}} = 245$$

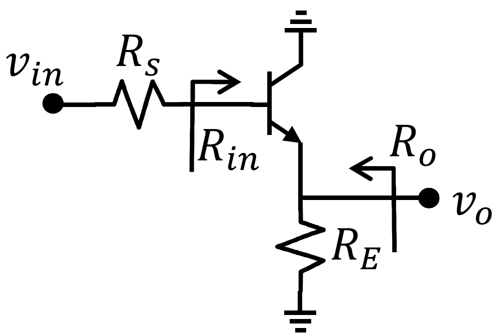
$$A_v = -245$$

Some Notes:


B
C
+
v_π
-
r_π
$g_m v_\pi$
r_0
E
B
C
+
v_π
-
r_π
$g_m v_\pi$
r_0
E
E
-
v_π
+
r_π
$g_m v_\pi$
r_0
B
C


Q_n
Q_p
≡
+
v_π
-
r_π
$g_m v_\pi$
r_0
≡
r_m

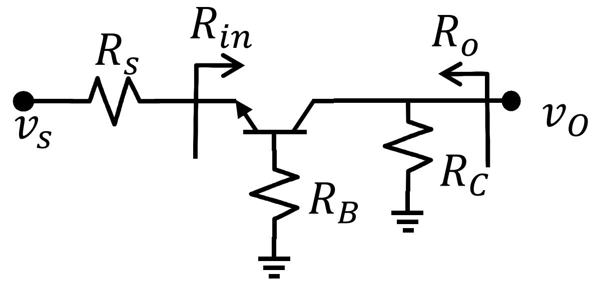
Summary



$$\frac{v_o}{v_{in}} = \frac{R_E}{\frac{R_S + r_\pi}{1+\beta} + R_E}$$

$$R_{in} = r_\pi + R_E(1+\beta)$$

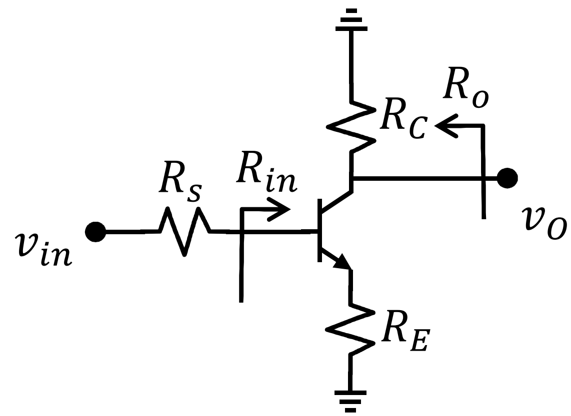
$$R_o = R_E \parallel \frac{R_S + r_\pi}{1+\beta}$$



$$\frac{v_o}{v_s} = \frac{R_C}{\frac{R_B + r_\pi}{\beta+1} + R_s}$$

$$R_{in} = \frac{R_B + r_\pi}{\beta + 1}$$

$$R_o = R_c$$

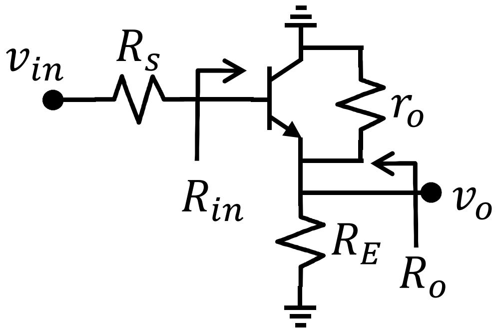


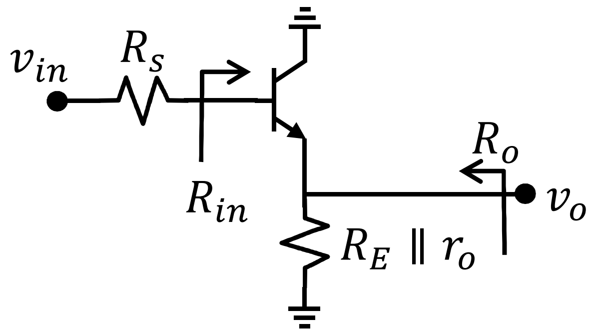
$$\frac{v_o}{v_s} = \frac{-R_C}{\frac{R_s + r_\pi}{\beta+1} + R_E}$$

$$R_{in} = r_\pi + R_E(1+\beta)$$

$$R_o = R_C$$

? V_A





$r_o = \infty$

$$\frac{v_o}{v_{in}} = \frac{R_E}{\frac{R_S + r_\pi}{1+\beta} + R_E}$$

$$\frac{v_o}{v_{in}} = \frac{R_E \parallel r_o}{\frac{R_S + r_\pi}{1+\beta} + R_E \parallel r_o}$$

$$R_{in} = r_\pi + R_E(1+\beta)$$

$$R_{in} = r_\pi + (R_E \parallel r_o)(1+\beta)$$

$$R_o = R_E \parallel \frac{R_S + r_\pi}{1+\beta}$$

$$R_o = R_E \parallel r_o \parallel \frac{R_S + r_\pi}{1+\beta}$$

Input / Output Impedances