٣٤

$$I_{avg} = \frac{1}{2\pi}\int_0^{\pi} \sin(t)\,dt = \frac{1}{2\pi}\Big[-\cos t\Big]_0^{\pi} = \frac{1}{\pi}$$

$$I_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} \sin^2(t)\,dt} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} \frac{1-\cos 2t}{2}\,dt} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

(۳٤)

$$Z_{in} = jL\omega + 1 + \frac{1 \times \frac{1}{j\omega C}}{1 + \frac{1}{j\omega C}} = jL\omega + 1 + \frac{1}{1 + j\omega C}$$

$$= 1 + jL\omega + \frac{1 - j\omega C}{1 + \omega^2 C^2}$$

$$\text{Im}\{Z_{in}\} = L\omega - \frac{\omega C}{1 + \omega^2 C^2} = 0$$

$$\longrightarrow L = \frac{C}{1 + \omega^2 C^2}$$

$$\omega^2 L C^2 = C - L \rightarrow \omega^2 = \frac{C - L}{LC^2}$$

$$\omega = \frac{1}{C}\sqrt{\frac{C}{L} - 1}$$

برای $\omega = 100$

$$\frac{C}{L} - 1 = 1 \longrightarrow \frac{C}{L} = 2$$

$$\Rightarrow C = 0.01 \,, \quad L = 0.005$$

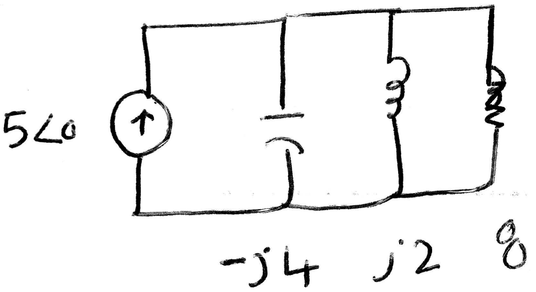
جواب منحصر به فرد نیست

50

فرکانس ها متفاوت هستند

$\omega = 2000$

اصل جمع آثار ①



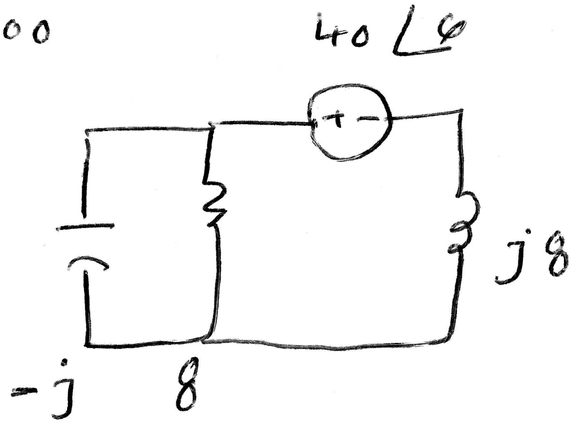
$$V = \frac{1 \times 5\angle 0}{\frac{1}{8} + \frac{1}{j2} + \frac{1}{-j4}} = \frac{5\angle 0}{0.125 - j0.25}$$

$$V = \frac{40}{1-j2} \rightarrow |V| = \frac{40}{\sqrt{5}}$$

$$P = \frac{1}{2}\frac{|V|^2}{8} = \frac{1}{2}\left(\frac{40}{\sqrt{5}}\right)^2 \times \frac{1}{8} = 20\ W$$

اصل جمع آثار ②

$\omega = 8000$



$$V = \frac{8 \| (-j)}{8 \| (-j) + j8} \times 40\angle 0$$

$$= \frac{\frac{-j8}{8-j}}{\frac{-j8}{8-j} + j8} \times 40\angle 0$$

$$= \frac{-j8}{8-j56} \times 40\angle 0$$

$$= \frac{-j}{1-j7} \times 40$$

$$\Rightarrow |V| = \frac{40}{\sqrt{50}}$$

$$P = \frac{1}{2}\frac{|V|^2}{8} = \frac{1}{2}\left(\frac{40}{\sqrt{50}}\right)^2 \times \frac{1}{8}$$

$$= 2W$$

$$\Longrightarrow P = 20 + 2 = 22W$$

۵۲

$$S_A = 10000(0.6+j0.8)$$
$$S_B = 5000(0.8-j6) \longrightarrow S_{AB} = 10000+j5000$$

$$|I_Z| = \frac{|S_{AB}|}{100} = \frac{5000\sqrt{5}}{100} = 50\sqrt{5}$$

$$\rightarrow S_Z = |I_Z|^2(0.1+j0.1) = 1250+j1250$$

$$\rightarrow S_T = S_Z + S_{AB} = 11250+j6250 = 1250(9+j5)$$

$$|V_S| = \frac{|S_T|}{|I_Z|} = \frac{1250\sqrt{9^2+5^2}}{50\sqrt{5}} = 25\sqrt{21.2} = 115.11\ V_{rms}$$

$$Z_{AB} = \frac{S_A+S_B}{|I_Z|^2} = \frac{10000+j5000}{12500}$$

راه دقیق: لازم است که مؤلفه موهومی امپدانس دیده شده توسط منبع ولتاژ صفر باشد

$$Z_T = 0.1+j0.1+\frac{10000+j5000}{12500} \parallel (-jx)$$

$$= 0.1+j0.1+(0.8+j0.4)\parallel(-jx)$$

$$= 0.1+j0.1+\frac{0.4x-j0.8x}{0.8+j(0.4-x)} = \frac{0.5x+0.04+j(-0.9x+0.12)}{0.8+j(0.4-x)}$$

برای حقیقی خالص شدن:

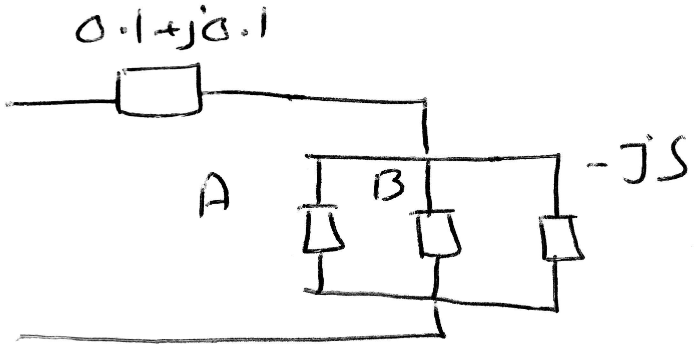
$$\frac{0.5x+0.04}{0.8}=\frac{-0.9x+0.12}{0.4-x} \longrightarrow x=1.6638$$

$$x=0.09616$$

امپدانس بزرگتر $-j1.6638$ را انتخاب می‌کنیم.

راه تقریبی (۱) فرض می‌کنیم با قرار دادن بار با توان مختلط $-jS$ ، ولتاژ

$100\,V_{rms}$ عوض نمی‌شود.



$$S_T=10000+j(5000-S)+(0.1+j0.1)\frac{10000^2+(5000-S)^2}{(100)^2}$$

$$5000-S+0.1\,\frac{10000^2+(5000-S)^2}{(100)^2}=\emptyset$$

$$\longrightarrow S=103989.79 \text{ و } 6010.21$$

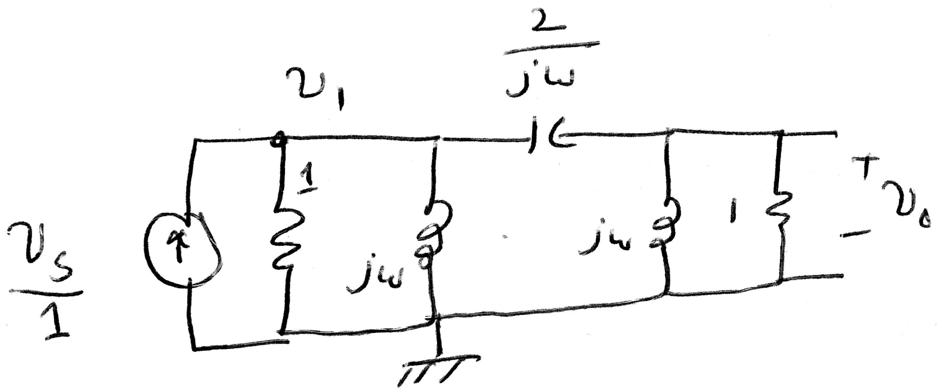
$$-j\,6010.21=\frac{100^2}{(-jX)^*}$$ توان موهومی کمتر را انتخاب می‌کنیم

$$\longrightarrow X=-j1.6638$$

راه تقریبی (۲) وجود بار خازنی نه ولتاژ در سر A و B و نه جریان آنها را عوض نکند

$$-j\,6250=\frac{100^2}{(-jX)^*} \longrightarrow X=-j1.6$$

ب - ۷۱



$$\begin{cases} \frac{v_1}{1} + \frac{v_1}{j\omega} + \frac{v_1 - v_o}{2/j\omega} = v_s \\ \frac{v_o}{1} + \frac{v_o}{j\omega} + \frac{v_o - v_1}{2/j\omega} = 0 \end{cases} \Rightarrow \begin{cases} (1 + \frac{1}{j\omega} + \frac{j\omega}{2}) v_1 - \frac{j\omega}{2} v_o = v_s \\ -\frac{j\omega}{2} v_1 + (1 + \frac{1}{j\omega} + \frac{j\omega}{2}) v_o = 0 \end{cases}$$

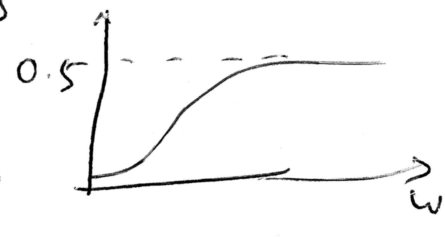
$$H(j\omega) = \frac{(j\omega)^3}{2(1 - 2\omega^2 + j\omega(2 - \omega^2))}$$

$$\rightarrow |H(j\omega)| = \frac{\omega^3}{2\left[(1-2\omega^2)^2 + \omega^2(2-\omega^2)^2\right]^{0.5}}$$

$$H(j\omega)\Big|_{\omega \approx 0} \approx 0 \qquad H(j\omega)\Big|_{\omega \approx \infty} \approx 0.5$$

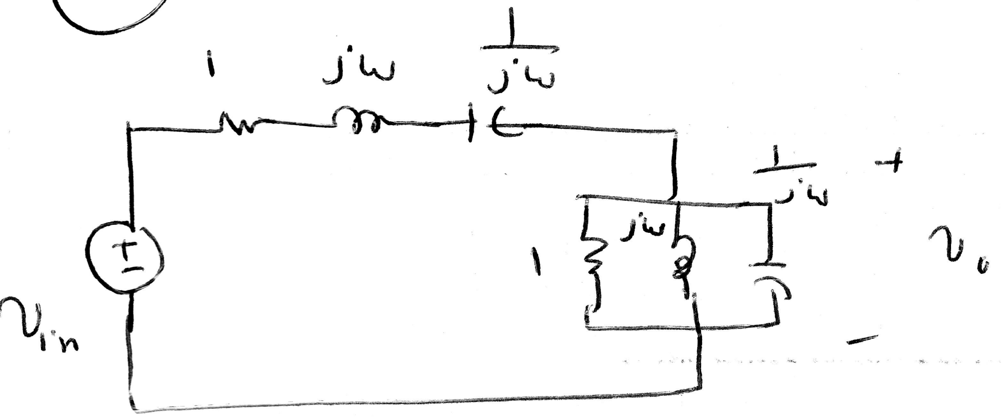
با رسم دقیق مشخص می‌شود که بالاگذر است (با حل معادله (دستور roots در Matlab)

$$|H(j\omega_o)| = \frac{1}{\sqrt{2}} \times \frac{1}{2} \longrightarrow \omega_o = 1 \ rad/s$$



توجه: با کاهش فرکانس خازن رابطه ورودی و خروجی را قطع می‌کند

۸۳



$$\frac{v_0}{v_{in}} = \frac{\dfrac{1}{1+j\omega+\dfrac{1}{j\omega}}}{1+j\omega+\dfrac{1}{j\omega}+\dfrac{1}{1+j\omega+\dfrac{1}{j\omega}}} =$$

$$= \frac{-\omega^2}{(1-4\omega^2+\omega^4)+2j\omega(1-\omega^2)}$$

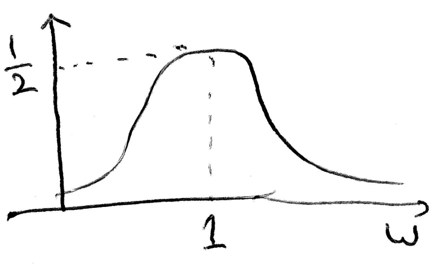
$$|H(j\omega)| = \frac{\omega^2}{\left[(1-4\omega^2+\omega^4)^2+4\omega^2(1-\omega^2)^2\right]^{0.5}}$$

$$H(j\omega)\big|_{\omega\approx 0} = 0 \quad , \quad H(j\omega)\big|_{\omega\approx\infty} = 0$$

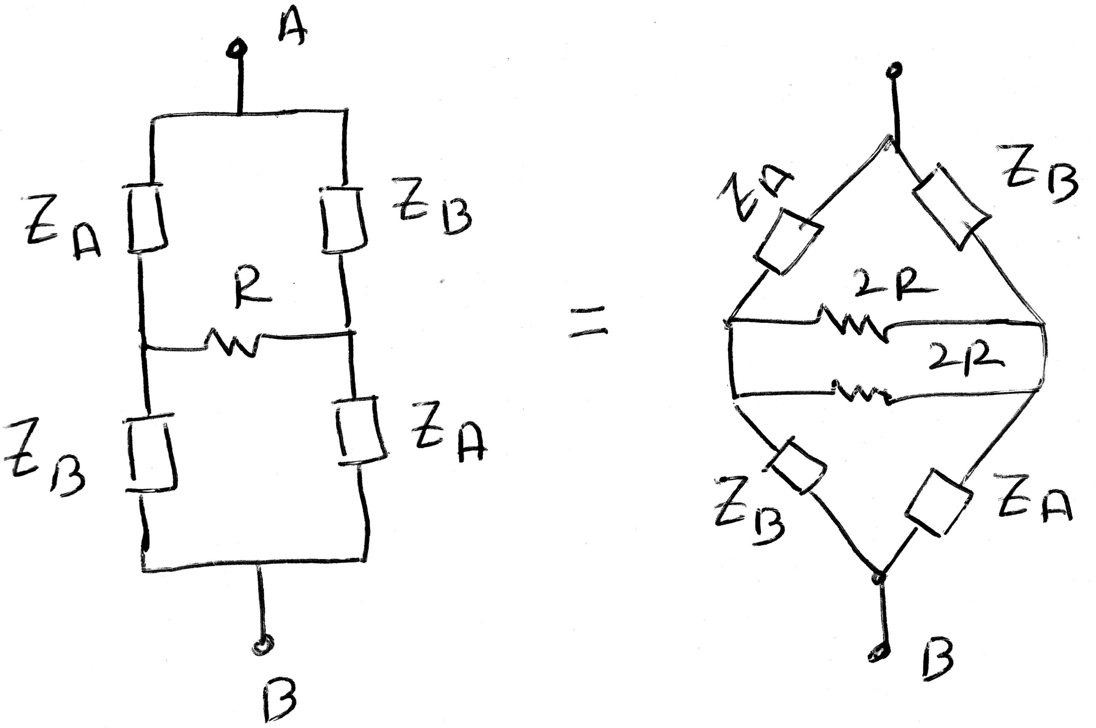
با رسم مشخص می‌شود حداکثر دامنه در ω=1 است (از روی مدار رفتار در این فرکانس را تحلیل کنید) - فیلتر میان گذر است.

با دستور roots در Matlab

$$\omega_{-3dB} = \sqrt{2 \pm \sqrt{3}}$$

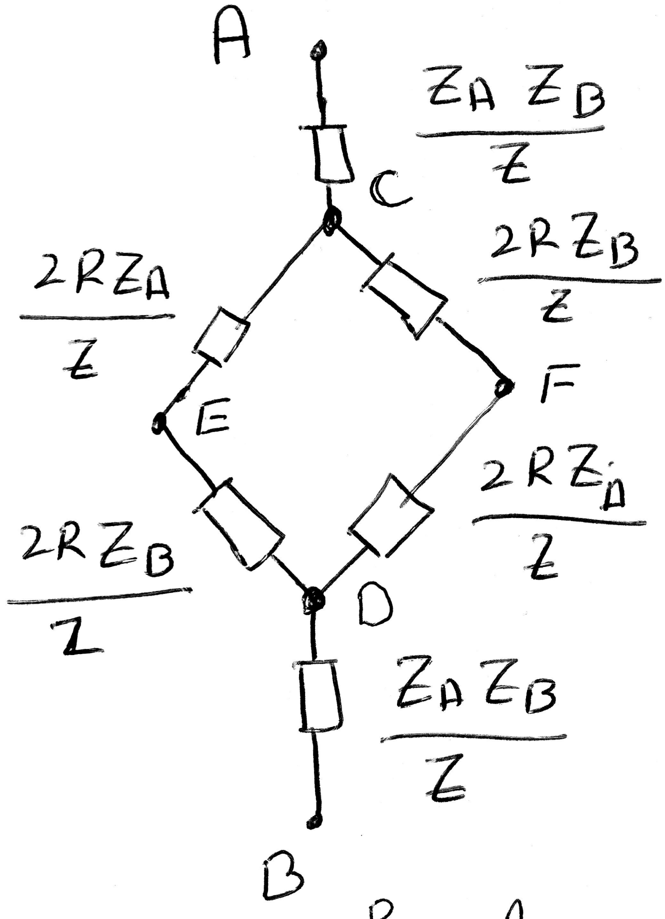


Z_{in} مقاومت (امپدانس) دو گره A ، B است



تبدیل مثلث به ستاره

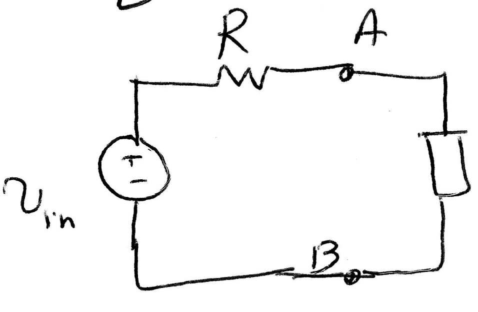
$$Z_A+Z_B+2R=Z$$



$$Z_T=\frac{2Z_A Z_B}{Z}+\frac{R(Z_A+Z_B)}{Z}$$

$$=\frac{2R^2+R(Z_A+Z_B)}{Z_A+Z_B+2R}$$

$$=R$$



$Z_T=R \longrightarrow v_{AB}=\dfrac{v_{in}}{2}$

ادامه

(119)

$$v_{CD} = \frac{\dfrac{R(Z_A+Z_B)}{Z}}{\dfrac{R(Z_A+Z_B)}{Z} + \dfrac{R^2}{Z} + \dfrac{R^2}{Z}} \times v_{AB}$$

$$v_{CD} = \frac{Z_A+Z_B}{Z} \times \frac{v_{in}}{2}$$

$$v_{EF} = v_{CD} \times \left[\frac{2RZ_B/Z}{2RZ_B/Z + 2RZ_A/Z} - \frac{2RZ_A/Z}{2RZ_A/Z + \dfrac{2RZ_B}{Z}} \right]$$

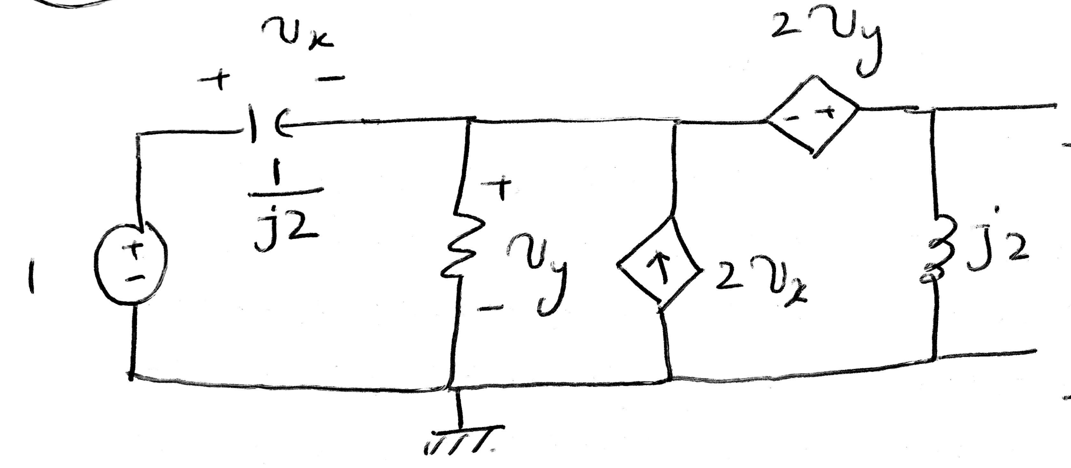
$$v_{EF} = \frac{Z_A+Z_B}{Z} \left[\frac{Z_B}{Z_B+Z_A} - \frac{Z_A}{Z_B+Z_A} \right] \frac{v_{in}}{2}$$

$$v_{EF} = \frac{Z_B - Z_A}{Z_A + Z_B + 2R} \times \frac{v_{in}}{2}$$

$$v_{EF} = \frac{\dfrac{R^2}{Z_A} - Z_A}{Z_A + \dfrac{R^2}{Z_A} + 2R} \times \frac{v_{in}}{2} = \frac{R^2 - Z_A^2}{Z_A^2 + R^2 + 2RZ_A} \times \frac{v_{in}}{2}$$

$$v_{EF} = v_0 = \frac{R - Z_A}{R + Z_A} \times \frac{1}{2} v_{in} \rightarrow \frac{v_o}{v_{in}} = \frac{1}{2}\,\frac{R - Z_A}{R + Z_A}$$

130



ابتدا v_{oc}

$v_x = 1 - v_y =$

$v_{oc} = 3v_y$

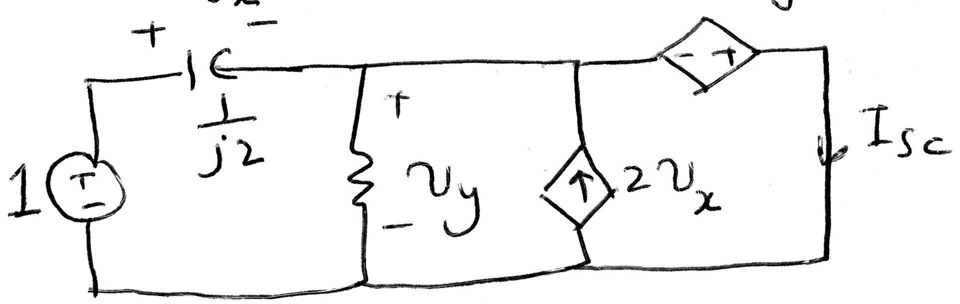
$$\frac{v_y - 1}{\frac{1}{j2}} + \frac{v_y}{1} - 2v_x + \frac{2v_y + v_y}{j2} = \emptyset$$

$$j2(v_y - 1) + v_y - 2(1 - v_y) + \frac{3v_y}{j2} = \emptyset$$

$$\left(j2 + 1 + 2 + \frac{3}{j2}\right)v_y = 2 + j2 \Rightarrow v_y = \frac{2 + j2}{3 + j0.5}$$

$$v_{oc} = \frac{6 + j6}{3 + j0.5} = 12\,\frac{j + 1}{6 + j} = 12\,\frac{\sqrt{2}}{\sqrt{37}}\,e^{j\varphi}$$

$$\varphi = tg^{-1}\frac{1}{1} - tg^{-1}\frac{1}{6}$$



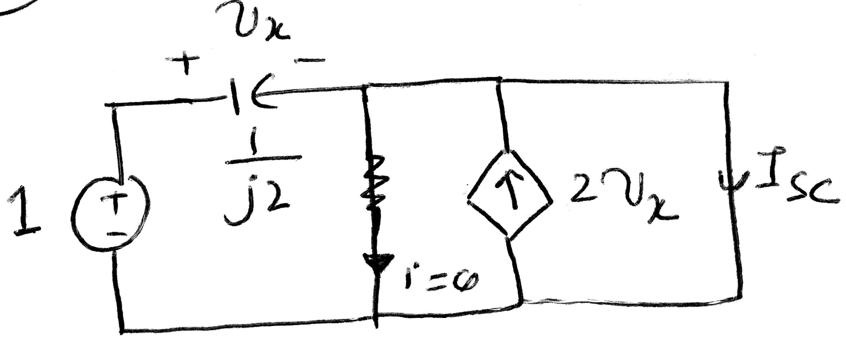
حل جریان I_{sc}

$v_y = -2v_y \rightarrow$

$v_y = \emptyset$

(130)

$v_y = 0$

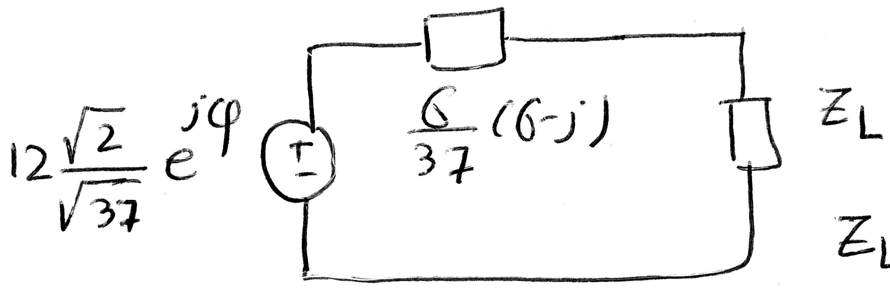


$v_x = 1 \longrightarrow I_{sc} = \frac{1}{1/j2} + 2v_x = 2 + j2$

$v_{oc} = 12\,\frac{j+1}{6+j}$

$\longrightarrow Z_{th} = \frac{6}{6+j} = \frac{6}{37}(6-j)$

$I_{sc} = 2(j+1)$



$Z_L = R_L + jX_L$

$X_L = -X_S \,,\; R_L = \sqrt{R_S^2 + (X_L + X_S)^2}$

① $X_L = \frac{6}{37} \,,\; R_L = \frac{36}{37}$ (الف)

② $X_L = 0 \,,\; R_L = \sqrt{\left(\frac{36}{37}\right)^2 + \left(\frac{6}{37}\right)^2}$ (ب)

③ $P = |V_{th}|^2 \times \frac{1}{2}\,\frac{R_L}{(R_L + R_S)^2 + (X_S + 0.5R_L)^2}$

$$\frac{\partial P}{\partial R_L} = (-)\,\frac{(R_L+R_S)^2 + (X_S+0.5R_L)^2 - 2R_L(R_L+R_S) - R_L(X_S+0.5R_L)}{(\rule{2cm}{0.4pt})^2}$$

$$(R_L+R_S)(R_S-R_L) + (X_S+0.5R_L)(X_S-0.5R_L) = 0$$

$$R_S^2 - R_L^2 + X_S^2 - 0.25R_L^2 = 0$$

$$R_L^2 = \frac{R_S^2 + X_S^2}{1.25} \rightarrow R_L = \frac{1}{\sqrt{1.25}}\sqrt{R_S^2 + X_S^2}$$

$$R_L = \frac{1}{\sqrt{1.25}} \times \frac{6}{37}\sqrt{6^2+1^2} = \frac{6}{\sqrt{37 \times 1.25}}$$